

Un po' di conti con R

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Consideriamo la matrice stocastica P

	1	2	3	4	5
1	1	0	0	0	0
2	0.2	0.8	0	0	0
3	0	0	0.2	0.3	0.5
4	0	0	0	0.6	0.4
5	0	0.2	0.4	0.4	0

```
CalcolaB=function(P){  
  X = diag(nrow(P))  
  Y = X  
  massima = nrow(P)-1  
  for(potenza in 1:massima)  
  {  
    X <- X %*% P  
    Y <- Y + X  
  }  
  return(Y)  
}
```

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	5.0000	0.0000	0.0000	0.0000	0.0000
[2,]	1.0214	1.4251	0.0000	0.0000	0.0000
[3,]	0.0548	0.2554	1.7216	1.5776	1.2536
[4,]	0.0464	0.2312	0.4288	3.0032	1.1744
[5,]	0.1620	0.4138	0.7456	1.4960	1.7776

```
> xtable(B, type = "latex", file = "B.tex")
```

	1	2	3	4	5
1	5.00	0	0	0	0
2	1.64	3.36	0	0	0
3	0.06	0.38	1.72	1.58	1.25
4	0.05	0.34	0.43	3.00	1.17
5	0.22	0.76	0.75	1.50	1.78

$$C_1 = \{1\}, \quad T = \{2, 3, 4, 5\},$$

P matrice stocastica 8×8

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0.5	0.5	0	0	0	0	0
4	0	0	0	0.6	0.4	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.53	1.97	2.40	0.79	0.25	0.07	0	0
2	0.18	2.57	3.46	1.23	0.42	0.14	0	0
3	0.08	2.10	4.60	0.86	0.27	0.08	0	0
4	0.90	0.64	0.96	3.64	1.36	0.50	0	0
5	1.26	1.14	1.72	1.68	1.59	0.61	0	0
6	2.47	1.72	1.97	0.62	0.18	1.05	0	0
7	0.73	0.45	0.69	3.41	1.26	0.46	1.00	0
8	0.55	0.28	0.46	3.15	1.15	0.40	1.00	1.00

$$C_1 = \{1, 2, 3, 4, 5, 6\} \quad T = \{7, 8\}$$

P matrice stocastica 8×8

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0.5	0.5	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.46	1.94	2.33	1.27	0	0	0	0
2	0	2.47	3.29	2.24	0	0	0	0
3	0	2.06	4.52	1.42	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	0.95	1.43	3.27	1.00	0.40	0	0
6	2.43	1.71	1.93	0.93	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

$$C_1 = \{4\}, \quad T = \{1, 2, 3, 5, 6, 7, 8\}$$

P

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	1	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.46	2.42	1.65	1.48	0	0	0	0
2	0	2.95	2.36	2.69	0	0	0	0
3	0	2.95	2.95	2.10	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	1.21	1.03	3.40	1.00	0.40	0	0
6	2.43	2.06	1.44	1.07	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

Anche qui $C_1 = \{4\}$ e tutti gli altri stati sono transienti.

	1	2	3
1	0.60	0.40	0.00
2	0.80	0.20	0.00
3	0.00	0.30	0.70

	[,1]	[,2]	[,3]
[1,]	2.28	0.72	0.00
[2,]	1.44	1.56	0.00
[3,]	0.24	0.57	2.19

```
> xtable(CalcolaB(Q))
```

B

	1	2	3
1	2.28	0.72	0.00
2	1.44	1.56	0.00
3	0.24	0.57	2.19

$$C_1 = \{1, 2\}, \quad T = \{3\}$$

```
Markov=function(P, nrepl, passi){  
  omega =1  
  Y = vector( length = passi)  
  for(omega in 1:nrepl)  
  {  
    xi=runif(passi, min=0, max=1)  
    dim(xi)=c(passi,1)  
    passo = 0  
    i <- sample(1:nrow(P), size = 1)  
    print(paste("Sorteggio X[0](omega)", i))  
    X = matrix(, ncol = passi, nrow = nrepl)  
    arresto = 0  
    k=1  
    j=1
```

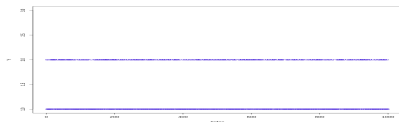
```
while(arresto < xi[1])  
{  
    arresto = arresto + P[i,j]  
    X[omega,1] = j  
    Y[1] = X[omega,1]  
    j= j+1  
}  
for(k in 2:passi)  
{
```

```

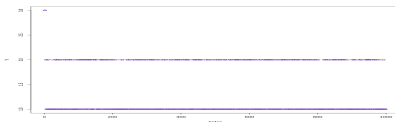
i=X[omega,k-1]
arresto = 0
j=1
while(arresto < xi[k])
{
    arresto = arresto + P[i,j]
    X[omega,k] = j
    j=j+1
}
Y[k] <- X[omega,k]
}
print(Y)
rosso = omega/nrepl
blu = 1 - rosso
plot(Y, col = rgb(rosso,0,blu))
}
}

```

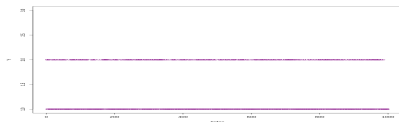
Simulazioni



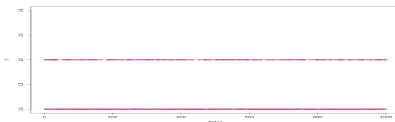
(a) $X_0(\omega) = 3$



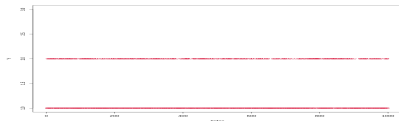
(b) $X_0(\omega) = 3$



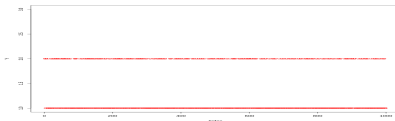
(c) $X_0(\omega) = 3$



(d) $X_0(\omega) = 3$



(e) $X_0(\omega) = 2$



(f) $X_0(\omega) = 1$

P

	1	2	3	4	5
1	1.00	0.00	0.00	0.00	0.00
2	0.20	0.30	0.00	0.00	0.00
3	0.00	0.00	0.20	0.30	0.50
4	0.00	0.00	0.00	0.60	0.40
5	0.00	0.20	0.40	0.40	0.00

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	5.0000	0.0000	0.0000	0.0000	0.0000
[2,]	1.6384	3.3616	0.0000	0.0000	0.0000
[3,]	0.0648	0.3824	1.7216	1.5776	1.2536
[4,]	0.0544	0.3392	0.4288	3.0032	1.1744
[5,]	0.2240	0.7568	0.7456	1.4960	1.7776

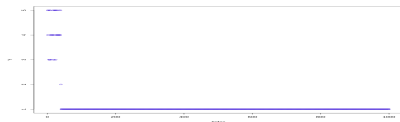
```
> xtable(CalcolaB(R))
```

B

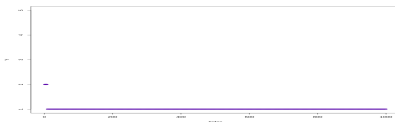
	1	2	3	4	5
1	5.00	0	0	0	0
2	1.64	3.36	0	0	0
3	0.06	0.38	1.72	1.58	1.25
4	0.05	0.34	0.43	3.00	1.17
5	0.22	0.76	0.75	1.50	1.78

$$C_1 = \{1\}, \quad T = \{2, 3, 4, 5\}$$

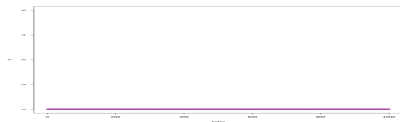
Simulazioni



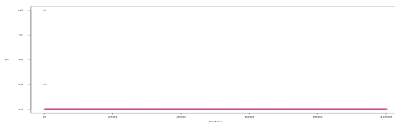
(a) $X_0(\omega) = 4$



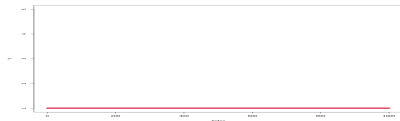
(b) $X_0(\omega) = 2$



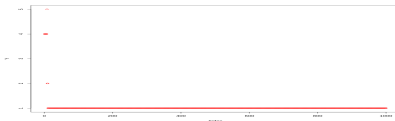
(c) $X_0(\omega) = 1$



(d) $X_0(\omega) = 4$



(e) $X_0(\omega) = 1$



(f) $X_0(\omega) = 3$

P

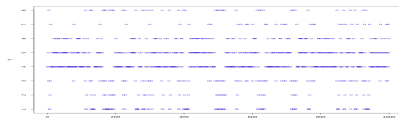
	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0	0	0	0	0	0.5	0.5
4	0	0	0	0.6	0.4	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	0	0	0	0.5	0.5	0	0	0
7	0	0	0	1	0	0	0	0
8	1	0	0	0	0	0	0	0

```
> CalcolaB(P)
```

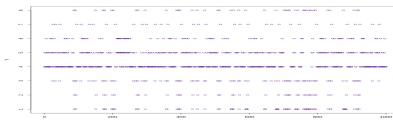
```
[,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]
[1,] 3.130010 1.183206 0.9361920 1.353344 0.4504064 0.1217536 0.412544 0.412544
[2,] 1.031360 1.355200 1.1873280 2.225562 0.8299264 0.2677760 0.551424 0.551424
[3,] 1.411680 0.501120 1.4723200 2.153120 0.8038400 0.2483200 0.704800 0.704800
[4,] 0.204480 0.058240 0.3614464 4.505818 1.9052032 0.6665728 0.149120 0.149120
[5,] 0.372800 0.117120 0.5269120 3.140339 2.4812288 0.9053440 0.228128 0.228128
[6,] 0.219200 0.064000 0.3772480 3.369920 1.9648960 1.6904960 0.157120 0.157120
[7,] 0.145600 0.035200 0.2982400 4.049408 1.6664320 0.5708800 1.117120 0.117120
[8,] 2.958016 1.094144 0.8250880 1.035776 0.3043840 0.0721920 0.355200 1.355200
```

Tutti gli elementi sono positivi quindi S è l'unica classe minimale. Tutti gli stati sono ricorrenti.

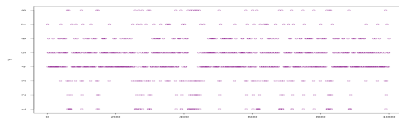
Simulazioni



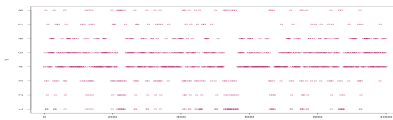
(a) $X_0(\omega) = 5$



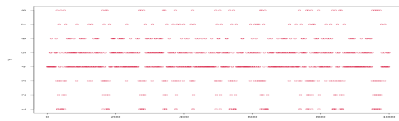
(b) $X_0(\omega) = 4$



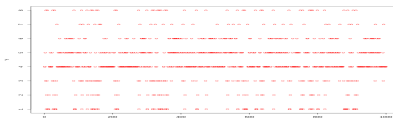
(c) $X_0(\omega) = 3$



(d) $X_0(\omega) = 7$



(e) $X_0(\omega) = 4$



(f) $X_0(\omega) = 4$

P

	1	2	3	4	5	6	7	8
1	0.6	0.40	0.00	0.00	0.00	0.00	0.00	0.00
2	0.0	0.00	0.80	0.20	0.00	0.00	0.00	0.00
3	0.0	0.50	0.50	0.00	0.00	0.00	0.00	0.00
4	0.0	0.00	0.00	0.60	0.40	0.00	0.00	0.00
5	0.00	0.00	0.20	0.40	0.00	0.40	0.00	0.00
6	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

```
> CalcolaB(P)
```

```
[1] 7
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]		
[1,]	2.528666	1.969046	2.3969760	0.786464	0.2462464	0.0726016			0	0
[2,]	0.181504	2.571316	3.4619720	1.230770	0.4170944	0.1373440			0	0
[3,]	0.083040	2.095047	4.5999315	0.864793	0.2747400	0.0824480			0	0
[4,]	0.899072	0.637748	0.9575624	3.638642	1.3625792	0.5043968			0	0
[5,]	1.260992	1.139403	1.7173950	1.681453	1.5947728	0.6059840			0	0
[6,]	2.465856	1.722464	1.9654080	0.615616	0.1815040	1.0491520			0	0
[7,]	0.734720	0.451880	0.6877200	3.406448	1.2609920	0.4582400			1	0
[8,]	0.550400	0.283600	0.4634400	3.152480	1.1456000	0.4044800			1	1

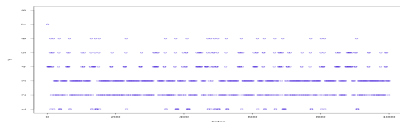
```
> xtable(CalcolaB(P))
```

B

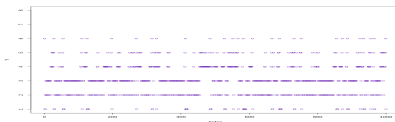
	1	2	3	4	5	6	7	8
1	2.53	1.97	2.40	0.79	0.25	0.07	0	0
2	0.18	2.57	3.46	1.23	0.42	0.14	0	0
3	0.08	2.10	4.60	0.86	0.27	0.08	0	0
4	0.90	0.64	0.96	3.64	1.36	0.50	0	0
5	1.26	1.14	1.72	1.68	1.59	0.61	0	0
6	2.47	1.72	1.97	0.62	0.18	1.05	0	0
7	0.73	0.45	0.69	3.41	1.26	0.46	1.00	0
8	0.55	0.28	0.46	3.15	1.15	0.40	1.00	1.00

$$C_1 = \{1, 2, 3, 4, 5, 6\}, \quad T = \{7, 8\}$$

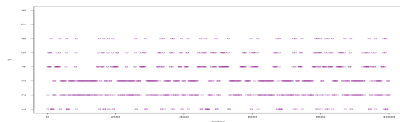
Simulazioni



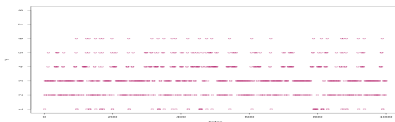
(a) $X_0(\omega) = 8$



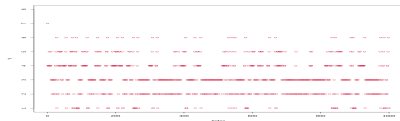
(b) $X_0(\omega) = 5$



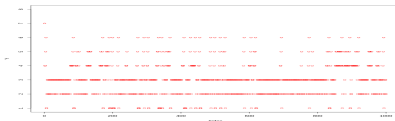
(c) $X_0(\omega) = 1$



(d) $X_0(\omega) = 1$



(e) $X_0(\omega) = 8$



(f) $X_0(\omega) = 8$

P3

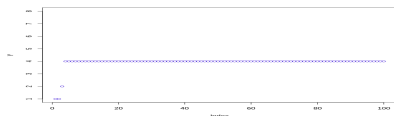
	1	2	3	4	5	6	7	8
1	0.60	0.40	0	0	0	0	0	0
2	0	0	0.80	0.20	0	0	0	0
3	0	1.00	0	0	0	0	0	0
4	0	0	0	1.00	0	0	0	0
5	0	0	0.20	0.40	0	0.40	0	0
6	1.00	0	0	0	0	0	0	0
7	0	0	0	1.00	0	0	0	0
8	0	0	0	0	0	0	1.00	0

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]
[1,]	2.458010	2.416358	1.647411	1.478221	0	0.0	0	0
[2,]	0.000000	2.952000	2.361600	2.686400	0	0.0	0	0
[3,]	0.000000	2.952000	2.952000	2.096000	0	0.0	0	0
[4,]	0.000000	0.000000	0.000000	8.000000	0	0.0	0	0
[5,]	0.953344	1.210176	1.032768	3.403712	1	0.4	0	0
[6,]	2.430016	2.059264	1.444352	1.066368	0	1.0	0	0
[7,]	0.000000	0.000000	0.000000	7.000000	0	0.0	1	0
[8,]	0.000000	0.000000	0.000000	6.000000	0	0.0	1	1

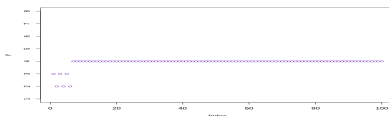
B di Pter

	1	2	3	4	5	6	7	8
1	2.46	2.42	1.65	1.48	0	0	0	0
2	0	2.95	2.36	2.69	0	0	0	0
3	0	2.95	2.95	2.10	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	1.21	1.03	3.40	1.00	0.40	0	0
6	2.43	2.06	1.44	1.07	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

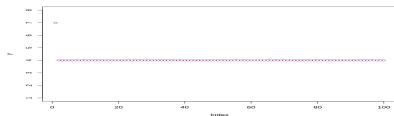
Simulazioni



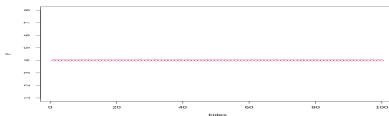
(a) $X_0(\omega) = 6$



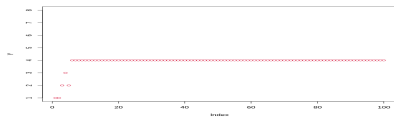
(b) $X_0(\omega) = 2$



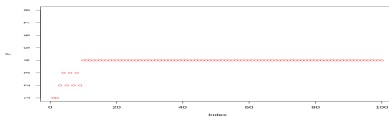
(c) $X_0(\omega) = 8$



(d) $X_0(\omega) = 5$



(e) $X_0(\omega) = 6$



(f) $X_0(\omega) = 1$

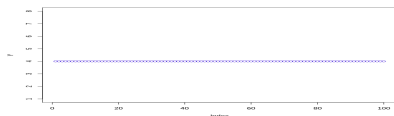
P4

	1	2	3	4	5	6	7	8
1	0.60	0.40	0	0	0	0	0	0
2	0	0	0.80	0.20	0	0	0	0
3	0	0.50	0.50	0	0	0	0	0
4	0	0	0	1.00	0	0	0	0
5	0	0	0.20	0.40	0	0.40	0	0
6	1.00	0	0	0	0	0	0	0
7	0	0	0	1.00	0	0	0	0
8	0	0	0	0	0	0	1.00	0

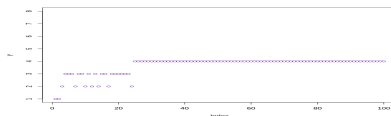
B di P_4

	1	2	3	4	5	6	7	8
1	2.46	1.94	2.33	1.27	0	0	0	0
2	0	2.47	3.29	2.24	0	0	0	0
3	0	2.06	4.52	1.42	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	0.95	1.43	3.27	1.00	0.40	0	0
6	2.43	1.71	1.93	0.93	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

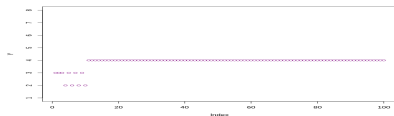
Simulazioni



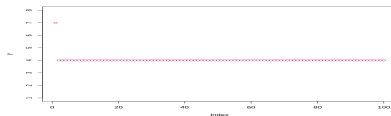
(a) $X_0(\omega) = 7$



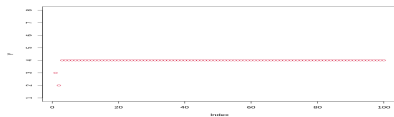
(b) $X_0(\omega) = 1$



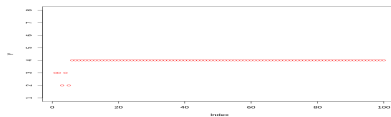
(c) $X_0(\omega) = 2$



(d) $X_0(\omega) = 8$



(e) $X_0(\omega) = 2$



(f) $X_0(\omega) = 2$

P5

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0.5	0.5	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0.5	0.5
8	0	0	0	0	0	0	1	0

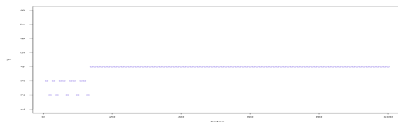
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]
[1,]	2.458010	1.936822	2.331779	1.273389	0	0.0	0.000000	0.000000
[2,]	0.000000	2.467500	3.290700	2.241800	0	0.0	0.000000	0.000000
[3,]	0.000000	2.056687	4.524188	1.419125	0	0.0	0.000000	0.000000
[4,]	0.000000	0.000000	0.000000	8.000000	0	0.0	0.000000	0.000000
[5,]	0.953344	0.954011	1.426963	3.265682	1	0.4	0.000000	0.000000
[6,]	2.430016	1.708704	1.929632	0.931648	0	1.0	0.000000	0.000000
[7,]	0.000000	0.000000	0.000000	0.000000	0	0.0	5.554688	2.445312
[8,]	0.000000	0.000000	0.000000	0.000000	0	0.0	4.890625	3.109375

B di $P5$

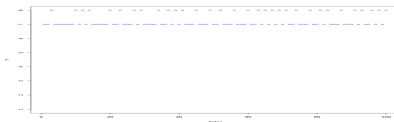
	1	2	3	4	5	6	7	8
1	2.46	1.94	2.33	1.27	0	0	0	0
2	0	2.47	3.29	2.24	0	0	0	0
3	0	2.06	4.52	1.42	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	0.95	1.43	3.27	1.00	0.40	0	0
6	2.43	1.71	1.93	0.93	0	1.00	0	0
7	0	0	0	0	0	0	5.55	2.45
8	0	0	0	0	0	0	4.89	3.11

N.B. gli elementi sono approssimati alla seconda cifra decimale

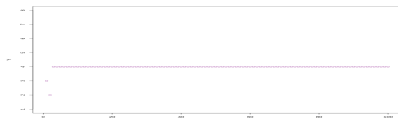
Simulazioni



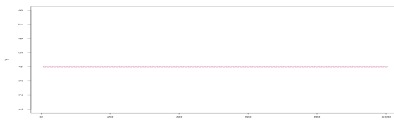
(a) $X_0(\omega) = 3$



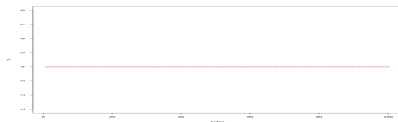
(b) $X_0(\omega) = 8$



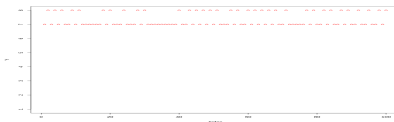
(c) $X_0(\omega) = 5$



(d) $X_0(\omega) = 4$



(e) $X_0(\omega) = 4$



(f) $X_0(\omega) = 8$