lunedì 9 novembre 2020   08:53

$$D = \left\{ (x,y) \in \mathbb{R}^2 : \sqrt{x^2+y^2} \leq 1 + \frac{x}{\sqrt{x^2+y^2}} \right\}$$

Disegnare D, calcolarne l'Area e il baricentro

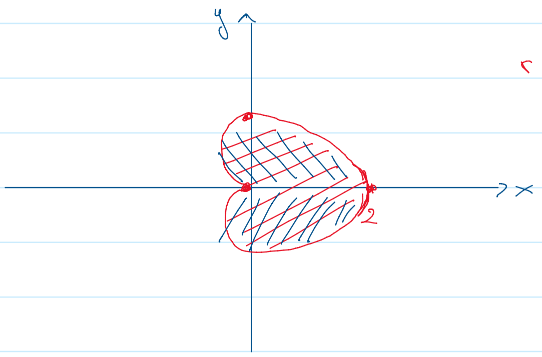
$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \qquad r \geq 0 \qquad \varphi \in [0, 2\pi)$$

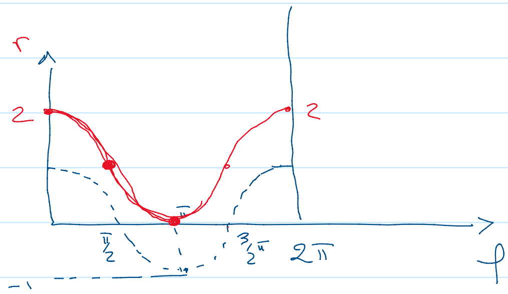
$$\sqrt{x^2+y^2} = r$$

$$\sqrt{x^2+y^2} \leq 1 + \frac{x}{\sqrt{x^2+y^2}}$$

$$r \leq 1 + \frac{r\cos\varphi}{r} = 1 + \cos\varphi$$

Cosa è la curva sul piano Oxy che in coordinate polari ha equazione $r = 1+\cos\varphi$?





$r = 1+\cos\varphi$

$\nexists$ la retta Tg al sostegno della curva sse $r(\varphi) = \dot r(\varphi) = 0$

$\dot r(\varphi) = -\sin\varphi$

$\nexists$ retta Tg al sostegno della curva sse $\begin{cases} 1+\cos\varphi = 0 \\ -\sin\varphi = 0 \end{cases}$ $\begin{cases} \cos\varphi = -1 \\ \sin\varphi = 0 \end{cases} \iff \varphi = \pi$

$$\Phi : (r,\varphi) : (0,+\infty) \times [0, 2\pi) \longmapsto (r\cos\varphi, r\sin\varphi)$$

$$\Phi^{-1}(D \setminus \{0,0\}) = \{(r,\varphi) : \varphi \in [0,2\pi) \quad 0 < r < 1+\cos\varphi\} = E$$

$$\text{Area}(D) = \int_D 1 \, dx dy = \int_E 1 \cdot r \cdot dr d\varphi = \int_0^{2\pi} \left(\int_0^{1+\cos\varphi} r \, dr \right) d\varphi =$$

$$E_\varphi = (0, 1+\cos\varphi) \quad \forall \varphi \in [0, 2\pi)$$

$$= \int_0^{2\pi} \frac{r^2}{2} \Big|_{r=0}^{r=1+\cos\varphi} d\varphi = \frac{1}{2} \int_0^{2\pi} (1+\cos\varphi)^2 \, d\varphi = \frac{1}{2} \int_0^{2\pi} (1 + 2\cos\varphi + \cos^2\varphi) \, d\varphi$$

$$\cos(2\varphi) = 2\cos^2\varphi - 1 \qquad \cos^2\varphi = \frac{1+\cos(2\varphi)}{2}$$

$$= \frac{1}{2}\int_0^{2\pi}\left(1 + 2\cos\varphi + \frac{1}{2} + \frac{1}{2}\cos(2\varphi)\right)d\varphi = \frac{1}{2}\left[\frac{3}{2}\varphi + 2\sin\varphi + \frac{1}{4}\sin(2\varphi)\right]\Bigg|_{\varphi=0}^{\varphi=2\pi}$$

$$= \frac{1}{2}\,\frac{3}{2}\,2\pi = \frac{3\pi}{2}$$

$$(x_B, y_B) \qquad x_B = \frac{1}{\text{Area}(D)}\int_D x\,dx\,dy \qquad y_B = \underbrace{\frac{1}{\text{Area}(D)}\int_D y\,dx\,dy}_{=0 \text{ per simmetria}}$$

$$x_B = \frac{2}{3\pi}\int_D x\,dx\,dy = \frac{2}{3\pi}\int_E r\cos\varphi \cdot r\,dr\,d\varphi =$$

$$= \frac{2}{3\pi}\int_0^{2\pi}\left(\int_0^{1+\cos\varphi} r^2\cos\varphi\,dr\right)d\varphi = \frac{2}{3\pi}\int_0^{2\pi}\frac{r^3}{3}\cos\varphi\,\Bigg|_{r=0}^{r=1+\cos\varphi} d\varphi =$$

$$= \frac{2}{9\pi}\int_0^{2\pi}\cos\varphi\,(1+\cos\varphi)^2\,d\varphi = \frac{2}{9\pi}\int_0^{2\pi}\cos\varphi\,(1 + 2\cos\varphi + \cos^2\varphi)\,d\varphi =$$

$$= \frac{2}{9\pi}\int_0^{2\pi}\left(\cos\varphi + 2\cos^2\varphi + \cos\varphi\,(1 - \sin^2\varphi)\right)d\varphi =$$

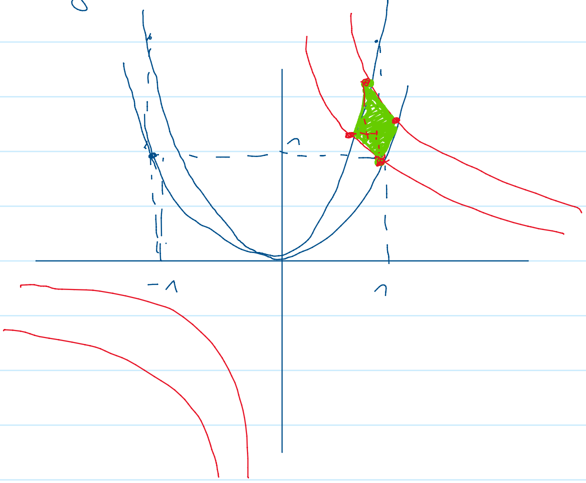
$$\cos(2\varphi) = 2\cos^2(\varphi) - 1$$
$$2\cos^2(\varphi) = 1 + \cos(2\varphi)$$

$$= \frac{2}{9\pi}\int_0^{2\pi}\left(2\cos\varphi + 1 + \cos(2\varphi) - \sin^2\varphi\cos\varphi\right)d\varphi =$$

$$= \frac{2}{9\pi}\left[2\sin\varphi + \varphi + \frac{1}{2}\sin(2\varphi) - \frac{1}{3}\sin^3\varphi\right]_{\varphi=0}^{\varphi=2\pi} = \frac{2}{9\pi}(2\pi - 0) = \frac{4}{9}$$

$$D = \{(x,y)\in\mathbb{R}^2 : \quad x^2 \le y \le 2x^2, \quad 1 \le xy \le 4\}$$

Disegnare D, calcolare Area (D) e $\int_D x^3y^2\,dx\,dy$



$y = x^2 \qquad y = 2x^2$

$xy = 1 \qquad xy = 4$

$$x^2 \le y \le 2x^2$$
$$\Updownarrow$$
$$1 \le yx^{-2} \le 2$$
$$1 \le yx \le 4$$

$$\Phi(\ldots) = \{ u = yx^{-2} \qquad (u,v) \in [1,2]\times[1,4]$$

$1 \le yx \le 4$

$$\Phi(x,y) = \begin{cases} u = yx^{-2} \\ v = yx \end{cases} \qquad (u,v) \in [1,2) \times [1,4] \iff (x,y) \in D$$

$$\begin{cases} u = yx^{-2} \\ v = yx \end{cases} \longrightarrow \quad vu^{-1} = \frac{\cancel{y}x}{\cancel{y}x^{-2}} = x^3 \qquad x = v^{1/3}\, u^{-1/3}$$

$$y = vx^{-1} = v \cdot v^{-1/3} \cdot u^{1/3} \qquad y = u^{1/3} \cdot v^{2/3}$$

$$\Phi : (u,v) \in [1,2] \times [1,4] \longmapsto \left(\underline{v^{1/3} u^{-1/3}} \, , \; u^{1/3} v^{2/3} \right) \in D$$

$$\text{Area}(D) = \int_D 1 \, dx dy = \int_{[1,2]\times[1,4]} 1 \cdot |\det J_\Phi(u,v)| \, du dv$$

$$J_\Phi(u,v) = \begin{pmatrix} -\frac{1}{3} u^{-4/3} v^{1/3} & \frac{1}{3} v^{-2/3} u^{-1/3} \\ \frac{1}{3} u^{-2/3} v^{2/3} & u^{1/3} \frac{2}{3} v^{-1/3} \end{pmatrix}$$

$$\det J_\Phi(u,v) = -\frac{1}{3} u^{-4/3} \cancel{v^{1/3}} \cdot u^{1/3} \frac{2}{3} \cancel{v^{-1/3}} - \frac{1}{3} u^{-2/3} \cancel{v^{2/3}} \cdot \frac{1}{3} \cancel{v^{-2/3}} u^{-1/3}$$

$$= -\frac{2}{9} u^{-1} - \frac{1}{9} u^{-1} = -\frac{1}{3} u^{-1} \qquad (u,v) \in [1,2] \times [1,4]$$

$$\text{Area}(D) = \int_{[1,2]\times[1,4]} \frac{1}{3} u^{-1} \, du dv = \frac{1}{3} \int_1^2 \left(\int_1^4 u^{-1} dv \right) du =$$

$$= \frac{1}{3} \int_1^2 \left(u^{-1} v \Big|_{v=1}^{v=4} \right) du = \cancel{\frac{1}{3}} \int_1^2 u^{-1} \, \cancel{(4-1)} \, du = \ln|u| \Big|_{u=1}^{u=2} = \ln(2)$$

$$\int_D x^3 y^2 \, dx dy = \int_{[1,2]\times[1,4]} v \cdot u^{-1} \left(u^{1/3} v^{2/3} \right)^2 \frac{1}{3} u^{-1} \, du dv =$$

$$= \int_1^2 \left(\int_1^4 \frac{1}{3} v \, \underbrace{u^{-2} \cdot u^{2/3}}_{} \, v^{4/3} \, dv \right) du = \frac{1}{3} \int_1^2 \left(\int_1^4 u^{-4/3} v^{7/3} dv \right) du =$$

$$= \frac{1}{3} \int_1^2 \left(u^{-4/3} \frac{1}{\frac{7}{3}+1} \underset{\substack{\shortparallel \\ v^{10/3}}}{v^{\frac{7}{3}+1}} \Big|_{v=1}^{v=4} \right) du = \frac{1}{\cancel{3}} \frac{\cancel{3}}{10} \int_1^2 u^{-4/3} \left(4^{\frac{10}{3}} - 1 \right) du$$

$$= \frac{1}{} \left(4^{\frac{10}{3}} - 1 \right) \frac{1}{\phantom{-\frac{4}{3}+1}} u^{-\frac{4}{3}+1} \Big|^{u=2} = \frac{1}{} \left(64 \sqrt[3]{4} - 1 \right) (-3) \left(2^{-1/3} - 1 \right.$$

$$= \frac{1}{10}\left(4^{\frac{10}{3}}-1\right)\frac{1}{-\frac{4}{3}+1}\, u^{-\frac{4}{3}+1}\Big|_{u=1}^{u=2} = \frac{1}{10}\left(64\sqrt[3]{4}-1\right)(-3)\left(2^{-1/3}-1\right)$$

$\sqrt{\ }^{1/3}$

$$\frac{10}{3} = 3+\frac{1}{3} \qquad 4^3 = 64 \qquad = \frac{3}{10}\left(64\sqrt[3]{4}-1\right)\left(1-\frac{1}{\sqrt[3]{2}}\right)$$

$$r(t) = e^t\cos(t)\,\underline{i} + e^t\sin(t)\,\underline{j} + e^t\,\underline{k} \qquad t\in\mathbb{R}$$

$$s(t) = \int_0^t \|\dot r(\tau)\|\,d\tau \qquad t = t(s)$$

$$\dot r(t) = \left(e^t\cos(t) - e^t\sin(t)\right)\underline{i} + \left(e^t\cos(t) + e^t\sin(t)\right)\underline{j} + e^t\,\underline{k}$$

$$\|\dot r(t)\|^2 = \left(e^t\left(\cos(t)-\sin(t)\right)\right)^2 + \left(e^t\left(\cos(t)+\sin(t)\right)\right)^2 + e^{2t}$$

$$= e^{2t}\left(\underbrace{\cos^2(t)+\sin^2(t)}_{1}\cancel{-2\sin(t)\cos(t)} + \underbrace{\cos^2(t)+\sin^2(t)}_{1}\cancel{+2\sin(t)\cos(t)} + 1\right)$$

$$= 3e^{2t} = \left(\sqrt{3}e^t\right)^2$$

$$\to s(t) = \int_0^t \sqrt{3}\, e^{\tau}\,d\tau = \sqrt{3}\, e^{\tau}\Big|_{\tau=0}^{\tau=t} = \sqrt{3}\left(e^t-1\right)$$

$$s = \sqrt{3}e^t - \sqrt{3} \qquad e^t = \frac{s+\sqrt{3}}{\sqrt{3}} \qquad t = \ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)$$

$$r(t) = e^t\cos(t)\,\underline{i} + e^t\sin(t)\,\underline{j} + e^t\,\underline{k}$$

$$\to r(t(s)) = \frac{s+\sqrt{3}}{\sqrt{3}}\cos\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right)\underline{i} + \frac{s+\sqrt{3}}{\sqrt{3}}\sin\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right)\underline{j} + \frac{s+\sqrt{3}}{\sqrt{3}}\,\underline{k}$$

$$r(t) = e^t\,\underline{i} + e^{-t}\,\underline{j} + t\sqrt{2}\,\underline{k}$$

$$\dot r(t) = e^t\,\underline{i} - e^{-t}\,\underline{j} + \sqrt{2}\,\underline{k}$$

$$\|\dot r(t)\|^2 = e^{2t} + e^{-2t} + 2 = \left(e^t + e^{-t}\right)^2$$

$$s(t) = \int_0^t \|\dot r(\tau)\|\,d\tau = \int_0^t \left(e^{\tau}+e^{-\tau}\right)d\tau = \left(e^{\tau}-e^{-\tau}\right)\Big|_{\tau=0}^{\tau=t} = e^t - e^{-t}$$

$$s = e^t - e^{-t}$$

$$e^t - s - e^{-t} = 0 \qquad e^{2t} - s e^t - 1 = 0$$

$e^t = w > 0$ $\Downarrow$ $t = \ln(w)$

$$w^2 - sw - 1 = 0 \qquad \Delta = s^2 + 4 > 0$$

$$w_{1,2} = \frac{s \pm \sqrt{s^2+4}}{2} \quad \begin{cases} \dfrac{s+\sqrt{s^2+4}}{2} \\ \dfrac{s-\sqrt{s^2+4}}{2} \end{cases}$$

$ax^2+bx+c=0$

$a \neq 0$ $\quad \Delta > 0$

$a \quad b \quad c$

1	$-s$	-1
$+$ V $-$	P	$-$
$+$ P $+$	V	$-$

$s>0$ 1 radice positiva, 1 radice negativa

$s<0$ 1 radice positiva, 1 radice negativa

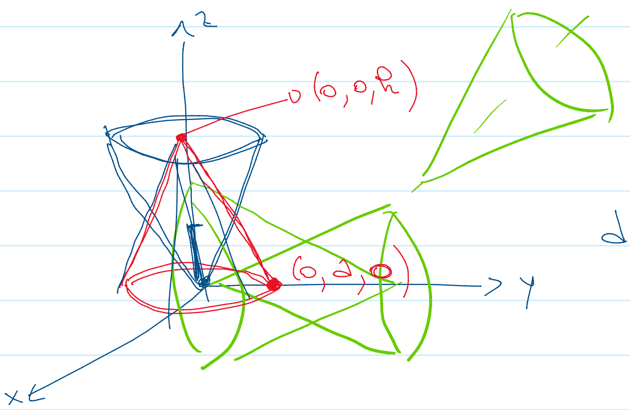
$$w = \frac{s+\sqrt{s^2+4}}{2} \qquad e^t = \frac{s+\sqrt{s^2+4}}{2}$$

$$t = \ln\left(\frac{s+\sqrt{s^2+4}}{2}\right) \qquad e^{-t} = \frac{2}{s+\sqrt{s^2+4}}$$

$$r(t) = e^t \underline{i} + e^{-t} \underline{j} + t\sqrt{2}\, \underline{k}$$

$$= \frac{s+\sqrt{s^2+4}}{2}\, \underline{i} + \frac{2}{s+\sqrt{s^2+4}}\, \underline{j} + \sqrt{2}\ln\left(\frac{s+\sqrt{s^2+4}}{2}\right) \underline{k}$$

Calcolare baricentro



$d((x,y,z), \text{base}) = |z|$

Sul piano $x=0$ ho la retta per $(0,h)$ e $(a,0)$

$$\frac{y-0}{a-0} = \frac{z-h}{0-h} \qquad \frac{y}{a} = \frac{-1}{h}(z-h) \qquad z-h = -\frac{h}{a} y$$

$$\begin{cases} x = 0 \\ \boxed{z = h - \frac{h}{a} y} \end{cases} \qquad \begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ \boxed{z = h - \frac{h}{a} r} \end{cases}$$

$r \geq 0$ $\varphi \in [0, 2\pi]$

$r \in [0, a]$

$0 \leq z \leq h$

$0 \leq h - \frac{h}{a} r \leq h$

$0 \leq 1 - \frac{r}{a} \leq 1$

$-1 \leq \frac{r}{a} - 1 \leq 0$

$0 \leq \frac{r}{a} \leq 1$ $\boxed{0 \leq r \leq a}$ $\leftarrow$

$$0 \leq z \leq h - \frac{h}{a} r = \frac{h}{a}(a - r)$$

$$C = \left\{ (x, y, z) : \underline{x = r\cos\varphi,\ y = r\sin\varphi,\ 0 \leq z \leq \frac{h}{a}(a-r)} \quad r \in [0, a],\ \varphi \in [0, 2\pi] \right\}$$

Il cono è riempito di un materiale con densità proporzionale alla distanza dalla base

$\longrightarrow \varrho(x, y, z) = K|z| = Kz \qquad K > 0$

$$\text{MASSA} = \int_C \varrho(x,y,z)\,dx\,dy\,dz = \int_C Kz\,dx\,dy\,dz =$$

$$= \int_{[0,a]\times[0,2\pi]} \left(\int_0^{h/a(a-r)} r \cdot Kz\,dz \right) dr\,d\varphi =$$

$$\int_0^a \left(\int_0^{2\pi} \left(\int_0^{h/a(a-r)} Krz\,dz \right) d\varphi \right) dr =$$

$$= \int_0^a \left(\int_0^{2\pi} \frac{K}{2} r z^2 \Big|_{z=0}^{z=\frac{h}{a}(a-r)} d\varphi \right) dr =$$

$$= \int_0^a \int_0^{2\pi} \frac{K}{2} r \frac{h^2}{a^2} (r^2 - 2ar + a^2) d\varphi\, dr =$$

$$= 2\pi \int_0^a \frac{Kh^2}{2a^2} (r^3 - 2ar^2 + a^2 r) dr = \frac{2\pi K h^2}{2a^2} \left(\frac{r^4}{4} - \frac{2}{3} a r^3 + \frac{a^2 r^2}{2} \right) \Big|_{r=0}^{r=a}$$

$$= \frac{\cancel{2}\pi K h^2}{\cancel{2} a^2} \left(\frac{a^4}{4} - \frac{2}{3} a^4 + \frac{a^4}{2} \right) = \frac{\pi K h^2}{\cancel{a^2}} a^{\cancel{4}\,2} \left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2} \right)$$

$$= \frac{\pi K h^2 a^2}{12}$$

$$\frac{1}{4} - \frac{2}{3} + \frac{1}{2} = \frac{3 - 8 + 6}{12}$$

$$= \frac{\pi K R^2 a^2}{12}$$

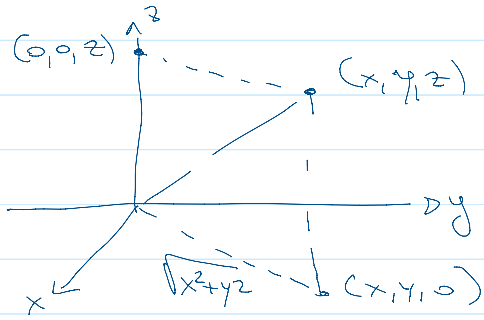
$$\frac{1}{4} - \frac{2}{3} + \frac{1}{2} = \frac{3-8+6}{12}$$

$$x_B = \frac{1}{MASSA} \int_C \underbrace{x\, k z}\, dx\,dy\,dz = 0 \qquad y_B = \frac{1}{MASSA} \int_C \underbrace{y\, k z}\, dx\,dy\,dz = 0$$

$$z_B = \frac{1}{MASSA} \int_C z\, k z\, dx\,dy\,dz = \frac{12}{\pi \cancel{K} R^2 a^2} \cancel{K} \int_C z^2\, dx\,dy\,dz$$

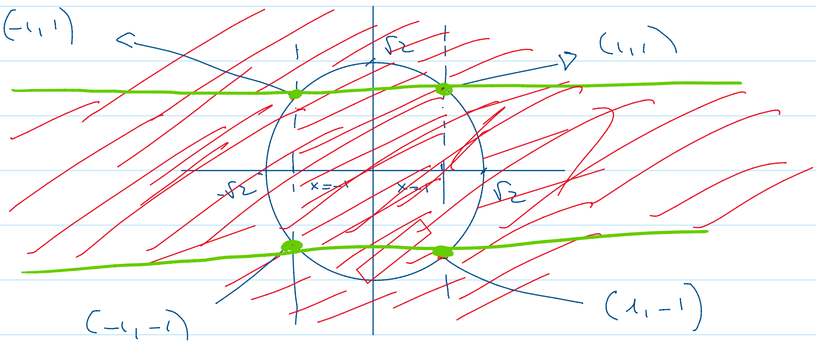
momento d' inerzia rispetto all' asse z

$$\int_C \varrho(x,y,z)\, d^2\big((x,y,z), \text{asse } z\big)\, dx\,dy\,dz = \int_C kz(x^2+y^2)\, dx\,dy\,dz$$



$f(x,y) = x$

$D = \{(x,y) \in \mathbb{R}^2 : |x| \geq 1,\ |y| \geq 1,\ x^2+y^2 \leq 2\} = \{(1,1), (1,-1), (-1,-1), (-1,1)\}$



$|x| = 1 \qquad x = 1 \ , \ x = -1$

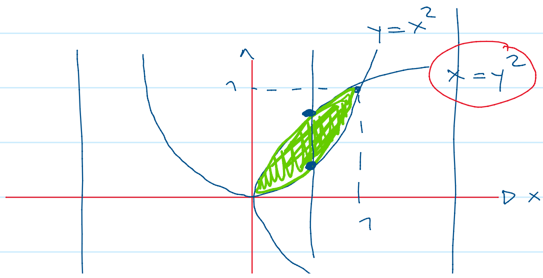
$|x| \geq 1, \qquad x \geq 1 \ \vee \ x \leq -1$

$|y| = 1 \qquad y = 1 \qquad y = -1$

$|y| \geq 1 \qquad y \geq 1 \ \vee \ y \leq -1$

$\mathcal{L}^1(D) = 0 \implies \int_D f(x,y)\,dx\,dy = 0$

$D = \{(x,y) \in \mathbb{R}^2 : \ y \geq x^2,\ x \geq y^2\} \qquad f(x,y) = xy^3$



$y > 0$

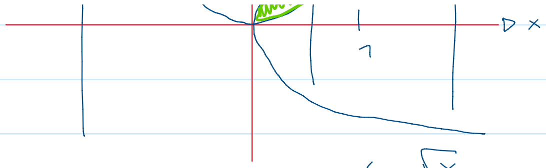
$y = \sqrt{x}$

$y = x^2 \qquad y \geq x^2$

$x = y^2 \qquad x \geq y^2$

$x < 0 \ \vee \ x > 1$

$E_x = \{ \emptyset$



$$E_x = \begin{cases} \emptyset & x<0 \lor x>1 \\ [x^2, \sqrt{x}] & x \in [0,1] \end{cases}$$

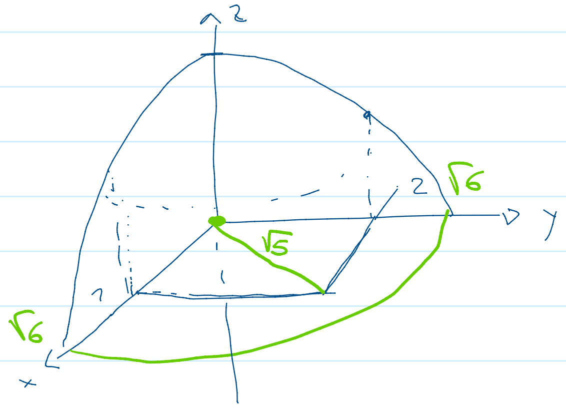
$$\int_D x y^3 \, dx\, dy = \int_0^1 \left(\int_{x^2}^{\sqrt{x}} x y^3 \, dy \right) dx =$$

$$= \int_0^1 \frac{x}{4} y^4 \Big|_{y=x^2}^{y=\sqrt{x}} dx = \frac{1}{4} \int_0^1 x \left(x^2 - x^8 \right) dx = \frac{1}{4} \int_0^1 \left(x^3 - x^9 \right) dx =$$

$$= \frac{1}{4} \left(\frac{x^4}{4} - \frac{x^{10}}{10} \right) \Big|_{x=0}^{x=1} = \frac{1}{4} \left(\frac{1}{4} - \frac{1}{10} \right) = \frac{1}{4} \, \frac{5-2}{20} = \frac{3}{80}$$

$E = \{ (x,y,z) \in \mathbb{R}^3 : \ 0 \le x \le 1, \ 0 \le y \le 2, \ 0 \le z \le 6 - x^2 - y^2 \}$

Disegnare E $\qquad \int_E x \ln(1+y) \, dx\, dy\, dz$

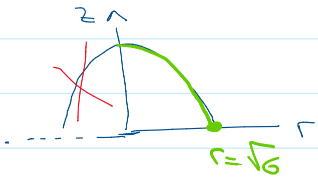


$z \geq 0$

$z = 6 - \underbrace{(x^2+y^2)}$

$x = r\cos\varphi, \quad y = r\sin\varphi$

$z = 6 - r^2$



$\forall (x,y) \in [0,1] \times [0,2] \qquad E_{(x,y)} = [0, \ 6 - (x^2+y^2)]$

$$\int_{[0,1]\times[0,2]} \left(\int_0^{6-(x^2+y^2)} \underline{x \ln(1+y) \, dz} \right) dx\, dy =$$

$$= \int_{[0,1]\times[0,2]} x \ln(1+y) \left(6 - x^2 - y^2 \right) dx\, dy =$$

$$= \int_0^1 \left(\int_0^2 \underbrace{\ln(1+y)}_{f(y)} \underbrace{\left(6x - x^3 - x y^2 \right)}_{g'(y)} dy \right) dx$$

$$= \int_0^1 \left[\ln(1+y) \left[(6x - x^3) y - \frac{x}{3} y^3 \right] \Big|^{y=2} - \int \frac{(6x - x^3) y - \frac{x}{3} y^3}{} dy \right] dx$$

$$= \int_0^1 \left[\ln(1+y) \left((6x-x^3)y - \frac{x}{3}y^3 \right) \Big|_{y=0}^{y=2} - \int_0^2 \frac{(6x-x^3)y - \frac{x}{3}y^3}{1+y}\, dy \right] dx$$

$$= \int_0^1 \left(\ln(3)\left(2(6x-x^3) - \frac{8}{3}x\right) + \int_0^2 \frac{\frac{x}{3}y^3 - (6x-x^3)y}{y+1}\, dy \right) dx$$

$$\begin{array}{l|l}
\cancel{\frac{x}{3}y^3} + 0y^2 + (x^3-6x)y + 0 & y+1 \\ \cline{2-2}
\cancel{-\frac{x}{3}y^3} \quad -\frac{x}{3}y^2 & \frac{x}{3}y^2 - \frac{x}{3}y + \left(x^3 - \frac{17}{3}x\right) \\ \cline{1-1}
\cancel{-\frac{x}{3}y^2} + (x^3-6x)y + 0 & \\
\cancel{+\frac{x}{3}y^2} \quad \frac{x}{3}y & \\ \cline{1-1}
\cancel{\left(x^3-\frac{17}{3}x\right)y} + 0 & \\
\cancel{-\left(x^3-\frac{17}{3}x\right)y} - x^3 + \frac{17}{3}x & \\ \cline{1-1}
-x^3 + \frac{17}{3}x &
\end{array}$$

$$\int_0^1 \left(\ln(3)\left(\frac{28}{3}x - 2x^3\right) + \int_0^2 \left(\frac{x}{3}y^2 - \frac{x}{3}y + x^3 - \frac{17}{3}x + \frac{-x^3+\frac{17}{3}x}{y+1}\right) dy \right) dx =$$

$$\int_0^1 \ln(3)\left(\frac{28}{3}x - 2x^3\right) + \left(\frac{x}{9}y^3 - \frac{x}{6}y^2 + \left(x^3 - \frac{17}{3}x\right)y + \left(-x^3+\frac{17}{3}x\right)\ln|1+y|\right)\Big|_{y=0}^{y=2} dx$$

$$= \int_0^1 \left(\ln(3)\left(\frac{28}{3}x-2x^3\right) + \frac{8}{9}x - \frac{2}{3}x + 2x^3 - \frac{34}{3}x + \ln(3)\left(-x^3+\frac{17}{3}x\right)\right) dx$$

$$= \ln(3)\left(\frac{28}{6}x^2 - \frac{1}{2}x^4\right) + \frac{4}{9}x^2 - \frac{1}{3}x^2 + \frac{1}{2}x^4 - \frac{17}{3}x^2 + \ln(3)\left(-\frac{x^4}{4} + \frac{17}{6}x^2\right) \Big|_{x=0}^{x=1}$$

= $x=0$ va via tutto, basta sostituire $x=1$

$$f(x,y,z) = \ln(x^2-y^2+z^2) + x - z + y$$

$$D = \{(x,y,z)\in\mathbb{R}^3 : x^2-y^2+z^2>0\} \qquad |y| < \sqrt{x^2+y^2}$$

$$x^2-y^2+z^2>0 \qquad \boxed{y^2 < x^2+z^2} \qquad t^2 < r^2$$

$x^2-y^2+z^2>0$

$$\boxed{y^2 < x^2+z^2}$$

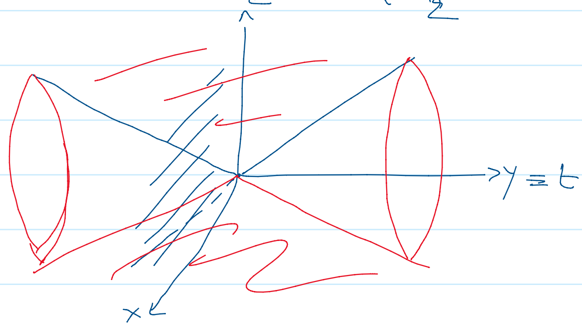
$t^2 < r^2$

$x = r\cos\varphi$

$y = t$

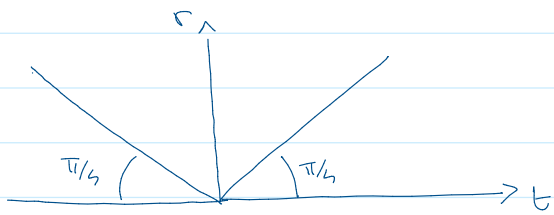
$z = r\sin\varphi$

$z = r$ se $\varphi = \pi/2$



$t^2 = r^2$

$t = r \quad \vee \quad t = -r$



$\varphi = \pi/2 \Rightarrow z = r \quad y = t$

$t - f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0) \cdot (x-x_0,\ y-y_0,\ z-z_0)$

è del tipo $t =$ costante SSE $\nabla f(x_0, y_0, z_0) = (0,0,0)$

$$\begin{cases} x^2-y^2+z^2>0 \\ f_x(x,y,z)=0 \\ f_y(x,y,z)=0 \\ f_z(x,y,z)=0 \end{cases}$$

$f(x,y,z) = \ln(x^2-y^2+z^2) + x - z + y$

$f_x = \dfrac{2x}{x^2-y^2+z^2} + 1$

$f_y = \dfrac{-2y}{x^2-y^2+z^2} + 1$

$f_z = \dfrac{2z}{x^2-y^2+z^2} - 1$

$$\begin{cases} x^2-y^2+z^2>0 \\ 2x + (x^2-y^2+z^2) = 0 \\ -2y + (x^2-y^2+z^2) = 0 \\ 2z - (x^2-y^2+z^2) = 0 \end{cases}$$

$$\begin{cases} x^2-y^2+z^2>0 \\ 2x+2y=0 \\ 2x+2z=0 \\ -2y+2z=0 \\ 2x+(x^2-y^2+z^2)=0 \end{cases}$$

$y = -x$

$z = -x$

$+2x - 2x = 0$

$$\begin{cases} x^2-y^2+z^2>0 \\ y=-x \\ z=-x \\ 2x + (\cancel{x^2} - \cancel{x^2} + x^2) = 0 \end{cases}$$

$2x + x^2 = 0$

$x(x+2) = 0$

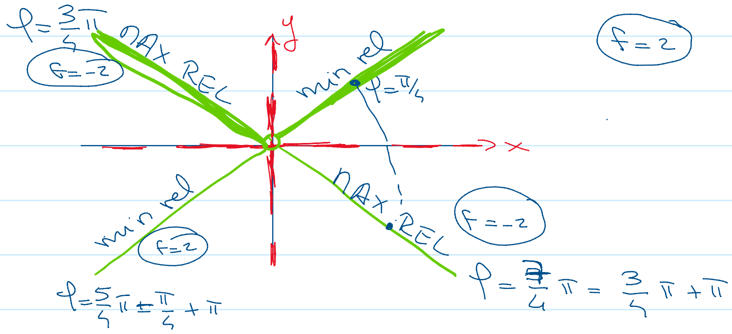
$x = 0 \qquad x = -2$

$$\begin{cases} x^2-y^2+z^2>0 \\ \cancel{(0,0,0)} \\ \boxed{(-2,2,2)} \end{cases}$$

$4-4+4>0$ sì

$$f(x,y) = \frac{x}{y} + \frac{y}{x} = \frac{x^2+y^2}{xy}$$

$$D = \{(x,y) \in \mathbb{R}^2 : x \neq 0,\ y \neq 0\}$$



$$\nabla f(x,y) = (0,0)$$

$$f_x = \frac{1}{y} - \frac{y}{x^2} = \frac{x^2-y^2}{x^2y}$$

$$f_y = \frac{-x}{y^2} + \frac{1}{x} = \frac{-x^2+y^2}{xy^2}$$

$$\begin{cases} x \neq 0 \\ y \neq 0 \\ x^2 - y^2 = 0 \\ -x^2+y^2 = 0 \end{cases}$$

$$x^2 - y^2 = 0 \qquad (x-y)(x+y) = 0$$

$$y = x \ \vee\ y = -x$$

$$f(x,y) = \frac{x}{y} + \frac{y}{x}$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$\frac{x}{y} + \frac{y}{x} = \frac{r\cos\varphi}{r\sin\varphi} + \frac{r\sin\varphi}{r\cos\varphi} =$$

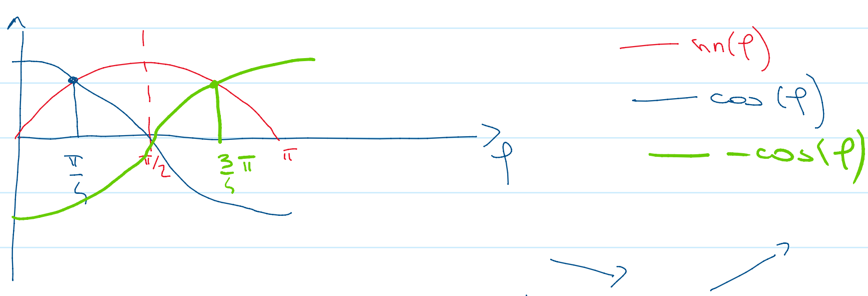
$$= \frac{\cos\varphi}{\sin\varphi} + \frac{\sin\varphi}{\cos\varphi} = g(\varphi)$$

$$\varphi \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3}{2}\pi\right) \cup \left(\frac{3}{2}\pi, 2\pi\right)$$

$$\boxed{g(\varphi) = \cot g(\varphi) + tg(\varphi)}$$

$$\varphi \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$$

$$g'(\varphi) = \frac{-1}{\sin^2(\varphi)} + \frac{1}{\cos^2(\varphi)} = \frac{\sin^2(\varphi) - \cos^2(\varphi)}{\cos^2(\varphi) \cdot \sin^2(\varphi)} = \frac{(\sin(\varphi) - \cos(\varphi))(\sin(\varphi) + \cos(\varphi))}{\cos^2(\varphi)\sin^2(\varphi)}$$



	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$		$\frac{3}{4}\pi$		π
$\sin\varphi - \cos\varphi$		$-$		$+$		$+$		$+$	
$\sin\varphi + \cos\varphi$		$+$		$+$		$+$		$-$	
$g'(\varphi)$		$-$		$+$		$+$		$-$	

$g(\varphi) = \cot g(\varphi) + Tg(\varphi)$

$g\left(\frac{\pi}{4}\right) = 1 + 1 = 2$ $\qquad$ $g\left(\frac{3}{4}\pi\right) = -1 - 1 = -2$