21/04/20

lunedì ~~20 aprile~~ 2020 14:45

QUADRICHE

Dato un pto C dello spazio e una quantità positiva r, si dice SFERA di CENTRO C e raggio r l'insieme dei pti. dello spazio la cui distanza da C è pari a r.

Oxyz sistema di coordinate cartesiane ortogonali e se $C = (x_0, y_0, z_0)$, allora la sfera

$$\mathcal{S} = \left\{ (x,y,z) \in \mathbb{R}^3 : \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = r \right\}$$

$$\mathcal{S} = \left\{ (x,y,z) \in \mathbb{R}^3 : (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2 \right\}$$

$$x^2 + y^2 + z^2 - 2x_0 x - 2y_0 y - 2z_0 z + x_0^2 + y_0^2 + z_0^2 - r^2 = 0$$

$$x^2 + y^2 + z^2 + b_1 x + b_2 y + b_3 z + c = 0$$

$$b_1 = -2x_0 \qquad b_2 = -2y_0 \qquad b_3 = -2z_0 \qquad c = x_0^2 + y_0^2 + z_0^2 - r^2$$

$$x_0 = \frac{-b_1}{2} \qquad y_0 = \frac{-b_2}{2} \qquad z_0 = \frac{-b_3}{2} \qquad r^2 = c - x_0^2 - y_0^2 - z_0^2$$

$$x^2 + y^2 + z^2 + \alpha x + \beta y + \gamma z + \delta = 0$$

$$\underbrace{x^2 + \alpha x + \frac{\alpha^2}{4}} - \frac{\alpha^2}{4} + \underbrace{y^2 + \beta y + \frac{\beta^2}{4}} - \frac{\beta^2}{4} + \underbrace{z^2 + \gamma z + \frac{\gamma^2}{4}} - \frac{\gamma^2}{4} + \delta = 0$$

$$\left(x + \frac{\alpha}{2}\right)^2 + \left(y + \frac{\beta}{2}\right)^2 + \left(z + \frac{\gamma}{2}\right)^2 - \frac{\alpha^2}{4} - \frac{\beta^2}{4} - \frac{\gamma^2}{4} + \delta = 0$$

$$\left(x + \frac{\alpha}{2}\right)^2 + \left(y + \frac{\beta}{2}\right)^2 + \left(z + \frac{\gamma}{2}\right)^2 = \underbrace{\frac{\alpha^2 + \beta^2 + \gamma^2}{4} - \gamma}$$

$$C\left(-\frac{\alpha}{2}, -\frac{\beta}{2}, -\frac{\gamma}{2}\right) \qquad \frac{\alpha^2 + \beta^2 + \gamma^2}{4} - \gamma > 0$$

$$\frac{\alpha^2 + \beta^2 + \gamma^2}{4} - \delta = 0 \qquad (x,y,z) = \left(-\frac{\alpha}{2}, -\frac{\beta}{2}, -\frac{\gamma}{2}\right)$$

$$\frac{\alpha^2 + \beta^2 + \gamma^2}{4} - \delta < 0 \qquad \nexists \text{ soluzioni}$$

— o —

L'insieme delle soluzioni $(x;y;z)$ di un'equazione del tipo

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$$
$$+ b_1 x + b_2 y + b_3 z + c = 0$$

— o —

1° CASO $\quad a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + c = 0$

$a_{11}a_{22}a_{33} \neq 0$ e sono concordi

$a_{11} > 0 \quad a_{22} > 0 \quad a_{33} > 0$ (senza perdere in generalità)

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 = -c$$

A) $-c < 0$ cioè $c > 0$ $\Rightarrow \nexists$ soluzioni

B) $c = 0$ $\Rightarrow \exists!$ soluzione $(x,y,z) = (0,0,0)$

C) $-c > 0$ cioè $c < 0$ (discorde dai 3 coefficienti)

$$\frac{x^2}{\frac{-c}{a_{11}}} + \frac{y^2}{\frac{-c}{a_{22}}} + \frac{z^2}{\frac{-c}{a_{33}}} = 1 \qquad \frac{-c}{a_{11}} =: \alpha^2$$

$$\frac{-c}{a_{22}} =: \beta^2$$

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1 \qquad \frac{-c}{a_{33}} =: \gamma^2$$

(x,y,z) soluz. $\Rightarrow$ $(x,y,-z)$ $\quad (x,-y,z)$ $\quad (-x,y,z)$

$(-x,-y,z)$ $\quad (-x,y,-z)$ $\quad (x,-y,-z)$

$(-x,-y,-z)$

$$\begin{cases} z = h \\ \dfrac{x^2}{\alpha^2} + \dfrac{y^2}{\beta^2} = 1 - \dfrac{h^2}{\gamma^2} \end{cases}$$

$|h| > \gamma \quad \nexists$ soluzioni

$|h| = \gamma \qquad (0,0,h)$

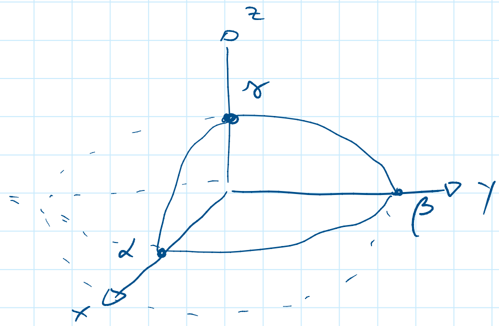
$|h| < \gamma \qquad$ ELLISSE

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = \frac{\gamma^2 - h^2}{\gamma^2}$$

$$\frac{x^2}{\frac{\alpha^2}{\gamma^2}\cdot(\gamma^2 - h^2)} + \frac{y^2}{\frac{\beta^2}{\gamma^2}\cdot(\gamma^2 - h^2)} = 1$$

ELLISSE riferita ai propri assi

$$\begin{cases} x = h \\ \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1 - \frac{h^2}{\alpha^2} \end{cases}$$



ELLISSOIDE

$a_{11}a_{22}a_{33} \neq 0$ ma uno dei 3 è discorde dagli altri due

$a_{11} > 0 \qquad a_{22} > 0 \qquad a_{33} < 0$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 = -c$$

$-c > 0$

$-c < 0$

$\boxed{-c > 0}$

$$\frac{x^2}{\underbrace{\frac{-c}{a_{11}}}_{>0}} + \frac{y^2}{\underbrace{\frac{-c}{a_{22}}}_{>0}} + \frac{z^2}{\underbrace{\frac{-c}{a_{33}}}_{<0}} = 1$$

$\alpha^2 = \frac{-c}{a_{11}}$

$\beta^2 = \frac{-c}{a_{22}}$

$\gamma^2 = \frac{+c}{a_{33}}$

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} - \frac{z^2}{\gamma^2} = 1$$

$$\begin{cases} z = h \\ \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1 + \frac{h^2}{\gamma^2} \end{cases} \qquad \begin{cases} z = h \\ \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = \frac{\gamma^2 + h^2}{\gamma^2} \end{cases}$$

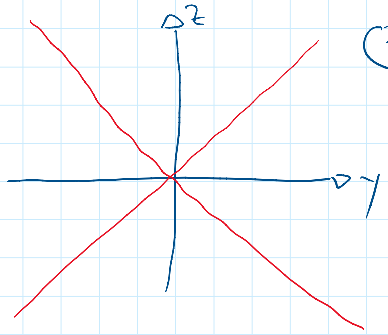
$$\frac{x^2}{\frac{\alpha^2(\gamma^2 + h^2)}{\gamma^2}} + \frac{y^2}{\frac{\beta^2(\gamma^2 + h^2)}{\gamma^2}} = 1$$

$\Rightarrow$ ELLISSE i cui semiassi crescono al crescere di $|h|$

$$\begin{cases} x = h \\ y^2 \quad z^2 \quad h^2 \end{cases}$$

$$\begin{cases} x = h \\ \dfrac{y^2}{\beta^2} - \dfrac{z^2}{\gamma^2} = 1 - \dfrac{h^2}{\alpha^2} \end{cases}$$

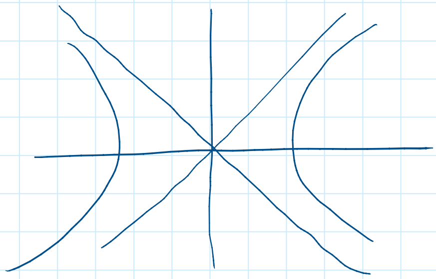
$$1 - \frac{h^2}{\alpha^2} = 0 \qquad \frac{y^2}{\beta^2} - \frac{z^2}{\gamma^2} = 0$$



$x = h$

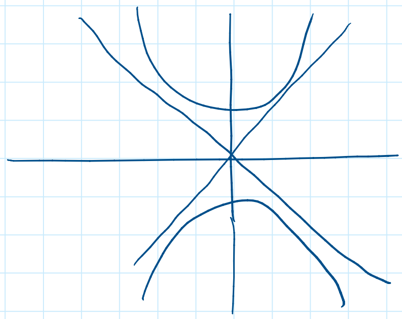
$$\left(\frac{y}{\beta} - \frac{z}{\gamma}\right)\left(\frac{y}{\beta} + \frac{z}{\gamma}\right) = 0$$

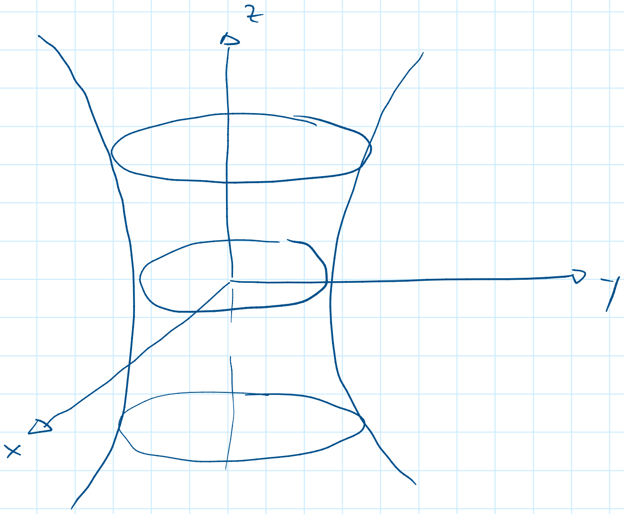
2 rette per l'origine



Se $1 - \frac{h^2}{\alpha^2} > 0$ iperbole

Se $1 - \frac{h^2}{\alpha^2} < 0$ iperbole





IPERBOLOIDE

A UNA FALDA

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 = -c$$

$a_{11} > 0$ $\quad a_{22} > 0$ $\quad a_{33} < 0$ $\quad -c < 0$

$$\frac{x^2}{\frac{c}{a_{11}}} + \frac{y^2}{\frac{c}{a_{22}}} + \frac{z^2}{\frac{c}{a_{33}}} = -1$$

$\alpha^2 := \frac{c}{a_{11}} > 0$

$\beta^2 := \frac{c}{a_{22}} > 0$

$$\frac{x^2}{\ldots} + \frac{y^2}{\ldots} - \frac{z^2}{\ldots} = -1$$

$\gamma^2 = c$ …

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}-\frac{z^2}{\gamma^2}=-1 \qquad \beta^2 := \frac{\quad}{a_{22}} > 0 \qquad \gamma^2 := \frac{-c}{a_{33}} > 0$$

$$\begin{cases} z = h \\ \dfrac{x^2}{\alpha^2}+\dfrac{y^2}{\beta^2} = \dfrac{h^2}{\gamma^2}-1 \end{cases} \qquad \begin{cases} z = h \\ \dfrac{x^2}{\alpha^2}+\dfrac{y^2}{\beta^2} = \dfrac{h^2-\gamma^2}{\gamma^2} \end{cases}$$

$|h| < \gamma$ $\quad \nexists$ soluzioni

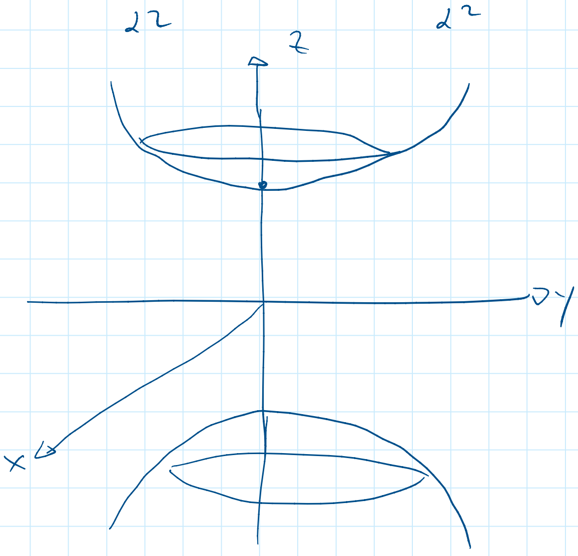
$|h| = \gamma$ $\quad \Rightarrow \quad (x,y,z) = (0,0,h)$

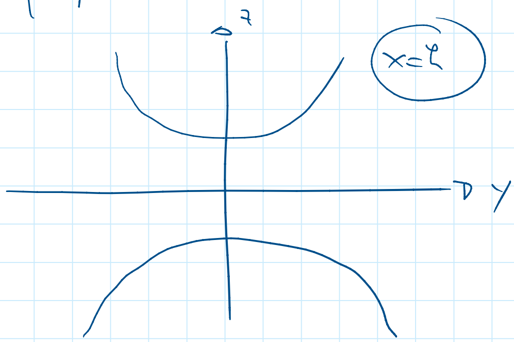
$|h| > \gamma$ $\quad \Rightarrow$ sul piano $z = h$ viene intercettata un'ellisse

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2} = \frac{h^2-\gamma^2}{\gamma^2} \qquad \frac{x^2}{\frac{\alpha^2(h^2-\gamma^2)}{\gamma^2}}+\frac{y^2}{\frac{\beta^2(h^2-\gamma^2)}{\gamma^2}} = 1$$

$$\begin{cases} x = h \\ \dfrac{y^2}{\beta^2}-\dfrac{z^2}{\gamma^2} = -1-\dfrac{h^2}{\alpha^2} \end{cases} \qquad \begin{cases} x = h \\ \dfrac{y^2}{\beta^2}-\dfrac{z^2}{\gamma^2} = -\dfrac{(\alpha^2+h^2)}{\alpha^2} \end{cases}$$

$$\frac{y^2}{\frac{\beta^2(\alpha^2+h^2)}{\alpha^2}}-\frac{z^2}{\frac{\gamma^2(\alpha^2+h^2)}{\alpha^2}} = -1$$





IPERBOLOIDE A 2 FALDE

$a_{11}a_{22}a_{33} \neq 0 \qquad c = 0$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 = 0$$

Se a_{11}, a_{22} e a_{33} sono concordi $\Rightarrow$ ho solo la soluzione $(x,y,z) = (0,0,0)$

$a_{11} > 0 \qquad a_{22} > 0 \qquad a_{33} < 0$

$$\frac{x^2}{\frac{1}{a_{11}}} + \frac{y^2}{\frac{1}{a_{22}}} + \frac{z^2}{\frac{1}{a_{33}}} = 0 \qquad \alpha^2 = \frac{1}{a_{11}} > 0$$

$$\beta^2 = \frac{1}{a_{22}} > 0$$

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} - \frac{z^2}{\gamma^2} = 0 \qquad \gamma^2 = \frac{-1}{a_{33}} > 0$$

$(0,0,0)$ è soluzione

Se $P_0(x_0,y_0,z_0)$ è soluzione $\Rightarrow$ tutte le rette per O e P_0 è soluzione

$P - O \parallel P_0 - O \qquad P(x,y,z)$

$$\begin{cases} x - 0 = s(x_0 - 0) \\ y - 0 = s(y_0 - 0) \\ z - 0 = s(z_0 - 0) \end{cases} \quad s \in \mathbb{R} \qquad \begin{cases} x = s x_0 \\ y = s y_0 \\ z = s z_0 \end{cases} \quad s \in \mathbb{R}$$

$$\frac{(s x_0)^2}{\alpha^2} + \frac{(s y_0)^2}{\beta^2} - \frac{(s z_0)^2}{\gamma^2} = 0$$

$$s^2 \left(\underbrace{\frac{x_0^2}{\alpha^2} + \frac{y_0^2}{\beta^2} - \frac{z_0^2}{\gamma^2}}_{=0} \right) = 0 \qquad \forall s \in \mathbb{R}$$

$$\begin{cases} z = h \\ \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = \frac{h^2}{\gamma^2} \end{cases}$$

Se $h = 0 \qquad (0,0,0)$

Se $h=0$ $\quad (0,0,0)$

$h\neq 0$ ellisse $\qquad \frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=\frac{h^2}{\gamma^2}$

$$\frac{x^2}{\frac{\alpha^2 h^2}{\gamma^2}}+\frac{y^2}{\frac{\beta^2 h^2}{\gamma^2}}=1$$

$\frac{\alpha}{\gamma}|h|$ e $\frac{\beta}{\gamma}|h|$ sono i semiassi

$$\begin{cases} x=h \\ \frac{y^2}{\beta^2}-\frac{z^2}{\gamma^2}=-\frac{h^2}{\alpha^2} \end{cases}$$

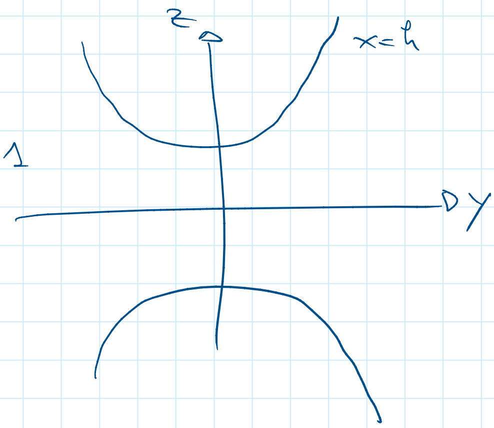
$h=0$ 2 rette $\qquad \frac{y^2}{\beta^2}-\frac{z^2}{\gamma^2}=0$

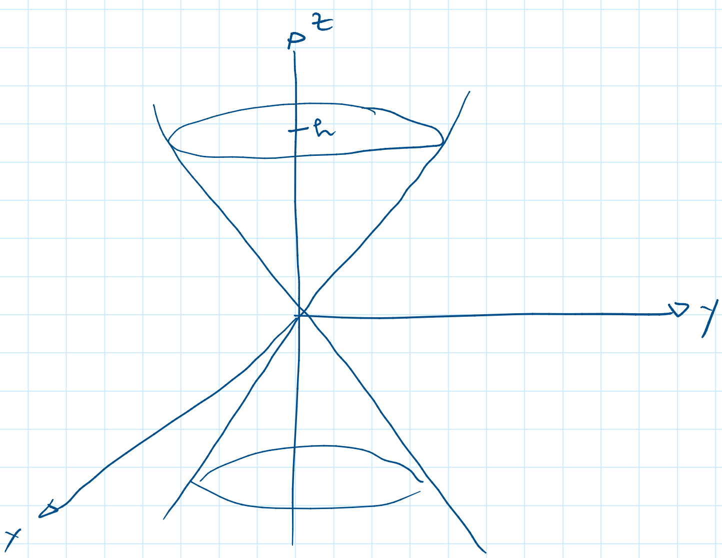
$$\left(\frac{y}{\beta}-\frac{z}{\gamma}\right)\left(\frac{y}{\beta}+\frac{z}{\gamma}\right)=0$$

$r_1 \begin{cases} x=h \\ \frac{y}{\beta}-\frac{z}{\gamma}=0 \end{cases} \qquad r_2 \begin{cases} x=h \\ \frac{y}{\beta}+\frac{z}{\gamma}=0 \end{cases}$

$h\neq 0$ $\qquad \frac{y^2}{\beta^2}-\frac{z^2}{\gamma^2}=-\frac{h^2}{\alpha^2}$

$$\frac{y^2}{\frac{\beta^2 h^2}{\alpha^2}}-\frac{z^2}{\frac{\gamma^2 h^2}{\alpha^2}}=-1$$





CONO DEL SECONDO ORDINE

$a_{11} a_{22} \neq 0 \quad c \neq 0 \quad a_{33} = 0$

$$a_{11} x^2 + a_{22} y^2 = -c$$

1) $a_{11}, a_{22}, -c$ sono positivi

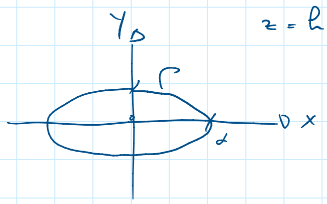
$$\frac{x^2}{\frac{-c}{a_{11}}} + \frac{y^2}{\frac{-c}{a_{22}}} = 1$$

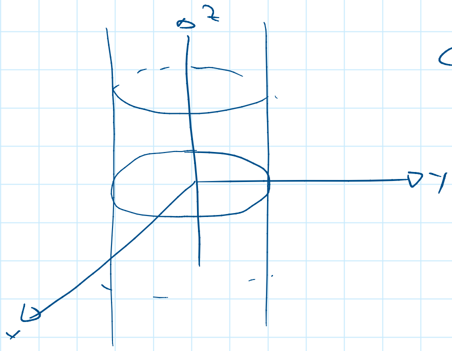
$$\alpha^2 := \frac{-c}{a_{11}} > 0$$

$$\beta^2 := \frac{-c}{a_{22}} > 0$$

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$$

$$\begin{cases} z = h \\ \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1 \end{cases}$$





CILINDRO A SEZIONE ELLITTICA

2) $a_{11} > 0 \quad a_{22} < 0 \quad -c > 0$

$$a_{11} x^2 + a_{22} y^2 = -c$$

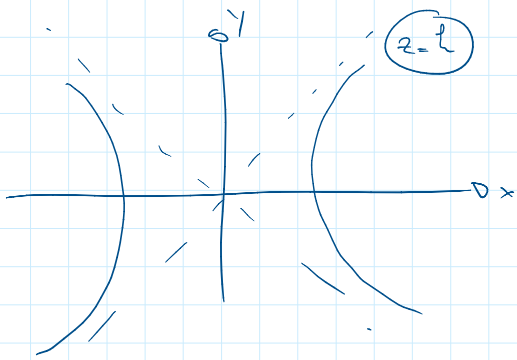
$$\frac{x^2}{\frac{-c}{a_{11}}} + \frac{y^2}{\frac{-c}{a_{22}}} = 1$$

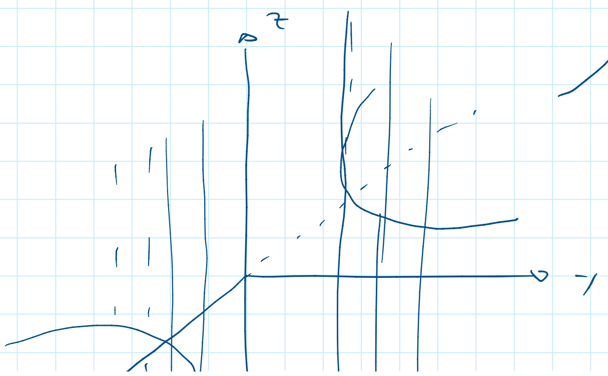
$$\alpha^2 := \frac{-c}{a_{11}} > 0$$

$$\beta^2 := \frac{+c}{a_{22}} > 0$$

$$\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$$

$$\begin{cases} z = h \\ \frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \end{cases}$$

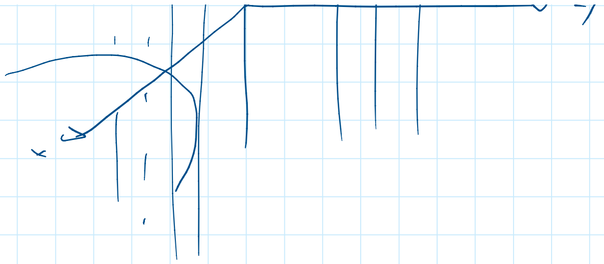




CILINDRO IPERBOLICO

o

CILINDRO A SEZIONE IPERBOLICA



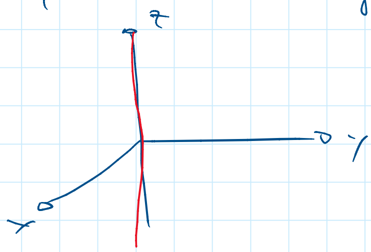
a_{11} e $a_{22} \neq 0$ Ne $a_{33} = c = 0$

$$a_{11}x^2 + a_{22}y^2 = 0$$

Se a_{11} e a_{22} sono concordi $\Rightarrow$ $x = y = 0$

L'insieme delle soluzioni dell'eq. è dato dagli

$(x,y,z) \in \mathbb{R}^3$: $\begin{cases} x = 0 \\ y = 0 \end{cases}$



Se a_{11} e a_{22} sono discordi,

$a_{11} > 0$ e $a_{22} < 0$ $\alpha^2 := a_{11}$ $\beta^2 := -a_{22}$

$$\alpha^2 x^2 - \beta^2 y^2 = 0 \qquad (\alpha x - \beta y)(\alpha x + \beta y) = 0$$

$\Pi_1 := \{(x,y,z) \in \mathbb{R}^3 : \alpha x - \beta y = 0\}$ $\Pi_1 \cup \Pi_2$

$\Pi_2 := \{(x,y,z) \in \mathbb{R}^3 : \alpha x + \beta y = 0\}$

Π_1 e Π_2 sono entrambi piani contenenti l'asse z

$a_{22} = a_{33} = 0$ $a_{11} \neq 0$ $c \neq 0$

$$a_{11}x^2 = -c \qquad x^2 = \frac{-c}{a_{11}}$$

Se c e a_{11} sono concordi $\Rightarrow$ $\frac{-c}{a_{11}} < 0$ $\Rightarrow \nexists$ soluzioni

Se c e a_{11} sono discordi $\Rightarrow$ $\frac{-c}{a_{11}} > 0$ $\alpha^2 := \frac{-c}{a_{11}}$

$$x^2 = \alpha^2$$

$\Pi_1 := \{(x,y,z) \in \mathbb{R}^3 : x = \alpha\}$

$\Pi_2 := \{ (x,y,z) \in \mathbb{R}^3 : x = -\alpha \}$

$\Rightarrow$ L'insieme delle soluzioni è $\Pi_1 \cup \Pi_2$

Π_1 e Π_2 sono due piani paralleli al piano condotto $x = 0$

$a_{33} = 0, \quad c = 0 \qquad b_3 \neq 0 \qquad a_{11} a_{22} \neq 0$

$$a_{11} x^2 + a_{22} y^2 + b_3 z = 0$$

$$z = \frac{-a_{11}}{b_3} x^2 - \frac{a_{22}}{b_3} y^2 \qquad z = f(x,y)$$

$a_{11} > 0 \qquad a_{22} > 0 \qquad b_3 < 0$

$\alpha^2 := \dfrac{-a_{11}}{b_3} > 0 \qquad z = \alpha^2 x^2 + \beta^2 y^2$

$\beta^2 := \dfrac{-a_{22}}{b_3} > 0$

$$\begin{cases} z = h \\ \alpha^2 x^2 + \beta^2 y^2 = h \end{cases} \qquad L_h \text{ della funzione } f$$

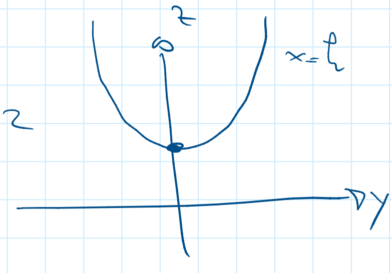
$h < 0 \qquad \nexists$ soluzioni

$h = 0 \qquad (0,0,0)$

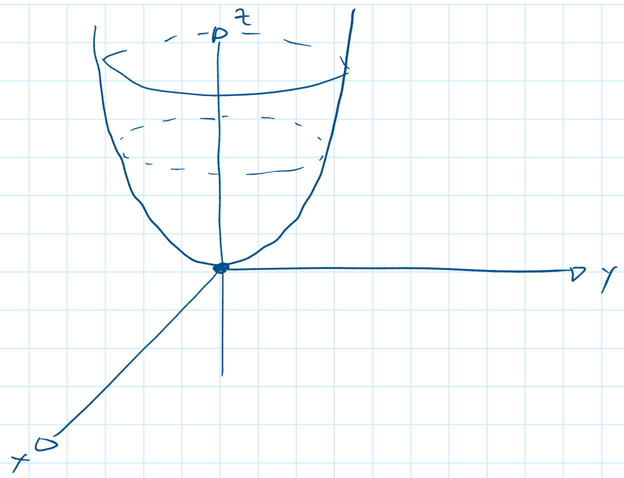
$h > 0 \qquad \alpha^2 x^2 + \beta^2 y^2 = h \qquad \dfrac{x^2}{\frac{h}{\alpha^2}} + \dfrac{y^2}{\frac{h}{\beta^2}} = 1$

ellisse di semiassi $\dfrac{\sqrt{h}}{\alpha}$ e $\dfrac{\sqrt{h}}{\beta}$

$$\begin{cases} x = h \\ \alpha^2 h^2 + \beta^2 y^2 = z \end{cases} \qquad z = \beta^2 y^2 + \alpha^2 h^2$$



PARABOLOIDE ELLITTICO



$a_{11} > 0 \qquad a_{22} < 0 \qquad b_3 < 0$

$$z = \frac{-a_{11}}{b_3} x^2 - \frac{a_{22}}{b_3} y^2$$

$$\alpha^2 := \frac{-a_{11}}{b_3} > 0$$

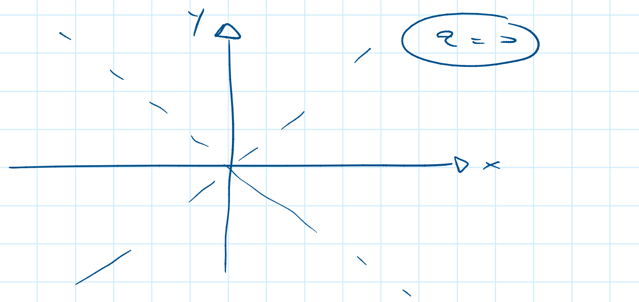
$$\beta^2 := \frac{+a_{22}}{b_3} > 0$$

$$z = \alpha^2 x^2 - \beta^2 y^2 \qquad z = f(x,y)$$

$$\begin{cases} z = h \\ \alpha^2 x^2 - \beta^2 y^2 = h \end{cases}$$

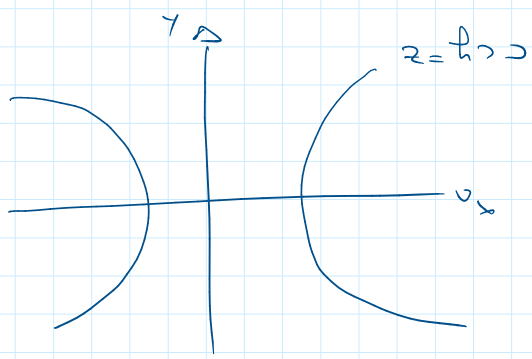
$h = 0 \qquad \alpha^2 x^2 - \beta^2 y^2 = 0$

$$(\alpha x - \beta y)(\alpha x + \beta y) = 0$$



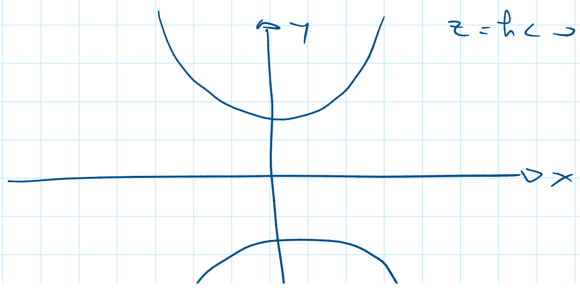
$h > 0 \qquad \alpha^2 x^2 - \beta^2 y^2 = h$

$$\frac{x^2}{\frac{h}{\alpha^2}} - \frac{y^2}{\frac{h}{\beta^2}} = 1$$



$h < 0$

$$\frac{x^2}{\frac{-h}{\alpha^2}} - \frac{y^2}{\frac{-h}{\beta^2}} = -1$$



PARABOLOIDE IPERBOLICO

PARABOLOIDE IPERBOLICO

ESERCIZI

Provare che l'insieme delle soluzioni dell'eq.

$$x^2+y^2+z^2-6x+9z=c$$

è una sfera SSE $c > \frac{-117}{4}$

Nel caso $c = \frac{139}{4}$ determinare l'intersezione della sfera con i piani coordinati.

$$x^2+y^2+z^2-6x+9z=c$$

$$x^2-6x+9+y^2+z^2+9z+\left(\frac{9}{2}\right)^2 = c+9+\left(\frac{9}{2}\right)^2$$

$$9 = 2\cdot\frac{9}{2}$$

$$(x-3)^2+(y-0)^2+\left(z+\frac{9}{2}\right)^2 = c+9+\frac{81}{4} = c+\frac{36+81}{4}$$

$$(x-3)^2+(y-0)^2+\left(z+\frac{9}{2}\right)^2 = \frac{4c+117}{4}$$

$\nexists$ soluzioni se $4c+117<0$ $\quad c < \frac{-117}{4}$

$\exists!$ $\left(3,0,-\frac{9}{2}\right)$ se $4c+117=0$ $\quad c = \frac{-117}{4}$

Sfera di centro $\left(3,0,-\frac{9}{2}\right)$ e raggio $r = \frac{\sqrt{4c+117}}{2}$

se $4c+117>0$ cioè

se $c > \frac{-117}{4}$

$c = \frac{139}{4}$ $\quad 4c = 139$ $\quad \frac{4c+117}{4} = \frac{139+117}{4}$

$= \frac{256}{4} = 64$

$$(x-3)^2+(y-0)^2+\left(z+\frac{9}{2}\right)^2 = 8^2$$

$$\begin{cases} x = 0 \\ y^2 + \left(z + \frac{9}{2}\right)^2 = 64 - 9 = 55 \end{cases}$$

Sul piano $x=0$ intercetta la circonferenza di centro $\left(0, -\frac{9}{2}\right)$ e raggio $\sqrt{55}$

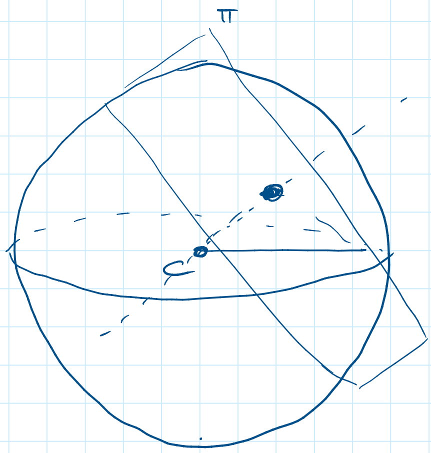
$$\begin{cases} y = 0 \\ (x-3)^2 + \left(z + \frac{9}{2}\right)^2 = 8^2 \end{cases}$$

contiene il centro

Viene intercettata la circonferenza di centro $\left(3, -\frac{9}{2}\right)$ e raggio 8

$$\begin{cases} z = 0 \\ (x-3)^2 + y^2 = 64 - \left(\frac{9}{2}\right)^2 = 64 - \frac{81}{4} = \frac{256-81}{4} = \frac{175}{4} \end{cases}$$

Viene intercettata la circonferenza di centro $(3,0)$ e raggio $\frac{\sqrt{175}}{2}$



S sfera

π piano

$S \qquad x^2 + y^2 + z^2 + \alpha x + \beta y + \gamma z + \delta = 0$

$\pi \qquad ax + by + cz + d = 0$

L'intersezione è una circonferenza è necessario e suff. che $d(C, \pi) < r$

è suff. che $d(C, \pi) < r$

$$C\left(-\frac{\alpha}{2}, -\frac{\beta}{2}, -\frac{\gamma}{2}\right) \qquad d(C,\pi) = \frac{\left|a\frac{-\alpha}{2} + b\frac{-\beta}{2} + c\frac{-\gamma}{2} + d\right|}{\sqrt{a^2+b^2+c^2}}$$

$$\overset{\parallel}{d}$$

$$r = \sqrt{\frac{\alpha^2+\beta^2+\gamma^2}{4} - \delta}$$

$S \cap \pi$ è una circonferenza SSE

$$\frac{\left|a\frac{-\alpha}{2} + b\frac{-\beta}{2} + c\frac{-\gamma}{2} + d\right|}{\sqrt{a^2+b^2+c^2}} < \sqrt{\frac{\alpha^2+\beta^2+\gamma^2}{4} - \delta}$$

Il raggio della circonferenza intercettata è

$$r' := \sqrt{r^2 - d^2}$$

Coordinate del centro della circonferenza

$$C = (x_c, y_c, z_c) = \left(-\frac{\alpha}{2}, -\frac{\beta}{2}, -\frac{\gamma}{2}\right)$$

$$\underline{n} = (a, b, c) \perp \text{ piano } \pi$$

$$\begin{cases} x - x_c = sa \\ y - y_c = sb \\ z - z_c = sc \end{cases} \quad s \in \mathbb{R} \qquad r \begin{cases} x = x_c + sa \\ y = y_c + sb \\ z = z_c + sc \end{cases} \quad s \in \mathbb{R}$$

$r \cap \pi \qquad ax + by + cz + d = 0$

$$a(x_c + sa) + b(y_c + sb) + c(z_c + sc) + d = 0$$

$$s(a^2+b^2+c^2) = -ax_c - by_c - cz_c - d$$

$$s = \frac{-(ax_c + by_c + cz_c + d)}{a^2+b^2+c^2}$$

$$s = \frac{-(a x_c + b y_c + c z_c + d)}{a^2+b^2+c^2}$$

ESERCIZIO

Individuare l'intersezione tra la sfera S di equazione $x^2+y^2+z^2+2x-4y-2z=0$ e il piano π di equazione $x-3y+z+1=0$

S $\quad x^2+y^2+z^2+2x-4y-2z=0$

$$x^2+2x+1+y^2-4y+4+z^2-2z+1 \quad = 0+1+4+1$$

$S)$ $\quad (x+1)^2+(y-2)^2+(z-1)^2=6$

S è la sfera di centro $C(-1,2,1)$ e raggio $r=\sqrt{6}$

$\pi)$ $\quad x-3y+z+1=0$

$$\underline{n} = (1,-3,1)$$

$$d := d(C,\pi) = \frac{|\cancel{-1}-3\cdot 2+\cancel{1}+1|}{\sqrt{1+9+1}} = \frac{|-6+1|}{\sqrt{11}} = \frac{5}{\sqrt{11}}$$

$\frac{5}{\sqrt{11}} < \sqrt{6}$ cioè $d<r$

$\Rightarrow$ $S \cap \pi$ è una circonferenza

Il raggio della circonferenza è $\sqrt{r^2-d^2} =$

$$= \sqrt{6-\frac{25}{11}} = \sqrt{\frac{66-25}{11}} = \sqrt{\frac{41}{11}}$$

Retta per C e $\parallel$ $\underline{n}$

$C=(-1,2,1)$ $\qquad \underline{n}=(1,-3,1)$

$$r\begin{cases} x = -1 + s\cdot 1 \\ y = 2 + s(-3) \\ z = 1 + s\cdot 1 \end{cases} \quad s\in\mathbb{R} \qquad \begin{cases} x = s-1 \\ y = -3s+2 \\ z = s+1 \end{cases} \quad s\in\mathbb{R}$$

$$\pi)\quad x-3y+z+1=0$$

$$(s-1)-3(-3s+2)+s+1+1=0$$

$$s(1+9+1)-1-6+2=0$$

$$11s=5 \qquad s=\frac{5}{11}$$

Il centro della circonferenza ha coordinate:

$$x=\frac{5}{11}-1 \qquad y=-3\cdot\frac{5}{11}+2 \qquad z=\frac{5}{11}+1$$

$$C'\left(\frac{-6}{11},\frac{7}{11},\frac{16}{11}\right)$$

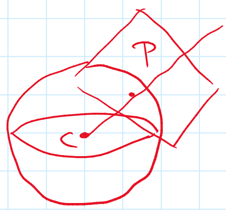
ESERCIZIO Data la sfera $\mathcal{S}$ di equazione

$$x^2+y^2+z^2-2x+2y=7$$

verificare che il pto $P(0,1,2)\in\mathcal{S}$ e scrivere l'eq del piano Tg ad $\mathcal{S}$ nel pto P.

$P\in\mathcal{S}$
$$x^2+y^2+z^2-2x+2y\Big|_{(x,y,z)=(0,1,2)}=7$$

$$1+4+2=7 \quad \text{vera} \quad \Rightarrow P\in\mathcal{S}$$



$P-C$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\ \text{t.c.}\quad \begin{pmatrix}x\\y\\z\end{pmatrix}-P \perp P-C$$

$$x^2+y^2+z^2-2x+2y-7=0$$

$$C:\left(\frac{+2}{2},\frac{-2}{2},0\right)=(1,-1,0)$$

$P(0,1,2)$

$\pi)\ \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\right) \circ \left(\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\right) = 0$

$\begin{pmatrix} x \\ y-1 \\ z-2 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$ $\qquad -x + 2(y-1) + 2(z-2) = 0$

$-x + 2y + 2z - 2 - 4 = 0$

$\pi)\ -x + 2y + 2z - 6 = 0$

$\mathcal{S})\ x^2 + y^2 + z^2 - 2x + 2y - 7 = 0$ $\qquad P(0,1,2)$

> 0

$z^2 = 7 - x^2 + 2x - y^2 - 2y$

$z = \sqrt{7 - x^2 + 2x - y^2 - 2y}$ $\qquad z = f(x,y)$

Sto cercando il piano tg al grafico di f nel pto $(0,1,f(0,1))$

$f_x = \dfrac{-2x+2}{2\sqrt{7-x^2+2x-y^2-2y}}$ $\qquad f_x(0,1) = \dfrac{2}{2\sqrt{7-1-2}} = \dfrac{2}{2\cdot 2} = \dfrac{1}{2}$

$f_y = \dfrac{-2y-2}{2\sqrt{7-x^2+2x-y^2-2y}}$ $\qquad f_y(0,1) = \dfrac{-2-2}{2\cdot 2} = -1$

$\pi)\ z = f(0,1) + f_x(0,1)(x-0) + f_y(0,1)(y-1)$

$\pi)\ z = 2 + \frac{1}{2}x - 1(y-1)$ $\qquad \pi)\ z = 2 + \frac{1}{2}x - y + 1$

$\pi)\ z = 3 + \frac{1}{2}x - y$

$\pi)\quad -x+2y+2z-6=0$

ESERCIZIO Scrivere l'eq. della sfera $\mathcal{S}$ passante per i p.ti $P_1(1,0,0)$ e $P_2(0,1,0)$ e tg al piano $\pi)\ x+z=0$ nel p.to P_2.

$$\mathcal{S})\quad x^2+y^2+z^2+\alpha x+\beta y+\gamma z+\delta=0 \qquad \alpha,\beta,\gamma,\delta\in\mathbb{R}$$

da determinare.

$P_1\in\mathcal{S} \qquad 1+\alpha+\delta=0 \quad \Leftarrow$

$P_2\in\mathcal{S} \qquad 1+\beta+\delta=0 \quad \Leftarrow$

$\pi)\ x+z=0$ tg a $\mathcal{S}$ nel p.to $P_2(0,1,0)$

$P_2-C\perp\pi \qquad \underline{n}=(1,0,1) \qquad \underline{n}\perp\pi$

$P_2-C\ /\!/\ \underline{n} \qquad \exists\lambda\in\mathbb{R}$ t.c. $P_2-C=\lambda\underline{n}$

$C\left(-\frac{\alpha}{2},-\frac{\beta}{2},-\frac{\gamma}{2}\right) \qquad P_2-C=\left(\frac{\alpha}{2},1+\frac{\beta}{2},\frac{\gamma}{2}\right)$

$\lambda\in\mathbb{R}$ t.c. $\left(\frac{\alpha}{2},1+\frac{\beta}{2},\frac{\gamma}{2}\right)=\lambda(1,0,1)$

$$\begin{cases}\frac{\alpha}{2}=\lambda \leftarrow\\ 1+\frac{\beta}{2}=0\\ \frac{\gamma}{2}=\lambda \leftarrow\\ 1+\alpha+\delta=0\\ 1+\beta+\delta=0\end{cases} \qquad \begin{cases}\beta=-2\\ 1-2+\delta=0 \qquad \delta=1\\ \alpha=-\delta-1=-2\\ \gamma=-2\end{cases}$$

$$\mathcal{S})\quad x^2+y^2+z^2-2x-2y-2z+1=0$$

$$x^2-2x+1+y^2-2y+1+z^2-2z+1=0+1+1$$

$$(x-1)^2 + (y-1)^2 + (z-1)^2 = 2$$

$$C\,(1,1,1) \qquad r = \sqrt{2}$$