Due: Thursday February 2 at the end of class. A portion of the homework will be graded (by Sam Carp) and returned to you at the end of the next class. Remember to staple your homework and put your name on it.

1. For $v \in \mathbb{R}^2$, $v = (v_1, v_2)$ and define a new norm

$$||v||_3 = \sqrt[3]{|v_1|^3 + |v_2|^3}$$

   You can use this norm the determine the length of a vector and use it compute the length of a curve. Prove that the distance defined by this norm is not invariant with respect to the isometries of the Euclidean space. Determine the length of the line segment joining the origin $(0,0)$ and the point (x_0, y_0). Describe the set of points that have unit distance from the origin.

2. Shifrin Ex. 1, pg. 31.

3. a) Compute the curvature and the torsion of the curve $\alpha(t) = (t + \sqrt{3}\sin(t), 2\cos(t), \sqrt{3}t - \sin(t))$. Conclude that this is an helix.

   b) [Extra credit] find an explicit isometry that maps α into a curve β parametrized in the 'canonical' form the we have used (Example 1.14 in the notes).

4. Let $\alpha : I \to \mathbb{R}^3$ be a bi-regular curve parametrized by arclength. Consider a positive orthonormal frame $(\underline{t}(t), \underline{e}_1(t), \underline{e}_2(t))$ along α (here $\underline{t}, \underline{n}, \underline{b}$ is the standard Frenet frame) such that

$$\underline{e}_1'(t) = -k_1(t)\underline{t}(t), \qquad \underline{e}_2'(t) = -k_2(t)\underline{t}(t)$$

   for some pair of functions $k_1(t)$ and $k_2(t)$.

   a) Construct the frame (Hint e_1 and e_2 are orthogonal to $\underline{t}$ hence they form an orthonormal basis of the plane spanned by $\underline{n}$ and $\underline{b}$).

   b) Express k_1 and k_2 as functions of the curvature of α and the function $\theta(t)$ where θ is defined by

$$\underline{n}(t) = \cos\theta(t)\underline{e}_1(t) + \sin\theta(t)\underline{e}_2(t)$$

   c) Express the curvature and the torsion of α as functions of k_1 and k_2.

   d) Prove that $\underline{t}' = k_1\underline{e}_1 + k_2\underline{e}_2$.

   e) Prove that α is a plane curve if and only if k_1/k_2 is constant.

5. (Extra, only for fun...) The Figure in the next page reproduces the trace of a bike passed through a mud patch. Is the bike traveling from the left to the right or viceversa? (See also Shifrin Ex. 27 pg. 22)