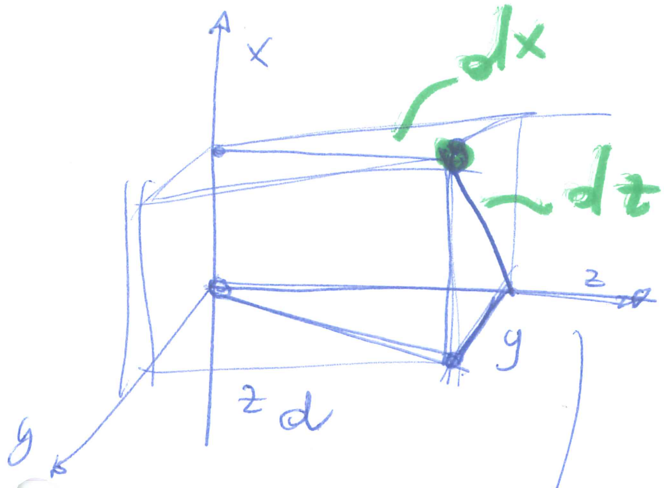
Momento d'inerzia

$$I_{xx} = \sum_i m_i (y_i^2 + z_i^2)$$



$d^2 = y_i^2 + z_i^2$

$\downarrow$

quad della dist. dei mi P_i

da asse x

$$I_{zz} = \Sigma m_i (x_i^2 + y_i^2)$$

limite al continuo

$$I_{zz} = \int_R \rho (x^2 + y^2) \, dV$$

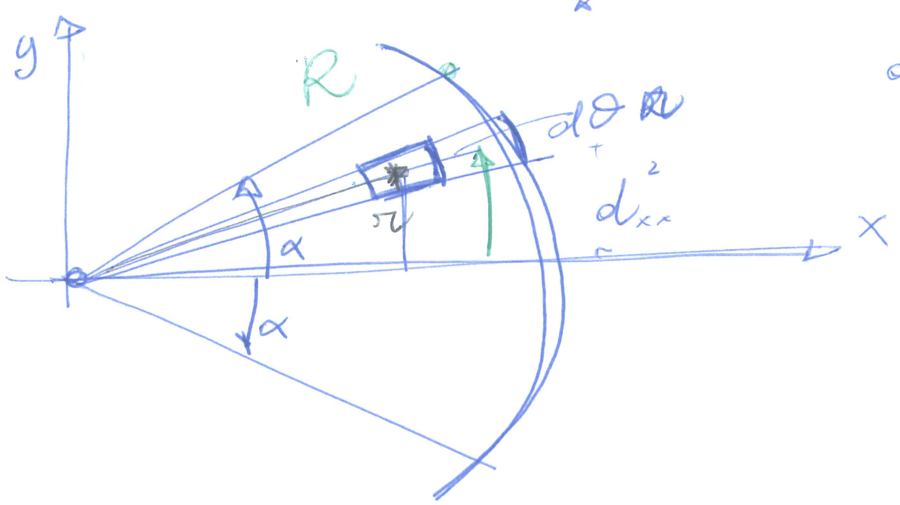
$$I_{yy} = \int_R \rho \cdot (x^2 + z^2) \, dV$$

Momento centrifughi

$$I_{xy} = \Sigma m_i x_i y_i \rightarrow \int_R \rho \, x y \, dV$$

Momento di inerzia di una lastra: settore circolare

Piano $z=0$



$dA = r\,d\theta\,dr$

$$\text{Area} = R^2\alpha \qquad \rho = \frac{m}{A} = \frac{m}{R^2\alpha}$$

Calcolo I_{xx}

$$I_{xx} = \rho \int_V (y^2 + \underbrace{z^2}_{z=0})\,dA =$$

$$= \frac{m}{R^2\alpha}\int_0^R \int_{-\alpha}^{\alpha} r(r\sin\theta)^2\,dr\,d\theta =$$

$$= \frac{m}{R^2\alpha}\int_0^R r^3 \int_{-\alpha}^{\alpha} \sin^2\theta\,d\theta$$

Ricorda $\displaystyle\int \sin^2\theta = \frac{\theta}{2} - \frac{\cos\theta\sin\theta}{2}$

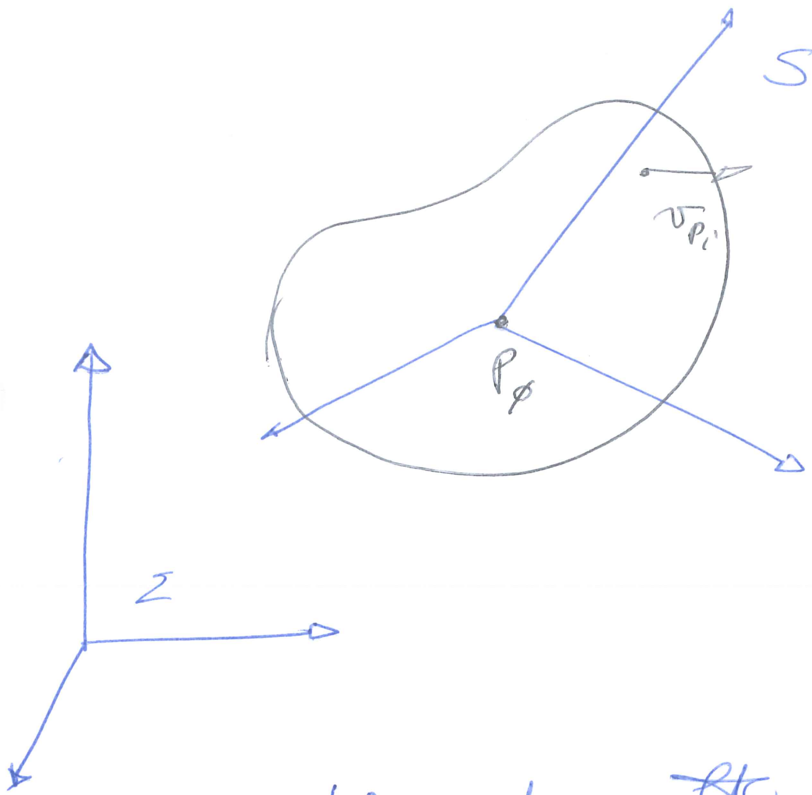
$$I_{xx} = \frac{m}{R^2 \alpha} \frac{R^4}{4} \left(\frac{\theta}{2} - \frac{\cos\theta \operatorname{sen}\theta}{2} \Big|_{-\alpha}^{\alpha} \right) =$$

$$= \frac{m}{\alpha} \frac{R^2}{4} \left(\frac{2\alpha}{2} - \frac{\cos\alpha \operatorname{sen}\alpha}{2} + \frac{\cos\alpha \operatorname{sen}(-\alpha)}{2} \right)$$

$$= \frac{m R^2}{\alpha \cancel{4}} \left(\alpha - \cos\alpha \operatorname{sen}\alpha \right)$$

T. di König applicato alla dinamica dei rigidi

Richiami e considerazioni



Il moto del rigido risulta semplificato se scritto rispetto al c.d.m (visto da un osserv. S solidale con origine nel c.d.m.)

Si divide lo studio del moto in 2 parti

- Moto del centro di massa
- Moto relativo al c.d.m

Alcune considerazioni

Dalla def di c.d.m. *

$$m\,(P_\phi - O) = \sum m_i (P_i - O) \Rightarrow m \cdot P_o = \sum m_i P_i$$

$$\Downarrow \; d/dt \,, \quad m = \sum m_i$$

$$\boxed{\sum_{\cancel{mi}} m_i \, \vec{v}_{P_o}\Big|_{\Sigma} = \sum_i m_i \, v_{P_i}\Big|_{\Sigma}} \quad (1)$$

Formula del moto di rigido risp. a S

$$\vec{v}(P_i)\Big|_{S} = v(P_o)\Big|_{\Sigma} + \cancel{v(P_i)\Big|_{S}}^{0} + \vec{\omega} \wedge (P_i - P_\phi) \quad (2)$$

$$\vec{v}(P_i) - \vec{v}(P_\phi) = \vec{\omega} \wedge (P_i - P_\phi)$$

Ricordo la def. di mom. angolare della q. di moto

$$K(O) = \sum (P_i - O) \wedge m_i v_i \Big|_{\Sigma} \quad ; \quad \vec{K}(P_\phi) = \underline{\underline{\sigma}}(P_\phi)\,\vec{\omega}$$

calcolo risp. a P_ϕ e uso (2)

$$\vec{K}(P_\phi) = \underbrace{\sum_i (P_i - P_\phi) \wedge v_{P_o} m_i}_{\displaystyle \sum_i (m_i P_i - m_i P_\phi) \wedge v_{P_o} = 0^{\,*}} + \sum (P_i - P_o)\, m_i \wedge [\vec{\omega} \wedge (P_i - P_\phi)]$$

Quindi

$$\boxed{\vec{K}(P_\emptyset) = \sum_i m_i (P_i - P_\emptyset) \wedge [\vec{\omega} \wedge (P_i - P_\emptyset)]} \quad (2)$$

ENERGIA CINETICA

Punto $\quad T(P_i) = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i v^2(P_i)\Big|_\Sigma$

Rigido $\quad T = \sum_i \frac{1}{2} m_i v^2(P_i) =$

$$= \sum_i \frac{1}{2} m_i \left(v(P_i) - v(P_\emptyset) + v(P_\emptyset) \right)^2 =$$

$$= \sum_i \frac{1}{2} m_i \big(\underbrace{v_{P_i} - v_{P_0}}_{= \vec{\omega} \wedge (P_i - P_\emptyset)} \big)^2 + \sum \frac{1}{2} m_i v_{P_0}^2$$

$$+ \sum_i m_i \underbrace{(v_{P_i} - v_{P_\emptyset}) v_{P_\emptyset}}_{= 0 \; (1)}$$

Ottengo

$$T = \underbrace{\frac{1}{2} m v_{P_0}^2}_{\text{E. cinetica del c.d.m.}} + \overbrace{\underbrace{\sum_i \frac{m_i}{2} |\vec{\omega} \wedge (P_i - P_0)|^2}_{\text{E. cinetica del moto relativo al c.d.m.}}}^{T_R}$$

$$T_R = \frac{1}{2}\sum_i m_i \,|\vec{\omega} \wedge (P_i - P_\emptyset)|^2 =$$

$$= \frac{1}{2}\sum_i m_i \,\underbrace{(\vec{\omega} \wedge (P_i - P_\emptyset))}_{\vec{a}} \cdot (\vec{\omega} \wedge (P_i - P_\emptyset))$$

$$\vec{a} \cdot (\vec{b} \wedge \vec{c}) = \vec{b} \cdot (\vec{c} \wedge \vec{a})$$

$$= \frac{1}{2}\sum_i m_i \,\vec{\omega} \cdot \left\{ (P_i - P_\emptyset) \wedge \left[\vec{\omega} \wedge (P_i - P_\emptyset) \right] \right\}$$

$$\stackrel{2}{=} \frac{1}{2}\,\vec{\omega} \cdot \vec{K}(P_\emptyset) = \frac{1}{2}\,\vec{\omega} \cdot \underline{\underline{\sigma}}(P_0)\,\vec{\omega}$$

ENERGIA CINETICA DI UN RIGIDO

$$T = \frac{1}{2} m v_{P_\emptyset}^2 + \frac{1}{2}\,\vec{\omega} \cdot \underline{\underline{\sigma}}(P_0)\,\vec{\omega}$$