NEW DIRECTIONS IN NONAUTONOMOUS DYNAMICAL SYSTEMS

In honor of Russell Johnson's 70th Birthday

Università di Firenze, January 31-February 3,2017.

—Abstracts—

Lectures

Flaviano Battelli, Università Politecnica delle Marche

Some remarks on implicit differential equations

We study the behavior of the solutions of a nonlinear differential equations such as

$$\omega(x)x' = F(x) + \varepsilon G(x,t,\varepsilon,\beta), \quad x \in \mathbb{R}^n,\ \beta \in \mathbb{R}^m,\ \varepsilon \in \mathbb{R}. \tag{1}$$

We suppose that the associated differential equations $\dot{x} = F(x)$ has a solution $\gamma(s)$ such that $\gamma(-\infty) = x_0$, $\gamma(1) = x_1$ where $\omega(x_0) = \omega(x_1) = 0$. We also suppose, besides other conditions, that $F(x_0) = 0$ and $G(x_0,t,\varepsilon,\beta) = 0$. We prove the existence of solutions $x(t)$ of equation (1) which are orbitally close to $\gamma(s)$ and such that $\lim_{t\to -T_*} x(t) = x_0$ (for some T_*). We also consider the case when $\gamma(s_0^*) = x_0$, $\gamma(s_1^*) = x_1$ for some $s_0^* < s_1^*$ in $\mathbb{R}$ and prove a similar result.

Finally we apply the results to the study of the equation modeling an RLC circuit

$$u + L(v)' + \varepsilon R(v) = \varepsilon e(t), \quad v = C(u)'$$

with small ohmic resistance and external force.

Join work with Michal Fečkan, Comenius University, Bratislava, Slovakia.

Tomás Caraballo, Universidad de Sevilla

Dynamics of 2D Navier-Stokes equations with delays

In this talk we will show several aspects related to the dynamics of 2D Navier-Stokes models when some hereditary characteristics (constant, distributed or variable delay, memory,etc.) appear in the formulation. First we report on some recent results concerning the local stability of steady-state solutions. This will be carried out by using several methods: the theory of Lyapunov functions, the Razumikhin-Lyapunov technique, by constructing appropriate Lyapunov functionals and funally using a method based in Gronwall-like inequalities. Then the global dynamics of solutions can be analyzed by using the theory of attractors. As the delay terms are allowed to be very general, the statement of the problem becomes nonautonomous in general. For this reason, the theory of nonautonomous pulback attractors appears to be appropriate for this end.

Fritz Colonius, Institut für Mathematik, Universität Ausburg, Germany
On the supports of invariant measures for Piecewise Deterministic Markov Processes

For a class of piecewise deterministic Markov processes, the support of the invariant measures are characterized. This is based on the analysis of controllability properties of an associated deterministic control system. Its invariant control sets determine the supports.

Sylvain Crovisier, Université Paris-Sud
About one-dimensional rotation sets of toral homeomorphisms

R. Johnson and M. Herman have shown that each quasi-periodic cocycles by 2×2 matrices rotate all vectors at the same frequency (called fibered rotation vector): the system induces a homeomorphism of the 2-torus which preserves a circle foliation. In this talk I will survey results and conjectures about the rotation set of toral homeomorphisms preserving a one-dimensional structure.

David Damanik, Rise University
Some surprises in the spectral theory of almost-periodic Schrödinger operators

We describe the general picture that has emerged in the study of spectral properties of almost-periodic Schrödinger operators, and then proceed to discuss some recent results that appear counter-intuitive at first sight. We also explain the mechanism behind these results.

Hakan Eliasson, Université Paris Diderot
Almost reducibility for the quasi-periodic linear wave equation

We shall discuss the almost reducibility of the linear wave equation on a torus (i.e. periodic boundary conditions) with a mass term and perturbed by a potential depending quasi-periodically on time. The potential is analytic and the quasi-periodic frequencies are supposed to be Diophantine.

Teresa Faria, Universidade de Lisboa
Positive periodic solutions for some classes of non-autonomous delay differential equations

For a family of n-dimensional periodic delay differential equations (DDEs) which encompasses a broad set of models used in structured population dynamics, the existence of a positive periodic solution is obtained under very mild conditions [1]. The proof uses the Schauder fixed point theorem and relies on the permanence of the system, proven in [2] for a larger families of DDEs not necessarily periodic. A general criterion for the existence of a positive periodic solution for Nicholson's blowflies periodic system (with both distributed and discrete time-varying delays) is derived as a simple application of our main result, generalizing the few existing results concerning multi-dimensional Nicholson models.

References

[1] T. Faria, *Periodic solutions for a non-monotone family of delayed differential equations with applications to Nicholson system,*submitted (2016).
[2] T. faria, R. Obaya, A.M. Sans, *Asymptotic behaviour for non-monotone delayed perturbations of monotone non-autonomous linear ODEs,* submitted (2016).

Raphaël Krikorian, Université Cergy-Pontoise
Almost reducibility in quasiperiodic dynamics

Almost reducibility is a very useful notion which allows to reduce the dynamics of a quasiperiodic system to a dynamics arbitrarily close to a simple one. I will

describe this notion for quasiperiodic cocycles, diffeomorphisms of the circle and pseudo rotation of the disk and state some new results.

Michael Li, University of Alberta
Orbital asymptotic stability of almost periodic solutions

We derive a criterion for the orbital asymptotic stability of almost periodic solution to autonomous systems in any finite dimension is derived. A principal tool used is the theory of compound matrices and compound linear nonautonomous differential systems. The result generalizes a stability criterion of Poincare for periodic solutions of planar autonomous systems. We also discuss a connection to a result of George Sell in 1977.

Sylvia Novo, Universidad de Valladolid
Nonautonomous linear Hamiltonian systems

This talk willbe devoted to provide a review of our contribution to the theory of nonautonomous linear Hamiltonian systems. In particular, fundamental methods of topological dynamics and of ergodic theory will be essential to prove our results concerning oscillation theory. Some applications to control theory will also be shown.
Joint work with R. Fabbri, R. Johnson, C. Núñez and R. Obaya.

Rafael Obaya, Universidad de Valladolid
A transition from the order to the chaos in non autonomous differential equations

We apply methods of topological dynamics and ergodic theory to investigate the transitions from the order that provides the uniform stability, to different scenarios of dynamical complexity including ingredients of chaotic dynamics.
We investigate the presence of Li-Yorke chaos in some of these scenarios and show how these ingredients of complexity appear in a natural way at the bifurcation points of the non autonomous version of classical bifurcation patterns.

Rafael Ortega, Universidad de Granada
The escaping set of a quasi-periodic map

We consider a planar map f of the type

$$\begin{cases} \theta_1 = \theta + F(\omega_1\theta, \dots, \omega_N\theta, r) \\ r_1 = r + G(\omega_1\theta, \dots, \omega_N\theta, r) \end{cases}$$

together with the associated family of maps g lying in the hull of f, $g \in \mathcal{H}$. We use recent ideas on recurrence due to Dolgopyat to prove that, under certain assumptions on f, the escaping set

$$E_g = \left\{ (\theta, r) \in \mathbb{R}^2 \ : \ \lim_{n\to\infty} r_n = +\infty \right\}$$

has zero measure for almost every $g \in \mathcal{H}$.

This result is employed to prove that there is no acceleration in a quasi-periodic Fermi-Ulam ping-pong. The frequencies $\omega_1\, dots, \omega_N$ are arbitrary, no Diophantine condition is needed.
Joint work with Markus Kunze.

Geneviève Raugel, Université Paris-Sud
Dynamics of the damped focusing subcritical Klein-Gordon equation

We first study the focusing subcritical Klein-Gordon equation with constant positive damping and radial data. In particular, we show that either the solutions blow up in finite time or they converge to an equilibrium point. Later we extend these

results to some variable positive dampings.
Joint work with N. Burq and W. Schlag.

Yingfei Yi,University of Alberta and Jilin University
Reducibility of quasi-periodic linear KdV equation

We consider the following one-dimensional, quasi-periodically forced, linear KdV equations

$$u_t + (1 + a_1(\omega t, x))u_{xxx} + a_2(\omega t, x)u_{xx} + a_3(\omega t, x)u_x + a_4(\omega t, x)u = 0$$

under the periodic boundary condition $u(t, x + 2\pi) = u(t, x)$, where ω's are frequency vectors lying in a bounded closed region $\Pi_* \subset \mathbb{R}^b$ for some $b > 1$, $a_i : \mathbb{T}^b \times \mathbb{T} \to \mathbb{R}$, $i = 1, \ldots, 4$, are bounded above by a small parameter $\varepsilon_* > 0$ under a suitable norm, real analytic in $\phi \in \mathbb{T}^b$ and sufficiently smooth in $x \in \mathbb{T}$, and a_1, a_3 are even, a_2, a_4 are odd. Under the real analyticity assumption of the coefficients, we show that there exists a Cantor set $\Pi_{\varepsilon_*} \subset \Pi_*$ with $|\Pi_* \setminus \Pi_{\varepsilon_*}| = O(\varepsilon_*^{\frac{1}{100}})$ such that for each $\omega \in \Pi_{\varepsilon_*}$, the corresponding equation is smoothly reducible to a constant-coefficients one. This problem is closely related to the existence and linear stability f quasi-periodic solutions in a nonlinear KdV equation.

Jiangogn You, Chern Institute of Mathematics, Nankai University.
Almost reducibility and apllications

I will briefly introduce the works on quantitative almost reducibility of analytic quasi-periodic linear systems and cocycles. The applications include: Dry Ten Martini problem in the non-critical case; Andre-Aubry-Jitomirskaya conjecture and the regularity of Lyapunov exponents of quasi-periodic cocycles.
Joint work with A. Avila, Q. Zhou and L. Ge.

Fabio Zanolin, Università di Udine.
Remarks on the periodic Ambrosetti-Prodi problem

With the term "Ambrosetti-Prodi type problems" we usually mean those nonlinear parameter-depending problems of the form $Lu + g(u) = s$ where $g'(u)$ crosses some eigenvalue of the linear differential operator L and, as the real parameter s varies, there is a change in the number of the solutions. Starting with the pioneering work of Ambrosetti and Prodi (Ann. Mat. Pura Appl., 1972) a great deal of researches has been devoted to this kind of problems, from different perspectives. In the first part of the talk we briefly survey some classical results about the AP-problem for nonautonomous equations in the periodic case. In the second part of the talk we present some recent developments with special emphasis to some typical features of interest in the area of dynamical systems, such as subharmonics solutions and chaotic dynamics.
Joint work with Elisa Sovrano (Udine).

Talks

Lluís Alsedà, Universidad Autònoma de Barcelona

Combinatorial dynamics of strip patterns of quasiperiodic skew products in the cylinder

We extend the results and techniques from [1] to study the combinatorial dynamics (*forcing*) and entropy of quasiperiodically forced skew-products on the cylinder. For these maps we prove that a cyclic permutation τ forces a cyclic permutation ν as interval patterns if and only if τ forces ν as cylinder patterns. This result gives as a corollary the Sharkovskiĭ Theorem for quasiperiodically forced skew-products on the cylinder proved in [1].

Next, the notion of s-horseshoe is defined for quasiperiodically forced skew-products on the cylinder and it is proved, as in the interval case, that if a quasiperiodically forced skew-product on the cylinder has an s-horseshoe then its topological entropy is larger than or equals to $\log(s)$.

Finally, if a quasiperiodically forced skew-product on the cylinder has a periodic orbit with pattern τ, then $h(F) \geq h(f_\tau)$, where f_τ denotes the *connect-the-dots* interval map over a periodic orbit with pattern τ. This implies that if the period of τ is $2^n q$ with $n \geq 0$ and $q \geq 1$ odd, then $h(F) \geq \frac{\log(\lambda_q)}{2^n}$, where $\lambda_1 = 1$ and, for each $q \geq 3$, λ_q is the largest root of the polynomial $x^q 2x^{q2}1$. Moreover, for every $m = 2^n q$ with $n \geq 0$ and $q \geq 1$ odd, there exists a quasiperiodically forced skew-product on the cylinder F_m with a periodic orbit of period m such that $h(F_m) = \frac{\log(\lambda_q)}{2^n}$. This extends the analogous result for interval maps to quasiperiodically forced skew-products on the cylinder.

Moreover, there is a natural question that arises in this setting: *Does Sharkovskiĭ Theorem holds when restricted to curves instead of general strips?*

We answer this question in the negative by constructing a counterexample: We construct a map having a periodic orbit of period 2 of curves (which is, in fact, the upper and lower circles of the cylinder) and without any invariant curve.

In particular this shows that there exist quasiperiodic skew products in the cylinder without invariant curves.

References

[1] Roberta Fabbri, Tobias Jäger, Russel Johnson, and Gerhard Keller. *A Sharkovskii-type theorem for minimally forced interval maps.*Topol. Methods Nonlinear Anal., 26(1):163–188, 2005.

Joint work with Francesc Mañosas and Leopoldo Morales.

Pierluigi Benevieri, Universidade de Sãu Paulo

On general properties of N-th order retarded functional differential equations

Consider the second order RFDE (retarded functional differential equation) $x''(t) = f(t, x_t)$, where f is a continuous real-valued function defined on the Banach space $\mathbb{R} \times C^1([-r, 0], \mathbb{R})$. The weak assumption of continuity on f (due to the strong topology of $C^1([-r, 0], \mathbb{R})$) makes not convenient to transform this equation into a first order RFDE of the type $z'(t) = g(t, z_t)$. In fact, in this case, the associated $\mathbb{R}^2$-valued function g could be discontinuous (with the C^0-topology) and, in addition, not necessarily defined on the whole space $\mathbb{R} \times C([-r, 0], \mathbb{R}^2)$. Consequently, in spite of what happens for ODEs, the classical results regarding existence, uniqueness, and continuous dependence on data for first order RFDEs could not apply.
Motivated by this obstruction, we provide results regarding general properties, such as existence, uniqueness, continuous dependence on data and continuation of solutions of RFDEs of the type $x^{(n)}(t) = f(t, x_t)$, where f is an $\mathbb{R}^k$-valued continuous

function on the Banach space $\mathbb{R} \times C^{(n-1)}([-r, 0], \mathbb{R}^k)$. Actually, for the sake of generality, our investigation will be carried out in the case of infinite delay.
Joint work with A. Calamai, M. Furi, M.P. Pera.

Luca Bisconti, Università di Firenze
Some recent results on the 2D dissipative Euler equations

We present some recent advances concerning the 2D dissipative Euler equations. This talk is mainly based on a joint work with Luigi Carlo Berselli (University of Pisa). First, we make a review of known results about the properties of weak solutions, their long-time behavior and the existence of certain global attractors. Then, we prove that, in the presence of a sufficiently large dissipative term, it is possible to obtain uniform estimates on the modulus of continuity of the vorticity. These allow us to show the existence of Stepanov almost-periodic solutions to the considered system.

Alberto Boscaggin, Università di Torino
Scattering parabolic solutions for the N-centre problem in the three dimensional space

For the N-centre problem in the three dimensional space, we prove the existence of entire parabolic trajectories having prescribed asymptotic directions. The proof relies on a variational argument of min-max type; Morse index estimates and regularization techniques are used in order to rule out the possible occurrence of collisions.
Joint work with W. Dambrosio and S. Terracini (Torino).

Francesca Dalbono , Università di Palermo
Multiplicity results for a class of asymptotically linear systems of second-order ordinary differential equations

We study multiplicity of solutions to an asymptotically linear Dirichlet problem associated with a planar system of second order ordinary differential equations. The multiplicity result is expressed in term of the Maslov indexes of the linearizations at zero and infinity: the gap between the Maslov indexes provides a lower estimate on the number of solutions. The proof is developed in the framework of the shotting methods and it is based on the concepts of phase angles and moments of verticality.

Cinzia Elia, Università di Bari
On periodic orbits of discontinuous dynamical systems

In this talk we consider an n-dimensional discontinuous dynamical system with an asymptotically stable periodic orbit. We first look at systems with an hyperplane of discontinuity and give theoretical results on the persistence of the periodic orbit under regularization.

The second part of the talk is work in progress. We consider the case in which the orbit has partial sliding on the intersection on two hyperplanes of discontinuity. Ambiguity of the sliding vector field in this case does not allow us to give general results, still, under certain conditions, persistence of the periodic orbit can be proven.

Alex Haro, Universidad de Barcelona
On the (computer-assisted) application of an a posteriori KAM theorem

We present a methodology to rigorously validate a given approximation of a quasi-periodic Lagrangia torus of an exact symplectic map. The approach consists in

verifying the hypotheses of an a-posteriori KAM theorem based on the parameterization method (following Rafael de la Llave and collaborators).

A crucial point of our implementation is an Approximation Lemma that allows us to control the norm of periodic functions using their discrete Fourier transform. An outstanding consequence of this approach is that the computational cost of the validation is asymptotically equivalent to the cost of the numerical computation of invariant tori using the parameterization method.

We illustrate the methodology with several examples, as the standard map,and the Froeschlé map.

Joint work with Jordi-Lluís Figueras and Alejandro Luque

R. Johnson, Università di Firenze

Remarks on nonautonomous bifurcation theory

What happens when the parameter in the classical saddle-node bifurcation pattern is subjected to a time-dependent perturbation? It turns out that a fairly clean-cut answer can be given. We will do this by (paraphrasing C. Poetzsche) "putting nonautonouns dynamics to work". We will also discuss the Andronov-Hopf bifurcation pattern when the parameter is made to oscillate rapidly. In this case some joint results with M. Franca and V. Munoz can be used to obtain the insight.

Angel Jorba, Universidad de Barcelona

Reducibility and fractalization of invariant curves

In this talk we will focus on invariant curves of quasi-periodically forced maps that depends on a parameter. In some situations, the invariant curve suffers a fractalization procedure that may lead to the creation of a Strange Non-Chaotic Attractor. In the talk I will discuss the role of the (lack of) reducibility of the invariant curve in some simple examples of fractalization.

Luisa Malaguti, Università di Modena e Reggio Emilia

Periodic and nonlocal trajectories in diffusion dynamics

The main part of this talk deals with the reaction-diffusion equation

$$u_t(t,x) = \Delta u(t,x) + f\left(t,x,u,\int_\Omega k(x,\xi)u(t,\xi)\,d\xi\right),\ t\in[0,T],\ x\in\Omega \tag{1}$$

with $\Omega \subset \mathbb{R}^n$, which is a model for several diffusive processes. The discussion concerns the existence of trajectories satisfying nonlocal conditions such as the Cauchy multi-point

$$u(0,x) = \sum_{i=1}^{p} \alpha_i\, u(t_i,x), \quad x\in\Omega$$

with $0 < t_1 < ...t_p \le T$ and $\alpha_i \in \mathbb{R}$, $i = 1,...p$ or the mean value condition

$$u(0,x) = \frac{1}{T}\int_0^T u(t,x)\,dt, \quad x\in\Omega.$$

The appearance of periodic solutions is treated, too.

By the standard reformulation of (1) in abstract setting some classical topological tools for the study of non-autonomous dynamical systems can be used, in this infinite-dimensional framework. The different strategies in order to overcome the lack of compactness in the infinite context, are discussed.

Two approaches are proposed, depending on the reaction term f. Both of them are based on some approximation scheme and make use of the invariance, through homotopic fields, of a suitably defined degree; Hartmann-type conditions are involved.

The discussion then proceeds with two further models where, differently than in (1), the linear part does not generate a compact semigroup; similar techniques as before can be used also here. The former describes the time evolution of the age-structure of a population

$$\begin{cases} u_t = -u_a - f\left(t, a, \int_0^B u(t,a)da\right)u, \quad 0 \le a \le B, \quad 0 \le t \le T \\ u(0,a) = u_0(a) \quad 0 \le a \le B \\ u(t,0) = \int_0^B b(a)u(t,a)da \quad 0 \le t \le T. \end{cases} \tag{2}$$

The latter is a second order equation

$$u_{tt} = cu_t + bu(t,\xi) + u(t,\xi)\int_\Omega k(\xi,\eta)u(t,\eta)d\eta + h(t,u(t,\xi)) \tag{3}$$

where a nonlocal diffusivity with integral form and degenerate nature replaces the classical Laplace operator; the appearance of periodic and nonlocal solutions also in (3) is showed.

References

[1] I. Benedetti, L. Malaguti, V. Taddei, *Nonlocal semilinear evolution equations without strong compactness: theory and applications*, Boundary Value Problems, 2013 (2013), 1–18.
[2] I. Benedetti, N. Loi, L. Malaguti, V. Taddei, *Nonlocal diffusion second order partial differential equations*, Journal of Differential Equations. 262 (2017) 14991523.
[3] I. Benedetti, N. Loi, V. Taddei, *Nonlocal diffusion second order partial differential equations*, submitted.
[4] I. Benedetti, L. Malaguti, V. Taddei, *The role of bounding functions in evolution problems with compact semigroups*, in preparation.

Pierpaolo Omari, Università di Trieste
Positive solution of a one-dimensional indefinite quasi-linear Neumann problem

We study the existence and multiplicity of positive solutions of the one-dimensional capillarity-type problem

$$-\left(u'/\sqrt{1+(u')^2}\right)' = a(x)f(u), \quad u'(0) = 0, u'(1) = 0,$$

where $a \in L^1(0,1)$ changes sign and $f : [0,+\infty) \to [0,+\infty)$ is continuous and has a power-like behavior at the origin and at infinity. The solutions we find are either classical or special functions of bounded variation. Our approach makes use of topological or variational methods and relies on a regularization procedure that yields bounded variation solutions which are of class $W^{2,1}_{\rm loc}$ on each open interval where the weight function a has a constant sign.

References

J. López-Gómez, P. Omari and S. Rivetti, Positive solutions of one-dimensional indefinite capillarity-type problems: a variational approach, J. Differential Equations (2017), in press (doi.org/10.1016/j.jde.2016.10.046)

J. López-Gómez, P. Omari and S. Rivetti, Bifurcation of positive solutions for a one-dimensional indefinite quasilinear Neumann problem, preprint (2016), submitted.

Raffaella Pavani, Politecnico di Milano
A new approach to characterization of D-stability

The concept of D-stability is relevant for stable square matrices of any order, especially when they appear in ordinary differential systems modeling physical problems. Indeed, D-stability was treated from different points of view in the last fifty years, but the problem of characterization of a D-stable matrix was solved for low order matrices only (i.e., up to order 4). Here a new approach is proposed within the context of numerical linear algebra. A new necessary and sufficient condition for D-stability is proved and implementation by computer algebra is shown to be encouraging.

Christian Poetzsche, Universität Klagenfurt
Global continuation of entire bounded solution

We investigate the structure of the entity of bounded entire solutions (with limit zero) for nonautonomous differential equations under not necessarily small perturbations. Out tools are global implicit function theorems bsed in an ambient degree theory for Fredholm operators.
Joint work with Robert Skiba (Nicolaus Copernicus University, Toruń, Poland.

Ana Sanz, Universidad de Valladolid
Uniform persistence of almost periodic ODEs and delay FDEs: an individual or collective property on the hull?

Uniform persistence is a dynamical property which has a great interest in mathematical modeling, in areas such as biological population dynamics, epidemiology or ecology. We show that this dynamical property is not robust in almost periodic ODEs or delay FDEs, meaning that in general it is not transferred from an individual almost periodic system to the family of systems over the hull. Besides, using classical results by Johnson and by Shneiberg, we prove that when the transfer fails to happen in models given by almost periodic cooperative and linear or sublinear ODEs or delay FDEs, the set of systems which do not gain the property is big, both from a topological and from a measure theory point of view. This means that in the non-robust situation it is highly improbable that we can experimentally or numerically detect uniform persistence.

Andrea Sfecci, Università Politecnica delle Marche
Invariant manifold theory in radial elliptic PDEs

Existence of entire radial solutions of elliptic PDEs can be investigated by the use of invariant manifold theory.

By the introduction of the Fowler transformation we can obtain a non-autonomous dynamical system having a saddle-type equilibrium at the origin. The existence of homoclinic trajectories is strictly related to the existence of regular fast decay solutions of the elliptic PDE.

Different asymptotic behaviors of the nonlinearity ruling the PDE, provide different dynamics. The presence of a sign-changing nonlinearity in the PDE brings a major difficulty: the non-continuability of the solutions of the dynamical system, but also in this situation existence result can be obtained.