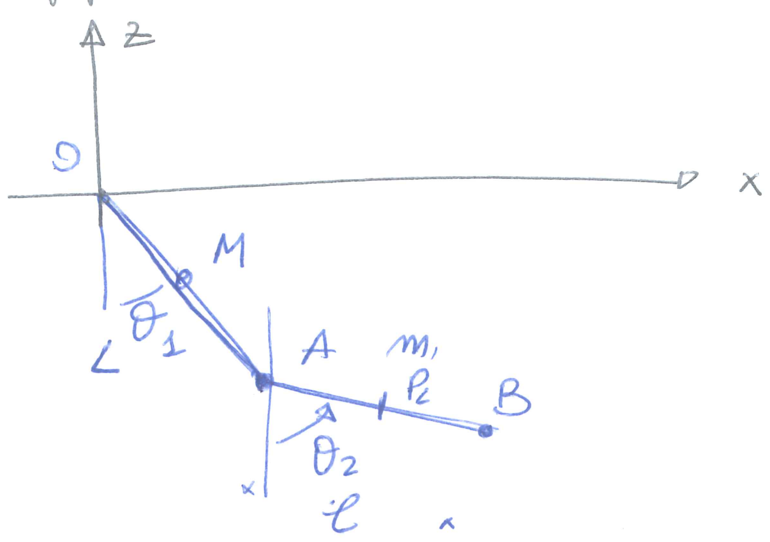
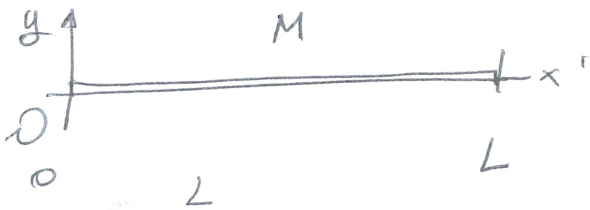
Doppio Pendolo



$$T_{OA} = \frac{1}{2} I_O^{yy} \omega^2 \quad \text{(ruota attorno a 1 pt. fisso)}$$

$$I_O^{yy} = \frac{1}{3} M L^2 \Rightarrow$$



$$I_O^{yy} = \frac{M}{L} \int_0^L (x')^2 dx' = \frac{ML^2}{3}$$

$$\omega^2 = \dot{\theta}_1^2$$

$$T_{AB} = \frac{1}{2} m v_{P_2}^2 + \frac{I_{P_2}^{yy}}{2} \omega_2^2$$

$$(P_2 - O) = (P_2 - A) + (A - O) =$$

$$(A - O) = L(\sin\theta_1 \hat{\imath} - \cos\theta_1 \hat{k})$$

$$(P_2 - A) = \frac{\ell}{2} (\sin\theta_2 \hat{\imath} - \cos\theta_2 \hat{k})$$

$$(P_2 - O) = \hat{\imath}\left(L \sin\theta_1 + \frac{\ell}{2} \sin\theta_2\right) +$$

$$- \hat{k}\left(L \cos\theta_1 + \frac{\ell}{2} \cos\theta_2\right)$$

$$v_{P_2} = \hat{\imath}\left(L\dot{\theta}_1\cos\theta_1 + \frac{l}{2}\dot{\theta}_2\cos\theta_2\right) + \hat{k}\left(L\dot{\theta}_1\sin\theta_1 + \frac{l}{2}\dot{\theta}_2\sin\theta_2\right)$$

$$v_{P_2}^2 = L^2\dot{\theta}_1^2 + \frac{l^2}{4}\dot{\theta}_2^2 + L l\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)$$

$\downarrow$

dov $\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \cos(\theta_1-\theta_2)$

$$I_{P_2}^{yy} = \frac{1}{12}ml^2 \longrightarrow \frac{m}{l}\int_{-l/2}^{l/2}(x')^2\,dx'$$

$$\omega_2^2 = \dot{\theta}_2^2$$

$$T_{AB} = \frac{1}{2}m\left(L^2\dot{\theta}_1^2 + \frac{l^2}{3}\dot{\theta}_2^2 + L l\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)\right)$$

$$V = Mg\underbrace{\left(-\frac{L}{2}\cos\theta_1\right)}_{z_{P_1}} + mg\underbrace{\left(-L\cos\theta_1 - \frac{l}{2}\cos\theta_2\right)}_{z_{P_2}}$$

$$L = T - V$$

EDL (BoRb. 124)

$$\mathcal{L} = \frac{1}{2}\left(\frac{M}{3} + m\right) L^2 \dot{\vartheta}_1^2 + \frac{1}{6} m \ell^2 \dot{\vartheta}_2^2 + \frac{1}{2} m L \dot{\vartheta}_1 \dot{\vartheta}_2 \cdot \cos(\vartheta_1 - \vartheta_2)$$

$$+ \frac{1}{2} M g L \cos\vartheta_1 + mg\left(L \cos\vartheta_1 + \frac{1}{2} \ell \cos\vartheta_2\right)$$

Si ottengono le equazioni di evoluzione

Per $\alpha = 0$ le eq. doppio pendolo hanno la forma

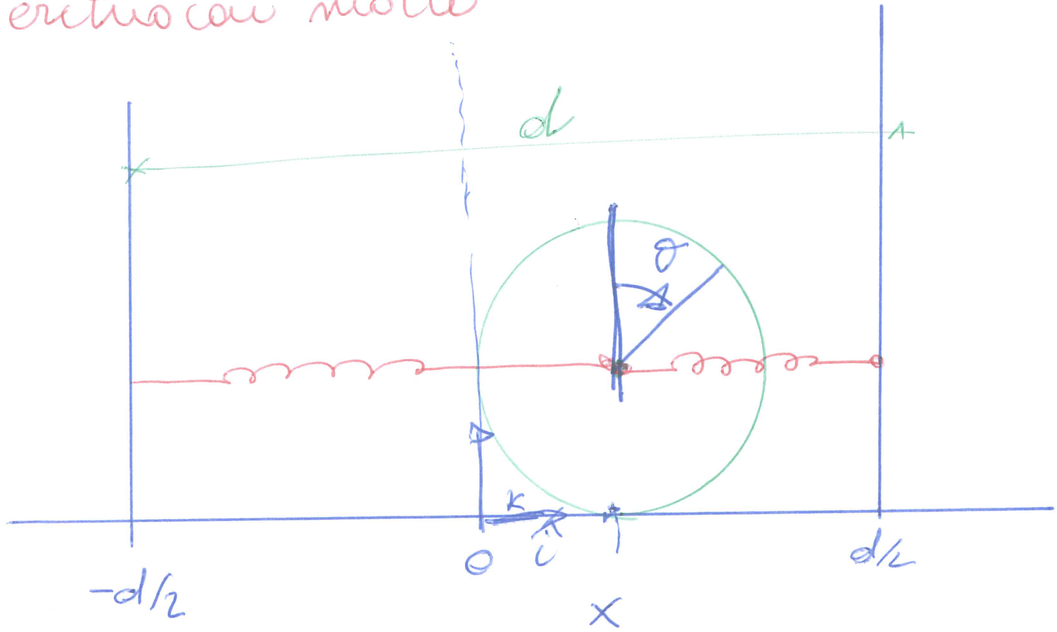
$$\begin{cases} A_1 \ddot{\theta}_1 + B_1 \cancel{\theta} \sin\theta_1 = 0 \\ A_2 \ddot{\theta}_1 + B_2 \sin\theta_1 = 0 \end{cases}$$

1 sol se le eq. sono dip.

$$\begin{cases} A_1 = \beta A_2 \\ B_1 = \beta B_2 \end{cases} \Rightarrow \quad \frac{A_1}{A_2} = \frac{B_1}{B_2} \Rightarrow A_1 B_2 = A_2 B_1$$

$$\det \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix} = 0$$

Cerchio con molle



Relazione di rotolamento puro

$$X = x_0 + R\vartheta$$

$\Downarrow$

conviene scegliere $x_0 = 0$, per $\vartheta = 0$ $x = 0$

il cerchio si trova al centro

$$T = \frac{1}{2} m v_{P_0}^2 + \frac{1}{2} I_{P_0}^{yy} \omega^2$$

$I_{P_0}^{yy} = mR^2$ $\qquad \omega^2 = \dot{\vartheta}^2$

$(P_0 - O) = \frac{R}{2}\hat{k} + x\hat{i} = \frac{R}{2}\hat{k} + R\vartheta\hat{i}$

$v_{P_0} = R\dot{\vartheta}\hat{i}$

$$T = \frac{1}{2} m R^2 \dot{\vartheta}^2 + \frac{m}{2} R^2 \dot{\vartheta}^2 = m R^2 \dot{\vartheta}^2$$

$$U = \frac{c}{2}\left(\frac{d}{2} + x\right)^2 + \frac{c}{2}\left(\frac{d}{2} - x\right)^2 = c\left(\frac{d^2}{2} + x^2\right)$$

$$L = m R^2 \dot{\theta}^2 - c\left(\frac{\theta^2}{2} + \theta^2 R^2\right)$$

Eq. evol.

$$2 m R^2 \ddot{\theta} = - c\, \theta R^2 2$$

$$\ddot{\theta} = - \frac{c}{m} \theta$$