**Due: Thursday January 26 at the end of class. A portion of the homework will be graded (by Sam Carp) and returned to you at the end of the next class. Remember to staple your homework and put your name on it.**

1. Let $\alpha : I \to \mathbb{R}^n$ be a $C^1$ curve in $\mathbb{R}^n$, where $I$ is a finite interval, and suppose that the curve does not pass through the origin. Let $t_0$ be such that $\alpha(t_0)$ is the point of the trace of $\alpha$ which is closest to the origin and assume $\alpha'(t_0) \neq 0$. Show that $\alpha(t_0)$ and $\alpha'(t_0)$ are orthogonal.

2. Let $\alpha : I \to \mathbb{R}^n$ be a $C^1$ curve in $\mathbb{R}^n$ and let $f : \mathbb{R}^n \to \mathbb{R}^n$ be an isometry. Let $\beta(t) = f(\alpha(t))$, show that $L(\beta) = L(\alpha)$.

3. Prove that the torsion of a biregular curve $\alpha$ is invariant under reparametrizations.

4. Let $\alpha(t)I \to \mathbb{R}^3$ be a biregular curve such that for every $t \in I$ the line through $\alpha(t)$ parallel to $\underline{n}(t)$ all pass through a single point $p$. Prove that the torsion of $\alpha$ is 0 and the curvature of $\alpha$ is constant.

5. Let $\alpha(t)I \to \mathbb{R}^3$ be a biregular curve and let
$$C(t) = \alpha(t) + \frac{1}{k(t)}\underline{n}$$
verify that if $\alpha$ is a circle then $C(t)$ is constant and coincides with the center of the circle. (to simplify the computations use exercise 2).

6. Let $\alpha(t)I \to \mathbb{R}^3$ be a biregular curve such that the trace of $\alpha$ lies on a sphere of radius $r$. Prove that the curvature of $\alpha$ satisfies $k(t) \geq \frac{1}{r}$. (parametrize by arclength and use $||\alpha(s) - c||^2 = r^2$, where $c$ is the center of the sphere).

7. Shifrin p. 18 Ex. 3.d

8. Shifrin p. 18 Ex. 4