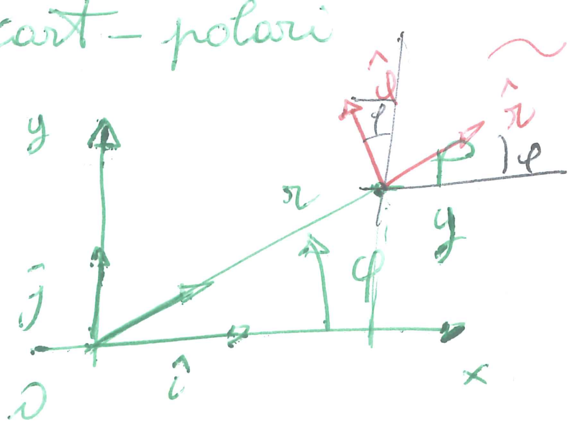
Cambio di coordinate piano

cart – polari ~ base locale



$P - O = x\hat{i} + y\hat{j}$ in coord cart

$$\hat{r} = \frac{(P-O)}{|P-O|}$$

$= \hat{r}\, r$

↳ distanza da O

$r = |P-O|$

$$\frac{(P-O)}{|P-O|}\,|P-O|$$

coord polari individ. P con r e φ

cambio di coordinate

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2} \\ \varphi = \operatorname{arctg} y/x \end{cases}$$

Dal un pt di vista matematico è una trasf. $\mathbb{R}^2 \to \mathbb{R}^2$

$$(x, y) \to (r, \varphi)$$

Per studiare le proprietà della trasforma (trasf. invertibile)

si utilizza lo Jacobiano

$$J \equiv \frac{\partial(x, y)}{\partial(r, \varphi)} = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{pmatrix} =$$

$$\overset{\text{polari}}{=} \begin{pmatrix} \cos\varphi & -r \operatorname{sen}\varphi \\ \operatorname{sen}\varphi & r \cos\varphi \end{pmatrix}$$

$$\det(J) = |J| = r \cos^2\varphi + r \sin^2\varphi = r$$

$$|J| \neq 0 \Rightarrow \text{trasf. invertibile}$$

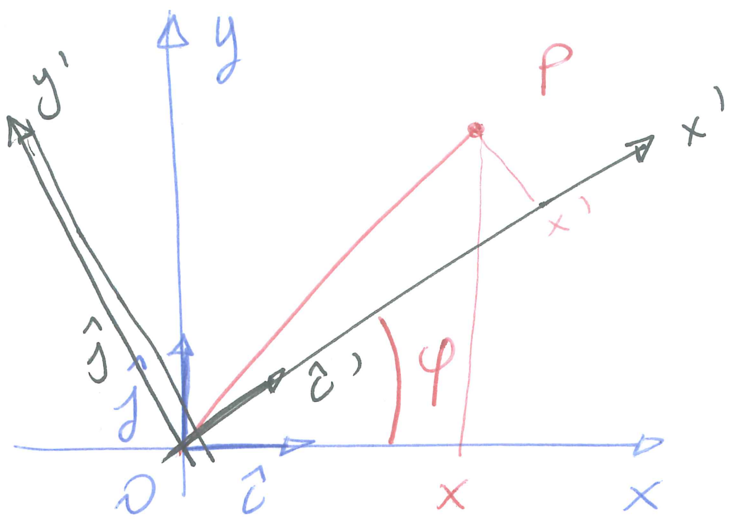
Conviene coordinate polari anche la base locale

$$\hat{r} = \hat{i}\cos\varphi + \hat{j}\sin\varphi$$

+ un versore ottenuto ruotando $\vec{r}$ in senso antiorario

$$\hat{\varphi} = \hat{j}\cos\varphi + \vec{i}\sin\varphi$$

cambio coord piano cart-cart



$$(P-O) = x\hat{\imath} + y\hat{\jmath} = x'\hat{\imath}' + y'\hat{\jmath}'$$

per procedere occorre esprimere una coppia di versori con l'altra

$$\hat{\imath}' = \hat{\imath}\cos\varphi + \hat{\jmath}\,\mathrm{sen}\,\varphi$$

$$\hat{\jmath}' = \hat{\jmath}\cos\varphi - \hat{\imath}\,\mathrm{sen}\,\varphi$$

esprimo (x, y) in funzione di (x', y')

$$(P-O)\cdot\hat{\imath} = x = x'\,\hat{\imath}\cdot\hat{\imath}' + y'\,\hat{\imath}\cdot\hat{\jmath}'$$

$$= x'\cos\varphi - y'\,\mathrm{sen}\,\varphi$$

$$y = y'\cos\varphi + x\,\mathrm{sen}\,\varphi$$

in forma matriciale $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\mathrm{sen}\,\varphi \\ \mathrm{sen}\,\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$