martedì 17 dicembre 2019 14:06

$\mathbb{P}_{X_i} = N(170, 100)$ unità di misura = cm.

$\mathbb{P}(X_i > 175) = \mathbb{P}\left(\frac{X_i - 170}{10} > \frac{175-170}{10}\right) = \mathbb{P}(Z > 0.5)$

$= 1 - \mathbb{P}(Z \leq 0.5) = 1 - \Phi\left(\frac{1}{2}\right) \simeq 1 - 0.69146 \cong 0.32$

— o —

$X_1 ——— X_{10}$ $\overline{X}_{10}$

$\mathbb{P}_{X_i} = N(170, 100)$ $\mathbb{P}_{\overline{X}_{10}} = N(170, 10)$

$\mathbb{P}(\overline{X}_{10} > 175) = \mathbb{P}\left(\frac{\overline{X}_{10} - 170}{\sqrt{10}} > \frac{175-170}{\sqrt{10}}\right) =$

$\mathbb{P}\left(Z > \frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}\right) = \mathbb{P}\left(Z > \frac{1}{2}\sqrt{10}\right) \simeq \mathbb{P}(Z > 1.58)$

$= 1 - \Phi(1.58) \simeq 1 - 0.94295 \simeq 0.06$

— o —

$X_1 —— X_{100}$ $\mathbb{P}(\overline{X}_{100} > 175)$

$\mathbb{P}_{X_i} = N(170, 100)$ $\mathbb{P}_{\overline{X}_{100}} = N(170, 1)$

$\mathbb{P}(\overline{X}_{100} > 175) = \mathbb{P}\left(\frac{\overline{X}_{100} - 170}{1} > \frac{175-170}{1}\right) = \mathbb{P}(Z > 5)$

$= 1 - \Phi(5) \simeq 0$.

— o —

$\underbrace{Y_1 —— Y_{200}}$ $\{2, 4, 6\}$ $f_2 = 68$ $f_4 = 72$ $f_6 = 60$

$\alpha = 0.05$ $1 - \alpha = 0.95$ $k = 3$ $p_2^0 = p_4^0 = p_6^0 = \frac{1}{3}$

$\frac{(f_2 - np_2^0)^2}{} + \frac{(f_4 - np_4^0)^2}{} + \frac{(f_6 - np_6^0)^2}{} < \chi^2$

$$\frac{(f_2 - np_2^0)^2}{np_2^0} + \frac{(f_4 - np_4^0)^2}{np_4^0} + \frac{(f_6 - np_6^0)^2}{np_6^0} < \chi^2_{2,\,0.95} \simeq \boxed{5.9915}$$

$$\frac{\left(68 - \frac{200}{3}\right)^2}{200\cdot\frac{1}{3}} + \frac{\left(72 - 200\cdot\frac{1}{3}\right)^2}{200\cdot\frac{1}{3}} + \frac{\left(60 - 200\cdot\frac{1}{3}\right)^2}{200\cdot\frac{1}{3}} =$$

$$= \frac{3}{200}\left[\left(\frac{204-200}{3}\right)^2 + \left(\frac{216-200}{3}\right)^2 + \left(\frac{180-200}{3}\right)^2\right] =$$

$$= \frac{3}{200}\cdot\frac{1}{9}\left[16 + 256 + 400\right] = \frac{1}{3\cdot 200}(672) =$$

$$= \frac{1}{3\cdot 100}\,336 < 5.9915$$

——— o ———

$Y_1, Y_2, \ldots, Y_{1000}$ $\quad \{0,1,2,3,4\}$

$f_0 = 59 \quad f_1 = 251 \quad f_2 = 377 \quad f_3 = 244 \quad f_4 = 69$

$\alpha = 0.05 \quad P_{Y_i} = Bin\left(4, \frac{1}{2}\right)$

$$Bin\left(4,\tfrac{1}{2}\right)(\{k\}) = \binom{4}{k}\left(\frac{1}{2}\right)^k\left(1-\frac{1}{2}\right)^{4-k} = \binom{4}{k}\left(\frac{1}{2}\right)^4$$

$$p_0^0 = p_4^0 = \binom{4}{0}\left(\frac{1}{2}\right)^4 = \frac{1}{16} \qquad p_2^0 = \binom{4}{2}\left(\frac{1}{2}\right)^4 = \frac{6}{16} = \frac{3}{8}$$

$$p_1^0 = p_3^0 = \binom{4}{1}\left(\frac{1}{2}\right)^4 = 4\cdot\frac{1}{16} = \frac{1}{4}$$

$$\sum_{i=0}^{4}\frac{(f_i - np_i^0)^2}{np_i^0} < \chi^2_{4,\,0.95} \simeq 9.4877$$

$$\frac{\left(59 - \cancel{1000}^{\,125}\cdot\frac{1}{\cancel{16}_{\,2}}\right)^2}{1000\cdot\frac{1}{\ldots}} + \frac{\left(251 - \cancel{1000}^{\,250}\cdot\frac{1}{\cancel{4}}\right)^2}{1000\cdot\frac{1}{\ldots}} + \frac{\left(377 - \cancel{1000}^{\,125}\cdot\frac{3}{\cancel{8}}\right)^2}{1000\cdot\frac{3}{\ldots}}$$

$$\frac{\ldots}{1000\cdot\frac{1}{16}} + \frac{\ldots}{1000\cdot\frac{1}{4}} + \frac{\ldots}{1000\cdot\frac{3}{8}}$$

$$+\frac{\left(244-\cancel{1000}^{\,250}\cdot\frac{1}{\cancel{4}}\right)^2}{1000\cdot\frac{1}{4}} + \frac{\left(69-\cancel{1000}^{\,125}\cdot\frac{1}{\cancel{16}_{\,2}}\right)^2}{1000\cdot\frac{1}{16}} =$$

$$=\frac{1}{1000}\left(\cancel{16}^{\,4}\cdot\frac{49}{\cancel{4}} + 4\cdot 1^2 + \frac{8}{3}\cdot 2^2 + 4\cdot 6^2 + \frac{\cancel{16}^{\,4}\cdot 13^2}{\cancel{4}}\right)$$

$$=\frac{4}{1000}\left(49+1+\frac{8}{3}+36+169\right) = \frac{138}{250} < 9.4877$$

—— o ——

$n=20$ $\quad \bar{x}=19.89$ $\quad s=0.4666365$

$\alpha=0.1$ $\quad H_0)\ \mu_0=20$ $\quad \Big|\ H_0)\ \sigma_0=0.5$

$\sigma_0^2=0.25$

$$\mu_0-\frac{t_{n-1,1-\frac{\alpha}{2}}\, s}{\sqrt{n}} \le \bar{x} \le \mu_0+\frac{t_{n-1,1-\frac{\alpha}{2}}\cdot s}{\sqrt{n}} \qquad t_{19,0.95}\simeq 1.7291$$

$$20-\underbrace{\frac{1.7291\cdot 0,4666365}{\sqrt{20}}}_{0.18} \le 19.89 \le 20+\underbrace{\frac{1.7291\cdot 0.4666365}{\sqrt{20}}}_{0.18}$$

$19.82 \le 19.89 \le 20.18$ ok.

—— o ——

$$\chi^2_{n-1,\frac{\alpha}{2}}\,\frac{\sigma_0^2}{n-1} < s^2 < \frac{\sigma_0^2}{n-1}\cdot\chi^2_{n-1,1-\frac{\alpha}{2}} \qquad \alpha=0.1$$

$\chi^2_{19,0.05}\simeq 10.1170$ $\qquad \chi^2_{19,0.95}\simeq 30.1435$

$$10{,}1170 \cdot \frac{0.25}{19} < (0.4666365)^2 < 30{,}1435 \cdot \frac{0.25}{19}$$

$$\begin{array}{ccccc} \wr\wr & & \wr\wr & & \wr\wr \\ 0.1331 & & 0.2178 & & 0.3966 \end{array}$$

$n = 10$ $\quad \bar{x} = 10{,}01063$ $\quad S = 2.281742$

$\alpha = 0.05$ $\quad H_0)\ \mu_0 = 10$ $\quad H_0)\ \sigma_0 \leq 2$

$\sigma_0^2 \leq 4$

$$\mu_0 - \frac{t_{n-1,1-\frac{\alpha}{2}} \cdot S}{\sqrt{n}} < \bar{x} < \mu_0 + \frac{t_{n-1,1-\frac{\alpha}{2}} \cdot S}{\sqrt{n}}$$

$$t_{9,0.975} \simeq 2.2622$$

$$10 - \underbrace{\frac{2{,}2622 \cdot 2{,}281742}{\sqrt{10}}}_{1{,}63226} < 10.01063 < 10 + \underbrace{\frac{2{,}2622 \cdot 2{,}281742}{\sqrt{10}}}_{\simeq 1.63226}$$

$$S^2 \leq \frac{\sigma_0^2}{n-1} \chi^2_{n-1,1-\alpha} = \frac{4}{9} \cdot \chi^2_{9,0.95}$$

$$(2.281742)^2 < \frac{4}{9} 16.9196$$

$$\simeq 5.206347 < 7.519546$$