$$f(x)=\frac{x^3}{x^2+16}$$

$D=\mathbb{R}$

$$f(-x)=\frac{(-x)^3}{(-x)^2+16}=\frac{-x^3}{x^2+16}=-f(x)$$

$\Rightarrow f$ è dispari $\Rightarrow$ Basta studiare $f|_{[0,+\infty)}$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to\infty}\frac{x}{1+\frac{16}{x^2}}=+\infty$$

$$\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{x^2}{x^2+16}=\lim_{x\to+\infty}\frac{1}{1+\frac{16}{x^2}}=1$$

$$\lim_{x\to+\infty} f(x)-x=\lim_{x\to+\infty}\frac{\cancel{x^3}-\cancel{x^3}-16}{x^2+16}=0$$

$\Rightarrow y=x$ AS OBL PER $x\to+\infty$

$$f'(x)=\frac{3x^2(x^2+16)-x^3\cdot 2x}{(x^2+16)^2}=\frac{x^4+48x^2}{(x^2+16)^2}=$$

$$=\frac{x^2(x^2+48)}{(x^2+16)^2}>0 \quad \forall x\in(0,+\infty)$$

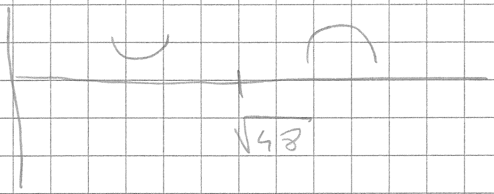
$$f''(x)=\frac{(4x^3+96x)(x^2+16)^{\cancel{2}}-(x^4+48x^2)2\cancel{(x^2+16)}2x}{(x^2+16)^{\cancel{4}\,3}}$$

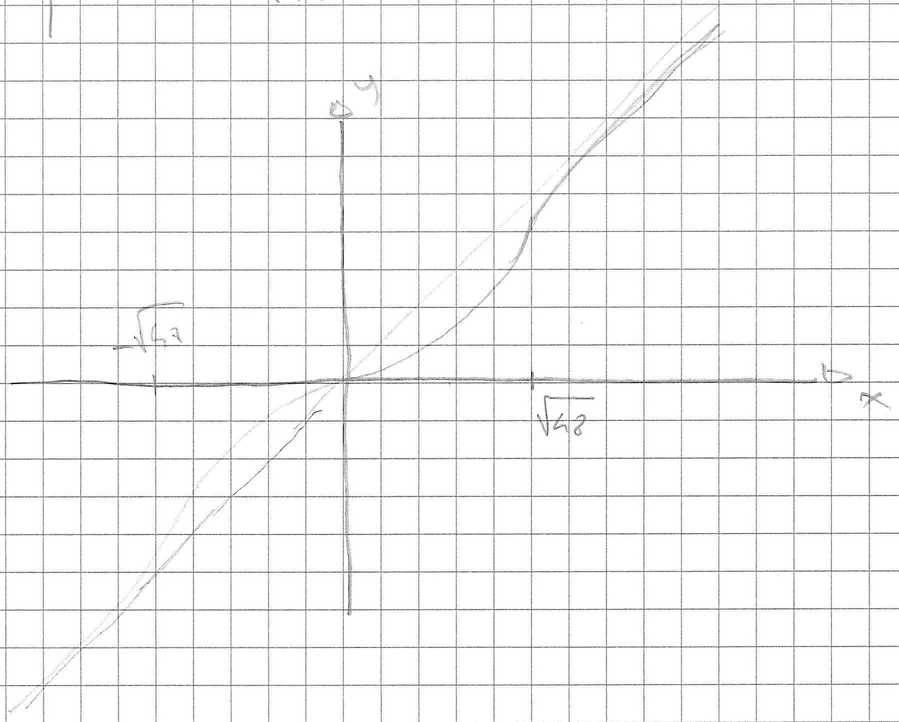
$$=\frac{4x\left[(x^2+24)(x^2+16)-(x^4+48x^2)\right]}{(x^2+16)^3}=$$

$$=\frac{4x\left(\cancel{x^4}+40x^2+24\cdot16-\cancel{x^4}-48x^2\right)}{(x^2+16)^3}=$$

$$=\frac{4x(-8)(-x^2+48)}{(x^2+16)^3}=\frac{32x(48-x^2)}{(x^2+16)^3}$$

$$= \frac{32x\left(\sqrt{48}-x\right)\left(\sqrt{48}+x\right)}{\left(x^2+16\right)^3}$$





$$f(x) = \frac{x^3}{x^2+16} = \frac{x^3+16x-16x}{x^2+16} = x - 8\frac{2x}{x^2+16}$$

$$F(x) = \left.\frac{t^2}{2} - 8\ln\left(t^2+16\right)\right|_{t=0}^{t=x} =$$

$$= \frac{x^2}{2} - 8\ln\left(x^2+16\right) + 8\ln(16)$$

$$= \frac{x^2}{2} - 8\ln\left(\frac{x^2+16}{16}\right)$$

F é pari $\Rightarrow \lim_{x\to+\infty} F(x) = \lim_{x\to-\infty} F(x)$, se esiste.

$$\lim_{x\to+\infty} \frac{x^2}{2} - 8\ln\left(\frac{x^2+16}{16}\right) = +\infty$$

$$\begin{cases} y'' + 2y = e^{-2x} + e^{x} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Il pb. è sovradeterminato.

Osserviamo però che valutando l'eq. differenziale in $x=0$ e sostituendo le condizioni richieste si ottiene

$$0 + 2 \cdot 1 = e^{-2 \cdot 0} + e^{0} \qquad \text{cioè } 2 = 2$$

E' dunque sufficiente risolvere il pb

$$\begin{cases} y' + 2y = e^{-2x} + e^{x} \\ y(0) = 1 \end{cases}$$

OMOGENEA $\quad y' + 2y = 0 \qquad y_0(x) = C e^{-2x}$

Cerco la solut. nella forma $\quad y(x) = C(x) e^{-2x}$

$$y(x) = C(x) e^{-2x} \Rightarrow y'(x) = C'(x) e^{-2x} - 2C(x) e^{-2x}$$

$$C'(x) e^{-2x} \cancel{- 2C(x) e^{-2x}} + \cancel{2 \cdot C(x) e^{-2x}} = e^{-2x} + e^{x}$$

$$C'(x) = 1 + e^{3x}$$

$$C(x) = x + \frac{1}{3} e^{3x} + K$$

$$y(x) = \left(x + \frac{1}{3} e^{3x} + K \right) e^{-2x} = (x+K) e^{-2x} + \frac{1}{3} e^{x}$$

$$y(0) = 1 \Rightarrow 1 = K + \frac{1}{3} \Rightarrow K = \frac{2}{3}$$

$$\Rightarrow y(x) = \left(x + \frac{2}{3} \right) e^{-2x} + \frac{1}{3} e^{x}$$