Due: Thursday February 16 at the end of class. A portion of the homework will be graded (by Sam Carp) and returned to you at the end of the next class. Remember to staple your homework and put your name on it.

1. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a smooth function and let a be a regular value for f. Show that $f^{-1}(a)$ is orientable.

2. Let $f : S_1 \to S_2$ be a diffeomorphism between two regular surfaces. Prove that S_1 is orientabile if and only if S_2 is orientable.

3. a) Determine the first fundamental form of a catenoid $S = \{x(u,v), u \in \mathbb{R}, v \in [0, 2\pi]\}$, where $x(u,v) = (\cosh(u)\cos(v), \cosh(u)\sin(v), u)$.

   b) For $t > 0$ let $S_t \subset S$ be the image, via x, of $(-t,t) \times [0, 2\pi]$. Find the image of the Gauss map $N(S_t)$ and describe the set $\{p \in S^2 : p \notin N(S_t), \forall t\}$.

   c) Find the Weingarten map of the catenoid.

4. Let S be a regular surface and fix a point $p_0 \in \mathbb{R}^3$ such that $p_0 \notin S$. Define the function $f : S \to \mathbb{R}$ by $f(p) = d(p, p_0)$ where $p \in S$ and d is the standard Euclidean distance in $\mathbb{R}^3$. Prove that f is a smooth function $S \to \mathbb{R}$ and prove that p is a critical point of f if and only if the line joining p to p_0 is orthogonal to S.

5. a) Let $S = \{(u, v, f(u,v)), u, v \in \mathbb{R}\}$ be the graph of a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$. Determine a formula for the Gauss curvature and the mean curvature of S.

   b) Compute the mean curvature and the Gauss curvature of the surfaces

   $$S_1 = \{u, v, u^2 + v^2, u, v \in \mathbb{R}\}, \qquad S_2 = \{u, v, u^2 - v^2, u, v \in \mathbb{R}\}$$

   c) Find eigenvectors and eigenvalues for the second fundamental forms of S_1 and S_2 at $(0,0,0)$.

6. a) Suppose that a regular orientable surface S and a plane π are tangent along the trace of a curve $\alpha(t)$ (i.e. π is the tangent plane to S at the points of $\alpha(t) \subset S \cap \pi$). Show that the Gauss curvature of S is zero at the points of $\alpha(t)$.

   b) Prove that there are points p on the torus T^2 such that $K(p) = 0$.