Compitino Analisi II

1. **Retta tangente**

   Data la curva $\gamma\colon [0,\pi] \to \mathbb{R}^3$ definita da

   $$\gamma(t) = \big(4\sin(t), t + \cos(t), t^2 - \pi t\big)\,,$$

   la retta tangente al sostegno di γ nel punto $\gamma\left(\frac{\pi}{2}\right)$

   (a) non è ben definita ✓
   (b) è contenuta sul piano $z = 0$
   (c) è contenuta sul piano $x = z$
   (d) passa per l'origine degli assi cartesiani
   (e) non voglio rispondere a questa domanda

2. **Lunghezza di una curva**

   Data la curva

   $$\varphi\colon t \in [0,\pi] \mapsto (4\cos(t), \sin(t)) \in \mathbb{R}^2,$$

   la lunghezza della curva è

   (a) $\int_0^\pi \sqrt{1 + 15\sin^2(t)}\mathrm{d}t$ ✓
   (b) $\int_0^\pi (4\sin(t) + |\cos(t)|)\mathrm{d}t$
   (c) $\int_0^\pi \sqrt{15 + \sin^2(t)}\mathrm{d}t$
   (d) $\int_0^\pi (15 + \cos^2(t))\mathrm{d}t$
   (e) non voglio rispondere a questa domanda

3. **Integrale curvilineo**

   Data la curva

   $$\gamma\colon t \in [0,\pi] \mapsto \big(\cos(t), \mathrm{e}^t\big) \in \mathbb{R}^2,$$

   calcolare $\int_\gamma \sqrt{1 - x^2 + y^2}\,\mathrm{d}s$

   (a) $\frac{\pi + \mathrm{e}^{2\pi} - 1}{2}$ ✓
   (b) $\frac{\pi - \mathrm{e}^{2\pi} - 1}{2}$
   (c) $\frac{\pi - 1 + \mathrm{e}^{\pi}}{4}$
   (d) $\frac{\pi - 1 - \mathrm{e}^{\pi}}{4}$
   (e) non voglio rispondere a questa domanda

4. **Coordinate polari**

   Sul piano cartesiano Oxy sia C il cerchio centrato in $(0,2)$ e avente raggio $R = 2$. Se $(\rho,\theta) \in [0,+\infty) \times [0,2\pi)$ sono le coordinate polari centrate nell'origine, allora C è dato da

   (a) $\{(\rho,\theta)\colon \theta \in [0,\pi], \rho \leq 4\sin(\theta)\}$ ✓
   (b) $\{(\rho,\theta)\colon \theta \in [0,\pi/2], \rho \leq 4\sin(\theta)\}$
   (c) $\{(\rho,\theta)\colon \theta \in [0,\pi], \rho \leq 2\}$
   (d) $\{(\rho,\theta)\colon \theta \in [0,\pi/2], \rho \leq 2\sin(\theta)\}$
   (e) non voglio rispondere a questa domanda

5. **Topologia**

   Sia $f(x,y) = \ln(x^2 - y^2 + 4)$ e sia D il suo dominio naturale.

   (a) D è aperto, illimitato e connesso per archi ✓
   (b) D è chiuso, illimitato e connesso per archi
   (c) D è chiuso, limitato e connesso per archi
   (d) D è aperto, limitato e connesso per archi
   (e) non voglio rispondere a questa domanda

6. **Gradiente**

   Sia $f(x,y) = \cos(xy) + \sin\left(\frac{x}{y}\right)$.

   (a) $f_x\big(\frac{\pi}{3}, 2\big) = \frac{-3\sqrt{3}}{4}$ ✓
   (b) $f_y\big(\frac{\pi}{3}, 2\big) = \frac{-5\pi\sqrt{3}}{24}$ ✓
   (c) $f_y\big(\frac{\pi}{3}, 2\big) = \frac{-3\sqrt{3}}{4}$
   (d) $f_x\big(\frac{\pi}{3}, 2\big) = \frac{-5\pi\sqrt{3}}{24}$
   (e) $f_x\big(\frac{\pi}{3}, 2\big) = \frac{3\sqrt{3}}{4}$

(f) non voglio rispondere a questa domanda

7. **Piano tangente al grafico**

   Sia $f(x,y) = x^2 - y^3 + xy$. Il piano tangente al grafico di f nel punto $(2,1,f(2,1))$ ha equazione cartesiana

   (a) $z = 5x - y - 4$ ✓
   (b) $z = 5x - y + 4$
   (c) $z = 5x + y - 4$
   (d) $z = -5x + y - 4$
   (e) non voglio rispondere a questa domanda

8. **Funzioni composte**

   La funzione $f\colon \mathbb{R}^2 \to \mathbb{R}$ è di classe C^1. Date le curve
   $\alpha\colon t \in [0,\pi] \mapsto (2\cos(t), \sin(t)) \in \mathbb{R}^2$,
   $\beta\colon t \in [0,2] \mapsto \left(t\sqrt{2}, \frac{\sqrt{2}}{4}(t^2+1)\right) \in \mathbb{R}^2$,
   si sa che
   $\frac{\mathrm{d}}{\mathrm{dt}}(f \circ \alpha)\left(\frac{\pi}{4}\right) = 2, \quad \frac{\mathrm{d}}{\mathrm{dt}}(f \circ \beta)(1) = -3.$
   Ne posso concludere che

   (a) $f_x(\sqrt{2}, \frac{\sqrt{2}}{2}) = \frac{-5\sqrt{2}}{4}$ ✓
   (b) $f_y(\sqrt{2}, \frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}$
   (c) $f_y(\sqrt{2}, \frac{\sqrt{2}}{2}) = \frac{-5\sqrt{2}}{4}$
   (d) $f_x(\sqrt{2}, \frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}$
   (e) non voglio rispondere a questa domanda

9. **Integrali1**

   Sul piano cartesiano Oxy sia T il triangolo di vertici $(0,0)$, $(0,2)$, $(1,1)$ e sia f una funzione integrabile su T. Allora $\int_T f(x,y)\mathrm{d}x\mathrm{d}y$ è uguale a

   (a) $\int_0^1 \left(\int_x^{2-x} f(x,y)\mathrm{d}y\right) \mathrm{d}x$ ✓
   (b) $\int_0^2 \left(\int_y^{2-y} f(x,y)\mathrm{d}x\right) \mathrm{d}y$
   (c) $\int_0^1 \left(\int_0^2 f(x,y)\mathrm{d}y\right) \mathrm{d}x$
   (d) $\int_0^1 \left(\int_x^2 f(x,y)\mathrm{d}y\right) \mathrm{d}x$
   (e) non voglio rispondere a questa domanda

10. **Integrali2**

    Sul piano cartesiano Oxy sia C la porzione del cerchio $x^2 + y^2 \leq 4$ contenuta nel semipiano $y \geq 0$. Allora $\int_C y\,\mathrm{d}x\mathrm{d}y$ è uguale a

    (a) $\frac{16}{3}$ ✓
    (b) $\frac{16}{3}\pi$
    (c) $\frac{8}{3}$
    (d) $\frac{8}{3}\pi$
    (e) non voglio rispondere a questa domanda

11. **Jacobiano**

    Sia $\Psi(u,v) = \left(u^2 + v^2, \frac{u}{v}\right)$. Allora

    (a) $\det J_\Psi(2,1) = -10$ ✓
    (b) $\det J_\Psi(2,1) = 10$
    (c) $\det J_\Psi(2,1) = 8$
    (d) $\det J_\Psi(2,1) = -8$
    (e) non voglio rispondere a questa domanda

venerdì 27 novembre 2020 09:08

(1) $\gamma : [0,\pi] \to \mathbb{R}^3$ $\quad \gamma(t) = (4\sin(t),\ t+\cos(t),\ t^2-\pi t)$

Retta Tangente al sostegno di γ nel pto $\gamma\left(\frac{\pi}{2}\right)$

SOLUZIONE
$$\gamma'(t) = (4\cos(t),\ 1-\sin(t),\ 2t-\pi)$$
$$\gamma'\left(\frac{\pi}{2}\right) = \left(4\cdot 0,\ 1-1,\ 2\frac{\pi}{2}-\pi\right) = (0,0,0) \Leftarrow$$

NON È BEN DEFINITA

(2) $\varphi : t\in[0,\pi] \mapsto (4\cos(t), \sin(t)) \in \mathbb{R}^2$, lunghezza della curva = ?

$$\int_0^\pi \text{—}\ dt$$

SOL $\quad L(\varphi) = \int_0^\pi \|\varphi'(t)\|\ dt \qquad \varphi'(t) = (-4\sin(t), \cos(t))$

$$\|\varphi'(t)\| = \sqrt{(-4\sin(t))^2 + (\cos(t))^2} = \sqrt{16\sin^2(t) + \cos^2(t)} \qquad \sin^2(t)+\cos^2(t) = 1$$
$$= \sqrt{15\sin^2(t) + 1}$$

$$\Rightarrow L(\varphi) = \int_0^\pi \sqrt{1+15\sin^2(t)}\ dt$$

(3) $\gamma : t\in[0,\pi] \mapsto (\cos(t), e^t) \in \mathbb{R}^2 \qquad \gamma(t) = (x(t), y(t))$

$$\int_\gamma \sqrt{1-x^2+y^2}\ ds$$

SOL $\quad \int_\gamma \sqrt{1-x^2+y^2}\ ds = \int_0^\pi \sqrt{1-x^2(t)+y^2(t)}\ \|\gamma'(t)\|\ dt$

$$\gamma'(t) = (-\sin(t), e^t) \qquad \|\gamma'(t)\| = \sqrt{(-\sin(t))^2 + (e^t)^2} = \sqrt{\sin^2(t) + e^{2t}}$$

$$\sqrt{1-x^2(t)+y^2(t)} = \sqrt{1-\cos^2(t) + (e^t)^2} = \sqrt{1-\cos^2(t)+e^{2t}}$$

$$\int_\gamma \sqrt{1-x^2+y^2}\ ds = \int_0^\pi \sqrt{\underbrace{1-\cos^2(t)}_{=\sin^2(t)} + e^{2t}} \cdot \sqrt{\sin^2(t)+e^{2t}}\ dt =$$

$$\underbrace{\qquad\qquad}_{=\sin^2(t)}$$

$$\int_0^{\pi}\left(\sin^2(t)+e^{2t}\right)dt =$$

$$\cos(2t) = 2\cos^2(t)-1 = $$
$$= 1-2\sin^2(t)$$
$$\sin^2(t) = \frac{1}{2}\left(1-\cos(2t)\right)$$

$$= \int_0^{\pi}\left(\frac{1}{2}-\frac{1}{2}\cos(2t)+e^{2t}\right)dt =$$

$$= \frac{t}{2}-\frac{1}{2}\frac{1}{2}\sin(2t)+\frac{1}{2}e^{2t}\Bigg|_{t=0}^{t=\pi} = \left(\frac{\pi}{2}-0+\frac{1}{2}e^{2\pi}\right)-\left(0-0+\frac{1}{2}\right) =$$

$$= \frac{1}{2}\left(\pi+e^{2\pi}-1\right)$$

4 Sul piano cartesiano Oxy sia C il cerchio centrato in $(0,2)$ e avente raggio $R=2$.
Se $(\rho,\theta)\in[0,+\infty)\times[0,2\pi)$ sono le coordinate polari centrate nell'origine, allora C è dato da

$$C = \left\{(x,y)\in\mathbb{R}^2 : \sqrt{(x-0)^2+(y-2)^2} \le 2\right\} =$$

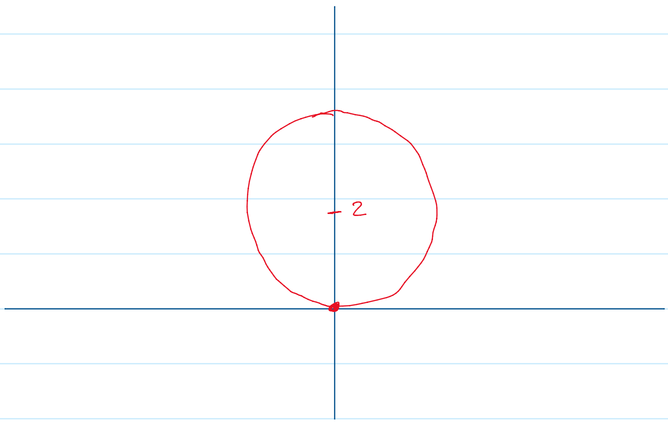
$$= \left\{(x,y)\in\mathbb{R}^2 : \underline{x^2+(y-2)^2\le 4}\right\}$$

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases} \qquad \rho\ge 0,\quad \theta\in[0,2\pi)$$

$$\begin{cases} (\rho\cos\theta)^2+(\rho\sin\theta-2)^2\le 4 \\ \rho\ge 0 \\ \theta\in[0,2\pi) \end{cases}$$

$$\begin{cases} \rho^2\cos^2\theta+\rho^2\sin^2\theta+\cancel{4}-4\rho\sin\theta\le \cancel{4}\,0 \\ \rho\ge 0 \\ \theta\in[0,2\pi) \end{cases}$$

$$\begin{cases} \rho^2-4\rho\sin\theta\le 0 \\ \rho\ge 0 \\ \theta\in[0,2\pi) \end{cases}$$



$$\rho^2-4\rho\sin\theta\le 0$$
$$\rho(\rho-4\sin\theta)\le 0$$

$\rho^2 - 4\rho\sin\theta \le 0$

$\rho(\rho - 4\sin\theta) \le 0$

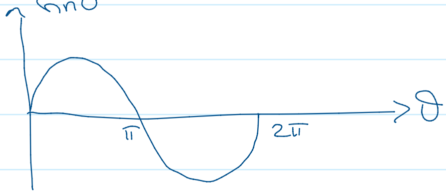
$\rho = 0 \quad \cup \quad \begin{cases} \rho - 4\sin\theta \le 0 \\ \rho \geq 0 \\ \theta \in [0, 2\pi) \end{cases}$

$\rho - 4\sin\theta \le 0 \qquad \rho \le 4\sin\theta$

$\begin{cases} 0 \le \rho \le 4\sin\theta \\ \cancel{\theta \in [0, 2\pi)} \end{cases} \leftarrow \quad \Longrightarrow \begin{cases} 4\sin\theta \geq 0 \\ \theta \in [0, 2\pi) \end{cases}$

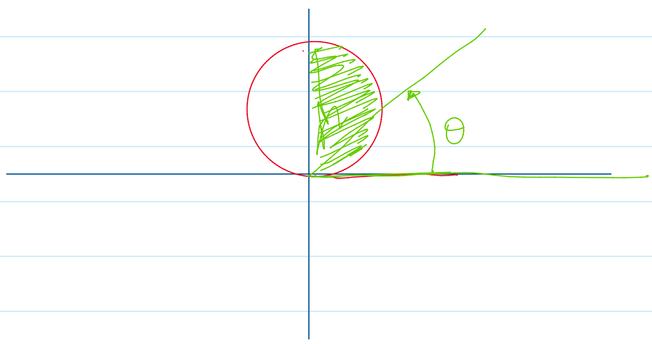
$\Updownarrow$

$\boxed{\theta \in [0, \pi]}$

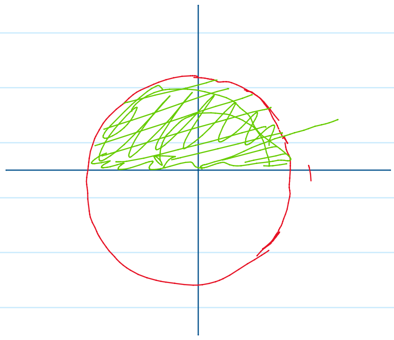


$\Rightarrow C$ è descritto da $\{(\rho, \theta) \in [0, +\infty) \times [0, 2\pi) : \theta \in [0, \pi], \ \rho \le 4\sin\theta\}$

N.B $\{(\rho, \theta) \in [0, +\infty) \times [0, 2\pi) : \theta \in [0, \frac{\pi}{2}], \ \rho \le 4\sin\theta\}$



$\{(\rho, \theta) : \theta \in [0, \pi) \quad , \ \rho \le 2\}$



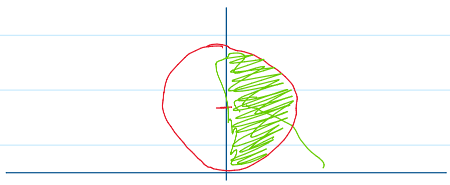
$\{(\rho, \theta) : \theta \in [0, \frac{\pi}{2}] \quad \rho \le 2\sin\theta\}$

$x = \rho\cos\theta$

$y = \rho\sin\theta$

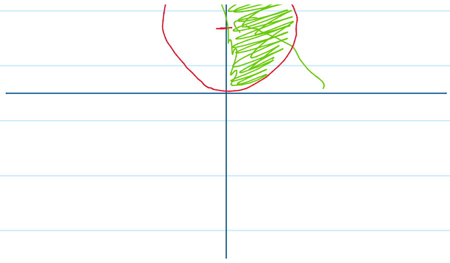
$\rho = \sqrt{x^2 + y^2}$

$\rho\sin\theta$



$y = \rho \sin\theta$

$$\boxed{\sin\theta = \frac{\rho \sin\theta}{\rho} = \frac{y}{\sqrt{x^2+y^2}}}$$



$\rho \le 2\sin\theta \qquad \sqrt{x^2+y^2} \le \frac{2y}{\sqrt{x^2+y^2}}$

$x^2+y^2 \le 2y \qquad x^2 + y^2 - 2y + 1 \le 0 + 1$

$x^2 + (y-1)^2 \le 1 \qquad$ centro $(0,1)$ $R = 1$

5 $\quad f(x,y) = \ln(x^2-y^2+4)$

$D = \{(x,y) \in \mathbb{R}^2 : x^2-y^2+4 > 0\}$

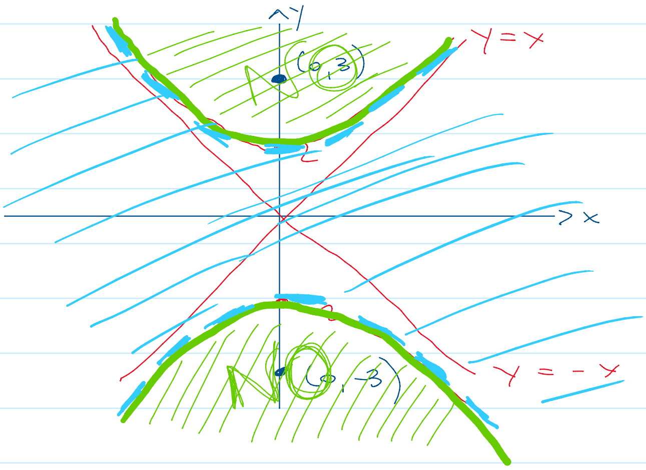
$x^2-y^2+4 > 0 \qquad x^2-y^2 > -4 \qquad \frac{x^2}{4} - \frac{y^2}{4} > -1$

$\frac{x^2}{4} - \frac{y^2}{4} = -1 \qquad \frac{x^2}{4} - \frac{y^2}{4} = 0 \qquad x^2-y^2 = 0$

$(x+y)(x-y) = 0$

$y=0 \quad \frac{x^2}{4} = -1$ imp

$x=0 \quad \frac{y^2}{4} = 1 \quad y = \begin{cases} 2 \\ -2 \end{cases}$



$D = \{(x,y) \in \mathbb{R}^2 : x^2-y^2+4 > 0\}$

$x^2-y^2+4 \big|_{(0,3)} = 0-9+4 < 0$

$x^2-y^2+4 \big|_{(0,-3)} = 0-9+4 < 0$

D è aperto, illimitato e connesso per archi

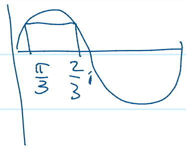
6 $\quad f(x,y) = \cos(xy) + \sin\left(\frac{x}{y}\right) \qquad P\left(\frac{\pi}{3}, 2\right)$

$f_x(x,y) = -\sin(xy)(y) + \cos\left(\frac{x}{y}\right)\left(\frac{1}{y}\right) = -y\sin(xy) + \frac{1}{y}\cos\left(\frac{x}{y}\right)$

$$f_x(x,y) = -\sin(xy)(y) + \cos\left(\frac{x}{y}\right)\left(\frac{1}{y}\right) = -y\sin(xy) + \frac{1}{y}\cos\left(\frac{x}{y}\right)$$

$$f_y(x,y) = -\sin(xy)(x) + \cos\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) = -x\left(+\sin(xy) + \frac{1}{y^2}\cos\left(\frac{x}{y}\right)\right)$$

$$f_x\left(\frac{\pi}{3},2\right) = -2\sin\left(\frac{2}{3}\pi\right) + \frac{1}{2}\cos\left(\frac{\pi}{3}\cdot\frac{1}{2}\right) =$$



$$= -2\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\sqrt{3}}{2} = \left(-2+\frac{1}{2}\right)\frac{\sqrt{3}}{2} = \frac{-3\sqrt{3}}{4}$$

$$f_y\left(\frac{\pi}{3},2\right) = -\frac{\pi}{3}\left(\sin\left(\frac{2}{3}\pi\right) + \frac{1}{4}\cos\left(\frac{\pi}{3}\cdot\frac{1}{2}\right)\right) =$$

$$= -\frac{\pi}{3}\left(\frac{\sqrt{3}}{2} + \frac{1}{4}\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}\frac{\sqrt{3}}{2}\left(1+\frac{1}{4}\right) = \frac{-\pi\sqrt{3}}{6}\,\frac{5}{4} = \frac{-5\pi\sqrt{3}}{24}$$

7) $f(x,y) = x^2 - y^3 + xy$ $\qquad P(2,1,f(2,1))$

Piano Tg al grafico di f in P = ?

$$z - f(2,1) = \nabla f(2,1)\cdot(x-2,\ y-1) \longleftarrow$$

$$f(2,1) = 4 - 1 + 2 = 5$$

$f_x(x,y) = 2x + y$ $\qquad f_x(2,1) = 4+1 = 5$

$f_y(x,y) = -3y^2 + x$ $\qquad f_y(2,1) = -3\cdot 1 + 2 = -1$

$$z - 5 = (5,-1)\cdot(x-2,\ y-1)$$

$$z = 5 + 5(x-2) - 1(y-1)$$

$$z = 5x - y + 5 - 10 + 1 \qquad \boxed{z = 5x - y - 4}$$

8) $f:\mathbb{R}^2 \to \mathbb{R}$ funzione di classe C^1

$\alpha: t\in[0,\pi] \mapsto (2\cos(t),\ \sin(t)) \in \mathbb{R}^2$

$\beta: t\in[0,2] \mapsto \left(t\sqrt{2},\ \frac{\sqrt{2}}{4}(t^2+1)\right) \in \mathbb{R}^2$

$\frac{d}{dt}(f\circ\alpha)\left(\frac{\pi}{4}\right) = 2$ $\qquad \frac{d}{dt}(f\circ\beta)(1) = -3$

$f_x\left(\sqrt{2},\frac{\sqrt{2}}{2}\right) = ?$ $\qquad f_y\left(\sqrt{2},\frac{\sqrt{2}}{2}\right) = ?$

Sol $\frac{d}{dt}(f\circ\alpha)\left(\frac{\pi}{4}\right) = \nabla f| \quad \cdot\ \dot{\alpha}\left(\frac{\pi}{4}\right)$ //

Sol $\qquad \frac{d}{dt}(f\circ\alpha)\left(\frac{\pi}{4}\right) = \nabla f\Big|_{\alpha\left(\frac{\pi}{4}\right)} \cdot \dot{\alpha}\left(\frac{\pi}{4}\right)$

$\frac{d}{dt}(f\circ\beta)(1) = \nabla f\Big|_{\beta(1)} \cdot \dot{\beta}(1)$

$\alpha(t) = (2\cos(t), \sin(t)) \Rightarrow \alpha\left(\frac{\pi}{4}\right) = \left(\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}, \frac{\sqrt{2}}{2}\right) = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$

$\beta(t) = \left(t\sqrt{2}, \frac{\sqrt{2}}{4}(t^2+1)\right) \Rightarrow \beta(1) = \left(1\cdot\sqrt{2}, \frac{\sqrt{2}}{4}(1+1)\right) = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$

$P := \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$

$$\begin{cases} (f_x(P), f_y(P)) \cdot \dot{\alpha}\left(\frac{\pi}{4}\right) = 2 \\ (f_x(P), f_y(P)) \cdot \dot{\beta}(1) = -3 \end{cases}$$

$\dot{\alpha}(t) = (-2\sin(t), \cos(t))$

$\dot{\beta}(t) = \left(\sqrt{2}, \frac{\sqrt{2}}{\cancel{4}_{\,2}}\cancel{2}t\right)$

$$\left|\begin{array}{l} \dot{\alpha}\left(\frac{\pi}{4}\right) = \left(-2\cdot\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(-\sqrt{2}, \frac{\sqrt{2}}{2}\right) \\ \dot{\beta}(1) = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right) \end{array}\right.$$

$$\begin{cases} (f_x(P), f_y(P)) \cdot \left(-\sqrt{2}, \frac{\sqrt{2}}{2}\right) = 2 \\ (f_x(P), f_y(P)) \cdot \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right) = -3 \end{cases}$$

$$\begin{cases} -\sqrt{2}\, f_x(P) + \frac{\sqrt{2}}{2} f_y(P) = 2 \\ \sqrt{2}\, f_x(P) + \frac{\sqrt{2}}{2} f_y(P) = -3 \end{cases} \qquad \begin{cases} \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} f_y(P) = -1 \\ -2\sqrt{2}\, f_x(P) = 5 \end{cases}$$

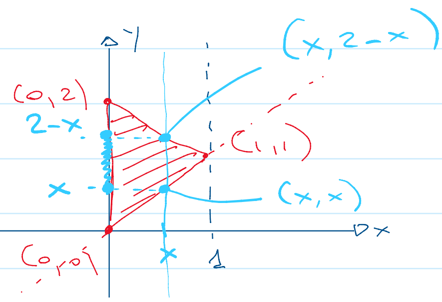
$$f_y(P) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$f_x(P) = \frac{5}{-2\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

9 $\quad T$ Triangolo di vertici $(0,0)$, $(0,2)$, $(1,1)$

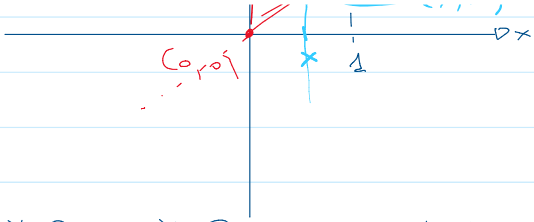
f funzione integrabile su T

$$\int_T f(x,y)\,dx\,dy$$



$x \in \mathbb{R} \quad T_x = \begin{cases} \emptyset & x<0 \vee x>1 \\ \ldots \end{cases}$

$x \in \mathbb{R} \quad T_x = \begin{cases} \emptyset & x<0 \vee x>1 \\ [x, 2-x] & x \in [0,1] \end{cases}$



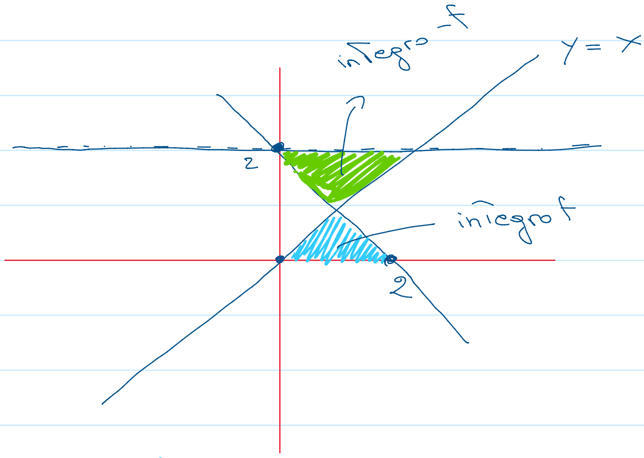
Retta per $(0,0)$ e $(1,1)$ $\qquad \dfrac{y-0}{1-0} = \dfrac{x-0}{1-0} \qquad y = x$

Retta per $(0,2)$ e $(1,1)$ $\qquad \dfrac{y-2}{1-2} = \dfrac{x-0}{1-0} \qquad y-2 = -x$

$y = 2-x$

$$\int_0^1 \left(\int_x^{2-x} f(x,y)\, dy \right) dx$$

N.B. $\quad \displaystyle\int_0^2 \left(\int_y^{2-y} f(x,y)\, dx \right) dy$



$\forall y \in [0,2] \qquad x \in [y, 2-y]$

$y \le x \le 2-y$

$\begin{cases} x \ge y \\ x \le 2-y \end{cases} \qquad \begin{cases} y \le x \\ y \le 2-x \end{cases} \qquad y \le 2-y \qquad y \le 1$

$y \le 2-y \iff y \le 1 \qquad \displaystyle\int_0^2 \left(\int_y^{2-y} f(x,y)\, dx \right) dy =$

$$= \underbrace{\int_0^1 \left(\int_y^{2-y} f(x,y)\, dx \right) dy}_{} + \int_1^2 \left(\int_y^{2-y} f(x,y)\, dx \right) dy$$

$y \in [1,2] \Rightarrow 2-y \le y$

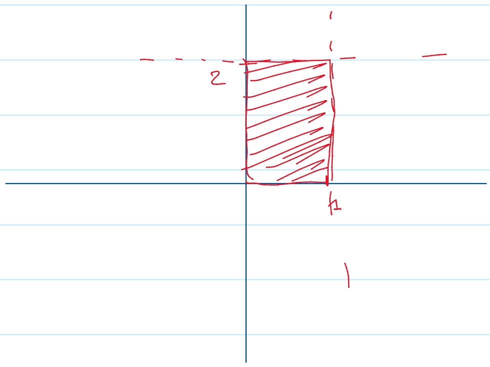
$$\int_1^2 \left(\int_{2-y}^{y} (-f(x,y))\, dx \right) dy \qquad y \in [1,2]$$

$2-y \le x \le y$

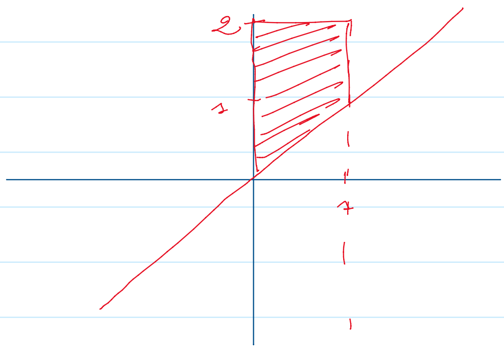
$\begin{cases} 2-y \le x \\ x \le y \end{cases} \qquad \begin{cases} y \ge 2-x \\ y \ge x \end{cases}$

$$\int_0^1 \left(\int_0^2 f(x,y)\, dy \right) dx \qquad \begin{matrix} y \in [0,2] \\ x \in [0,1] \end{matrix}$$

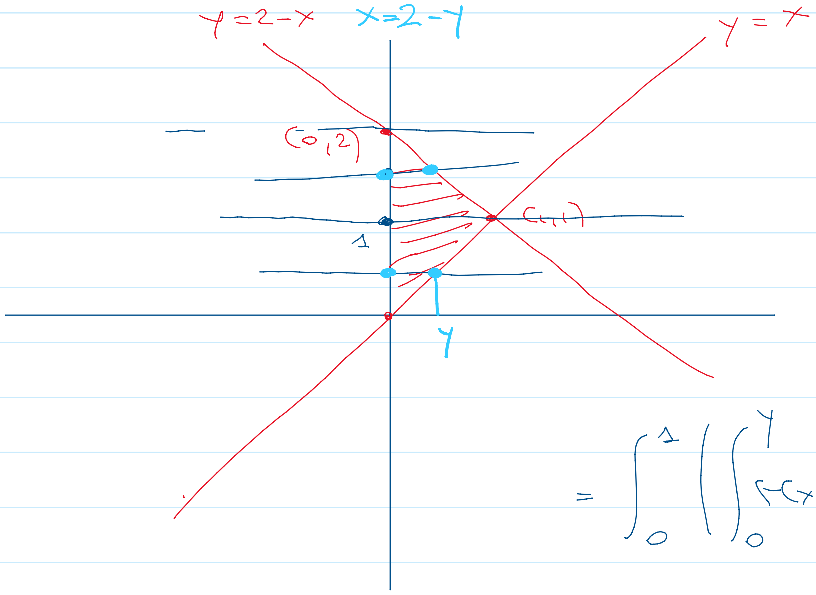
$\int_0 \quad {}_0$



$$\int_0^1 \left(\int_x^2 f(x,y)\,dy \right) dx$$



$$\begin{cases} 0 \le x \le 1 \\ x \le y \le 2 \end{cases}$$



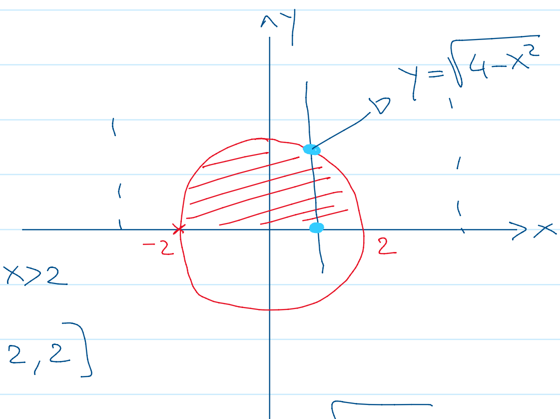
$$T_y = \begin{cases} \phi & y > 2 \vee y < 0 \\ [0, y] & y \in [0, 1] \\ [0, 2-y] & y \in (1, 2] \end{cases}$$

$$\int_T f(x,y)\,dx\,dy =$$

$$= \int_0^1 \left(\int_0^y f(x,y)\,dx \right) dy + \int_1^2 \left(\int_0^{2-y} f(x,y)\,dx \right) dy$$

10 $\quad C: \ x^2 + y^2 \le 4$, contenuta nel semipiano $y \ge 0$

$$\int_C y\,dx\,dy$$



$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$

$$C_x = \begin{cases} \phi & x < -2 \vee x > 2 \\ [0, \sqrt{4-x^2}] & x \in [-2, 2] \end{cases}$$

$$\int^2 \left(\int^{\sqrt{4-x^2}} \right) = \int^2 \left[\frac{y^2}{2} \right]^{y = \sqrt{4-x^2}}$$

$$\int_{-2}^{2}\left(\int_{0}^{\sqrt{4-x^2}} y\,dy\right)dx = \int_{-2}^{2} \frac{y^2}{2}\Big|_{y=0}^{y=\sqrt{4-x^2}} dx =$$

$$= \frac{1}{2}\int_{-2}^{2}\left(4-x^2-0\right)dx = \frac{1}{2}\left(4x-\frac{1}{3}x^3\right)\Big|_{x=-2}^{x=2} =$$

$$= \frac{1}{2}\left(8-\frac{8}{3}\right)+\frac{1}{2}\left(8-\frac{8}{3}\right) = 8-\frac{8}{3} = \frac{16}{3}$$

11

$$\Psi(u,v) = \left(u^2+v^2, \frac{u}{v}\right)$$

$$\begin{cases} x(u,v) = u^2+v^2 \\ y(u,v) = \frac{u}{v} \end{cases} \qquad J_\Psi(2,1) = \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}(2,1)$$

$$\begin{cases} x_u = 2u \\ x_v = 2v \\ y_u = \frac{1}{v} \\ y_v = \frac{-u}{v^2} \end{cases} \qquad J_\Psi(2,1) = \begin{pmatrix} 4 & 2 \\ 1 & -2 \end{pmatrix}$$

$$\det J_\Psi(2,1) = 4\cdot(-2) - 1\cdot 2 =$$
$$= -8-2 = -10$$