lunedì 16 dicembre 2019 09:36

$ay'' + by' + cy = g(x)$ $\quad a \neq 0 \quad a, b, c \in \mathbb{R}$ $\quad g$ funzione continua

$ay'' + by' + cy = 0$

$a\lambda^2 + b\lambda + c = 0 \qquad \Delta = b^2 - 4ac$

$\Delta > 0 \Rightarrow \lambda_1 < \lambda_2 \quad \lambda_1, \lambda_2 \in \mathbb{R}$

$$y_1(x) = e^{\lambda_1 x} \qquad y_2(x) = e^{\lambda_2 x}$$

sono soluz. dell' eq. omogenea

$\forall c_1, c_2 \in \mathbb{R} \qquad y(x) = c_1 y_1(x) + c_2 y_2(x)$ è soluzione. e non ne esistono altre

$\Delta = 0 \qquad \bar{\lambda} \in \mathbb{R}$ radice doppia $\quad a\lambda^2 + b\lambda + c = a(\lambda - \bar{\lambda})^2$

$$y_1(x) = e^{\bar{\lambda} x} \qquad y_2(x) = x\, e^{\bar{\lambda} x}$$

Le soluzioni dell' eq. omogenea sono tutte e sole delle forme

$$y_1(x) = c_1 y_1(x) + c_2 y_2(x) =$$
$$= c_1 e^{\bar{\lambda} x} + c_2 x e^{\bar{\lambda} x} = e^{\bar{\lambda} x}(c_1 + c_2 x)$$

$\Delta < 0 \qquad a\lambda^2 + b\lambda + c = 0 \qquad \lambda_{1,2} = \dfrac{-b \pm i\sqrt{-\Delta}}{2a}$

$$\lambda_{1,2} = \frac{-b}{2a} \pm i \frac{\sqrt{-\Delta}}{2a}$$

$$\frac{-b}{2a} =: \alpha \qquad \frac{\sqrt{-\Delta}}{2a} =: \omega$$

$$y_1(x) = e^{\alpha x}\cos(\omega x)$$
$$y_2(x) = e^{\alpha x}\sin(\omega x)$$

Le soluzioni dell' omogenea sono tutte e sole le funzioni:

$$y(x) = c_1 y_1(x) + c_2 y_2(x) = \underline{e^{\alpha x}\left(c_1 \cos(\omega x) + c_2 \sin(\omega x)\right)}$$

$c_1 \cos(\omega x) + c_2 \sin(\omega x)$

$$R := \sqrt{c_1^2 + c_2^2}$$

$$c_1 \cos(\omega x) + c_2 \sin(\omega x) = R\left(\frac{c_1}{R}\cos(\omega x) + \frac{c_2}{R}\sin(\omega x)\right)$$

$$\left(\frac{c_1}{R}\right)^2 + \left(\frac{c_2}{R}\right)^2 = \frac{c_1^2 + c_2^2}{R^2} = \frac{R^2}{R^2} = 1$$

$\exists! \varphi \in [0, 2\pi)$ t.c. $\cos(\varphi) = \frac{c_1}{R}$ $\quad \sin(\varphi) = \frac{c_2}{R}$

$\exists! \psi \in [0, 2\pi)$ t.c. $\sin(\psi) = \frac{c_1}{R}$ $\quad \cos(\psi) = \frac{c_2}{R}$

$$c_1 \cos(\omega x) + c_2 \sin(\omega x) = R\left(\cos(\varphi)\cos(\omega x) + \sin(\varphi)\sin(\omega x)\right)$$
$$= R \sin(\omega x + \varphi)$$

$$y(x) = R e^{\alpha x} \sin(\omega x + \varphi) \longleftarrow$$

$$a y'' + b y' + c y = g(x)$$

$\bar{y}(x)$ soluzione in (a,b)

$y(x)$ soluzione in (a,b)

$\forall x \in (a,b)$

$$\begin{cases} a\bar{y}''(x) + b\bar{y}'(x) + c\bar{y}(x) = g(x) \\ a y''(x) + b y'(x) + c y(x) = g(x) \end{cases}$$

$$a(\bar{y}''(x) - y''(x)) + b(\bar{y}'(x) - y'(x)) + c(\bar{y}(x) - y(x)) = 0 \Leftarrow$$

$$y_0(x) := \bar{y}(x) - y(x)$$

$$a y_0''(x) + b y_0'(x) + c y_0(x) = 0$$

$\forall \bar{y}, y$ soluzioni dell' eq. completa $\Rightarrow y_0 := \bar{y} - y$ è soluzione dell' eq. omogenea associata.

$$y_0 = \bar{y} - y \quad \Rightarrow \quad \bar{y} = y_0 + y$$

$y_0 = \bar{y} - y \quad \Rightarrow \quad \bar{y} = y_0 + y$

— o —

$y'' - 3y' = (x^3 - 1)e^x$

$y'' - 3y' = 0 \qquad \lambda^2 - 3\lambda = 0 \qquad \lambda(\lambda - 3) = 0$

$\lambda_1 = 0 \qquad \lambda_2 = 3$

$y_1(x) = e^{0x} = 1 \qquad y_2(x) = e^{3x}$

Le soluzioni dell'eq. omogenea sono tutte e sole le funzioni

$y_0(x) = c_1 \cdot 1 + c_2 e^{3x} = c_1 + c_2 e^{3x} \qquad c_1, c_2 \in \mathbb{R}$

Cerco $y(x)$ soluzione dell'eq. completa della forma

$y(x) = c_1(x) + c_2(x) e^{3x}$

$y'(x) = \boxed{c_1'(x) + c_2'(x) e^{3x}} + \underline{c_2(x) 3 e^{3x}}$

$c_1'(x) + c_2'(x) e^{3x} = 0$

$y''(x) = c_2'(x) 3 e^{3x} + c_2(x) 9 e^{3x}$

$y'' - 3y' = (x^3 - 1)e^x$

$c_2'(x) 3 e^{3x} + \cancel{c_2(x) 9 e^{3x}} - 3 \cancel{c_2(x) 3 e^{3x}} = (x^3 - 1)e^x$

$$\begin{cases} c_1'(x) + c_2'(x) e^{3x} = 0 \\ 3 c_2'(x) e^{3x} = (x^3 - 1) e^x \end{cases}$$

$\longrightarrow \quad c_2'(x) = \frac{1}{3}(x^3 - 1) e^{-2x}$

$c_1'(x) = -c_2'(x) e^{3x} = -\frac{1}{3}(x^3 - 1) e^x$

$$C_1'(x) = -C_2'(x)\,e^{3x} = -\frac{1}{3}(x^3-1)\,e^{x}$$

$$\longrightarrow C_2(x) = \int \frac{1}{3}(x^3-1)e^{-2x}\,dx$$

$$C_1(x) = \int -\frac{1}{3}(x^3-1)e^{x}\,dx$$

— o —

$$C_1(x) = -\frac{1}{3}\int \underbrace{(x^3-1)}_{f}\underbrace{e^{x}}_{h'}\,dx \qquad \begin{aligned} f(x) &= x^3-1 \\ h(x) &= e^{x}\end{aligned}$$

$$= -\frac{1}{3}\left\{(x^3-1)e^{x} - \int 3x^2 e^{x}\,dx\right\} =$$

$$= -\frac{1}{3}(x^3-1)e^{x} + \int \underbrace{x^2}_{f}\underbrace{e^{x}}_{h'}\,dx \qquad \begin{aligned} f(x) &= x^2 \\ h(x) &= e^{x}\end{aligned}$$

$$= -\frac{1}{3}(x^3-1)e^{x} + x^2e^{x} - \int \underbrace{2x}_{f}\underbrace{e^{x}}_{h'}\,dx \qquad \begin{aligned} f(x) &= 2x \\ h(x) &= e^{x}\end{aligned}$$

$$= -\frac{1}{3}(x^3-1)e^{x} + x^2e^{x} - \left(2xe^{x} - \int 2e^{x}\,dx\right) =$$

$$= -\frac{1}{3}(x^3-1)e^{x} + x^2e^{x} - 2xe^{x} + 2\int e^{x}\,dx$$

$$= -\frac{1}{3}(x^3-1)e^{x} + x^2e^{x} - 2xe^{x} + 2e^{x} + C_1$$

$$= e^{x}\left(-\frac{1}{3}x^3 + x^2 - 2x + \frac{7}{3}\right) + C_1$$

$$C_2(x) = \int -\frac{1}{3}(x^3-1)e^{-2x}\,dx = -\frac{1}{3}\int \underbrace{(x^3-1)}_{f}\underbrace{e^{-2x}}_{h'}\,dx$$

$$= -\frac{1}{3}\left\{(x^3-1)\cdot\left(-\frac{1}{2}e^{-2x}\right) - \int 3x^2\cdot\left(-\frac{1}{2}e^{-2x}\right)dx\right\} \qquad \Big|\quad f(x) = x^3-1$$

$$= -\frac{1}{3}\left\{ (x^3-1)\cdot\left(-\frac{1}{2}e^{-2x}\right) - \int 3x^2\cdot\left(-\frac{1}{2}e^{-2x}\right)dx \right\}$$

$$f(x) = x^3-1 \qquad h(x) = -\frac{1}{2}e^{-2x}$$

$$= \frac{1}{6}(x^3-1)e^{-2x} - \frac{1}{2}\int \underbrace{x^2}_{f}\,\underbrace{e^{-2x}}_{h'}\,dx$$

$$f(x)=x^2; \quad h(x) = -\frac{1}{2}e^{-2x}$$

$$= +\frac{1}{6}(x^3-1)e^{-2x} - \frac{1}{2}\left(x^2\cdot\left(-\frac{1}{2}e^{-2x}\right) + \int \cancel{2}x\cdot\left(+\frac{1}{\cancel{2}}e^{-2x}\right)dx\right)$$

$$= \frac{1}{6}(x^3-1)e^{-2x} + \frac{1}{4}x^2e^{-2x} - \frac{1}{2}\int \underbrace{x}_{f}\,\underbrace{e^{-2x}}_{h'}\,dx$$

$$f(x) = x \qquad h(x) = -\frac{1}{2}e^{-2x}$$

$$= \frac{1}{6}(x^3-1)e^{-2x} + \frac{1}{4}x^2e^{-2x} - \frac{1}{2}\left(x\cdot\left(-\frac{1}{2}e^{-2x}\right) + \int +\frac{1}{2}e^{-2x}dx\right)$$

$$= \frac{1}{6}(x^3-1)e^{-2x} + \frac{1}{4}x^2e^{-2x} + \frac{1}{4}xe^{-2x} - \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C_2$$

$$= e^{-2x}\left(\frac{1}{6}x^3 + \frac{1}{4}x^2 + \frac{1}{4}x + \frac{1}{12}\right) + C_2 \qquad \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12}$$

$$y(x) = e^x\left(-\frac{1}{3}x^3 + x^2 - 2x + \frac{7}{3} + C_1\right) + \left(e^{-2x}\left(\frac{1}{6}x^3 + \frac{1}{4}x^2 + \frac{1}{4}x + \frac{1}{12}\right) + C_2\right)\cdot e^{3x}$$

$$= \underbrace{C_1e^x + C_2e^{3x}} + e^x\left(-\frac{1}{3}x^3 + x^2 - 2x + \frac{7}{3}\right)$$

$$+ e^x\left(\frac{1}{6}x^3 + \frac{1}{4}x^2 + \frac{1}{4}x + \frac{1}{12}\right)$$

$$= \underbrace{C_1e^x} + C_2e^{3x} + e^x\left(-\frac{1}{6}x^3 + \frac{5}{4}x^2 - \frac{7}{4}x + \underbrace{\left(\frac{7}{3} + \frac{1}{12}\right)}\right)$$

—— o ——

$$y'' - 2y = (x-1)\sin(x) \quad \leftarrow$$

$$y'' - 2y = 0 \qquad \lambda^2 - 2 = 0 \qquad \lambda_1 = \sqrt{2} \qquad \lambda_2 = -\sqrt{2}$$

$$y_0(x) = C_1e^{x\sqrt{2}} + C_2e^{-x\sqrt{2}}$$

$$y_0(x) = C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}}$$

$$y(x) = C_1(x) e^{x\sqrt{2}} + C_2(x) e^{-x\sqrt{2}}$$

$$y'(x) = \boxed{C_1'(x) e^{x\sqrt{2}}} + \sqrt{2}\, C_1(x) e^{x\sqrt{2}} \boxed{+ C_2'(x) e^{-x\sqrt{2}}} - \sqrt{2}\, C_2(x) e^{-x\sqrt{2}}$$

$$C_1'(x) e^{x\sqrt{2}} + C_2'(x) e^{-x\sqrt{2}} = 0$$

$$y''(x) = \sqrt{2}\, C_1'(x) e^{x\sqrt{2}} + C_1(x)\, 2 e^{x\sqrt{2}} - \sqrt{2}\, C_2'(x) e^{-x\sqrt{2}} + C_2(x)\, 2 e^{-x\sqrt{2}}$$

$$= \sqrt{2}\, C_1'(x) e^{x\sqrt{2}} - \sqrt{2}\, C_2'(x) e^{-x\sqrt{2}} + 2\left(C_1(x) e^{x\sqrt{2}} + C_2(x) e^{-x\sqrt{2}}\right)$$

$$y'' - 2y = (x-1)\sin(x)$$

$$\sqrt{2}\, C_1'(x) e^{x\sqrt{2}} - \sqrt{2}\, C_2'(x) e^{-x\sqrt{2}} + 2\left(\cancel{C_1(x) e^{x\sqrt{2}} + C_2(x) e^{-x\sqrt{2}}}\right) - 2\left(\cancel{C_1(x) e^{x\sqrt{2}} + C_2(x) e^{-x\sqrt{2}}}\right) = (x-1)\sin(x)$$

$$\begin{cases} C_1'(x) e^{x\sqrt{2}} + C_2'(x) e^{-x\sqrt{2}} = 0 \\ \sqrt{2}\, C_1'(x) e^{x\sqrt{2}} - \sqrt{2}\, C_2'(x) e^{-x\sqrt{2}} = (x-1)\sin(x) \end{cases}$$

$$\begin{cases} C_2'(x) e^{-x\sqrt{2}} = -C_1'(x) e^{x\sqrt{2}} \\ \sqrt{2}\, C_1'(x) e^{x\sqrt{2}} + \sqrt{2}\, C_1'(x) e^{x\sqrt{2}} = (x-1)\sin(x) \end{cases}$$

$$2\sqrt{2}\, C_1'(x) e^{x\sqrt{2}} = (x-1)\sin(x)$$

$$\begin{cases} C_1'(x) = \dfrac{(x-1)\sin(x)}{2\sqrt{2}} e^{-x\sqrt{2}} \end{cases}$$

$2\sqrt{2}$

$$c_2'(x)\, e^{-x\sqrt{2}} = - \cancel{e^{x\sqrt{2}}} \cdot \frac{(x-1)\sin(x)}{2\sqrt{2}} \cancel{e^{-x\sqrt{2}}}$$

$$\begin{cases} c_1'(x) = \dfrac{(x-1)\sin(x)}{2\sqrt{2}} e^{-x\sqrt{2}} \\[2ex] c_2'(x) = \dfrac{-(x-1)\sin(x)\, e^{x\sqrt{2}}}{2\sqrt{2}} \end{cases}$$

Risolvere

$$ay'' + by' + cy = g(x) \qquad a,b,c \in \mathbb{R} \quad a \neq 0$$

1) $g(x) = P_n(x)\, e^{\gamma x}$ $P_n =$ polinomio di grado n, $\gamma \in \mathbb{R}$

$$a\lambda^2 + b\lambda + c = 0$$

(A) γ non è radice dell'eq. caratteristica $\Rightarrow \exists\, \bar{y}(x)$ soluzione della forma $\bar{y}(x) = Q_n(x)\, e^{\gamma x}$ dove Q_n è un polinomio di grado $\leq n$

(B) γ è radice semplice dell'eq. caratteristica $\Rightarrow \exists\, \bar{y}(x)$ soluzione della forma $\bar{y}(x) = x \cdot Q_n(x)\, e^{\gamma x}$ dove Q_n è un polinomio di grado $\leq n$

(C) γ è radice doppia dell'eq. caratteristica $\Rightarrow \exists\, \bar{y}(x)$ soluzione della forma $\bar{y}(x) = x^2 Q_n(x)\, e^{\gamma x}$ dove Q_n è un polinomio di grado $\leq n$.

(2) $g(x) = e^{\gamma x}\left(P_n(x)\cos(\omega x) + Q_m(x)\sin(\omega x)\right)$

(A) Se $\lambda := \gamma + i\omega$ non è radice del polinomio caratteristico $\Rightarrow \exists \bar{y}(x)$ soluzione di questo tipo

$$\bar{y}(x) = e^{\gamma x}\left(S_N(x)\cos(\omega x) + T_N(x)\sin(\omega x)\right)$$

dove $N := \max\{n, m\}$ e S_N e T_N sono polinomi di grado $\leq N$

(B) Se $\lambda := \gamma + i\omega$ è radice del polinomio caratteristico $\Rightarrow \exists \bar{y}(x)$ soluzione del tipo:

$$\bar{y}(x) = x e^{\gamma x}\left(S_N(x)\cos(\omega x) + T_N(x)\sin(\omega x)\right)$$

con N, S_N e T_N come sopra.

$$y'' - 2y = (x-1)\sin(x)$$

$$y'' - 2y = 0 \qquad \lambda^2 - 2 = 0 \qquad \lambda_1 = \sqrt{2} \quad \lambda_2 = -\sqrt{2}$$

$$y_0(x) = C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}}$$

$$g(x) = P_1(x)\sin(1\cdot x)\, e^{0x} \Leftarrow \qquad \lambda = 0 + i\cdot 1 = i$$

$$\bar{y}(x) = \left((\alpha x + \beta)\sin(x) + (\gamma x + \delta)\cos(x)\right) e^{0x}$$
$$= (\alpha x + \beta)\sin(x) + (\gamma x + \delta)\cos(x)$$

$$\bar{y}'(x) = \alpha\sin(x) + (\alpha x + \beta)\cos(x) + \gamma\cos(x) - (\gamma x + \delta)\sin(x)$$
$$= (-\gamma x - \delta + \alpha)\sin(x) + (\alpha x + \beta + \gamma)\cos(x)$$

$$\bar{y}''(x) = -\gamma\sin(x) + (-\gamma x - \delta + \alpha)\cos(x) + \alpha\cos(x)$$
$$- (\alpha x + \beta + \gamma)\sin(x)$$

$$-(\alpha x+\beta+\gamma)\sin(x)$$

$$=(-\alpha x-\beta-2\gamma)\sin(x)+(-\gamma x-\delta+2\alpha)\cos(x)$$

$$y''-2y=(x-1)\sin(x)$$

$$(-\alpha x-\beta-2\gamma)\sin(x)+(-\gamma x-\delta+2\alpha)\cos(x)$$
$$-2\Big((\alpha x+\beta)\sin(x)+(\gamma x+\delta)\cos(x)\Big)=(x-1)\sin(x)$$

$$\sin(x)\left(-\alpha x-\beta-2\gamma-2\alpha x-2\beta\right)+$$
$$+\cos(x)\left(-\gamma x-\delta+2\alpha-2\gamma x-2\delta\right)=(x-1)\sin(x)$$

$$\sin(x)\left(\underline{-3\alpha x}-3\beta-2\gamma\right)+\cos(x)\left(\underline{-3\gamma x-3\delta+2\alpha}\right)$$
$$=\sin(x)(1x-1)$$

$$-3\gamma=0$$
$$-3\delta+2\alpha=0$$

$$-3\alpha=1$$
$$-3\beta-2\gamma=-1$$

$$\begin{cases}-3\alpha=1\\ -3\beta-2\gamma=-1\\ -3\gamma=0\\ -3\delta+2\alpha=0\end{cases}$$

$$\alpha=-\frac{1}{3}$$
$$-3\beta=-1 \qquad \beta=\frac{1}{3}$$
$$\gamma=0$$
$$3\delta=2\alpha=-\frac{2}{3} \qquad \delta=-\frac{2}{9}$$

$$\bar{y}(x)=\left(-\frac{1}{3}x+\frac{1}{3}\right)\sin(x)+\left(-\frac{2}{9}\right)\cos(x)$$

$$y(x)=C_1e^{x\sqrt{2}}+C_2e^{-x\sqrt{2}}+\left(\frac{1}{3}-\frac{1}{3}x\right)\sin(x)-\frac{2}{9}\cos(x)$$

— o —

$$u''+u=xe^{x}+\cos(x)$$

$$y''+y=xe^{x}$$

$$\bar{y}_1(x)$$

$$y'' + y = \underbrace{x e^{x}}\ + \underbrace{\cos(x)}$$

$y'' + y = x e^{x}$ $\quad \bar{y}_1(x)$

$y'' + y = \cos(x)$ $\quad \bar{y}_2(x)$

y_1 soluzione di $\quad a y'' + b y' + c y = g_1(x)$

y_2 soluzione di $\quad a y'' + b y' + c y = g_2(x)$

$\alpha_1, \alpha_2 \in \mathbb{R}$ $\qquad y(x) := \alpha_1 y_1(x) + \alpha_2 y_2(x) \leftarrow$

$$\begin{cases} \alpha_1 \left(a\, y_1''(x) + b y_1'(x) + c y_1(x) \right) = g_1(x)\, \alpha_1 \\ \alpha_2 \left(a\, y_2''(x) + b y_2'(x) + c y_2(x) \right) = g_2(x)\, \alpha_2 \end{cases}$$

$$a \Big(\underbrace{\alpha_1 y_1''(x) + \alpha_2 y_2''(x)}_{y''(x)} \Big) + b \Big(\underbrace{\alpha_1 y_1'(x) + \alpha_2 y_2'(x)}_{y'(x)} \Big) +$$

$$+ c \Big(\underbrace{\alpha_1 y_1(x) + \alpha_2 y_2(x)}_{y(x)} \Big) = \alpha_1 g_1(x) + \alpha_2 g_2(x)$$

$\Rightarrow y(x) := \alpha_1 y_1(x) + \alpha_2 y_2(x)$ risolve l'EDO

$$a y'' + b y' + c y = \alpha_1 g_1(x) + \alpha_2 g_2(x)$$

— o —

$y'' + y = 0$ è l'omogenea associata

Polinomio caratteristico: $\lambda^2 + 1 = 0$ $\qquad \lambda_{1,2} = \pm i =$

$y_0(x) = C_1 \cos(x) + C_2 \sin(x)$ $\qquad = 0 \pm i \cdot 1$

$$y'' + y = x e^{x} + \cos(x)$$

— o —

$y'' + y = x e^{x}$ $\qquad g_1(x) = e^{1 \cdot x} \cdot P_1(x)$

$$\bar{y}_1(x) = e^{x}(\alpha x + \beta)$$

$$\bar{y}_1'(x) = e^{x} \cdot \alpha + e^{x}(\alpha x + \beta) = e^{x}(\alpha x + \alpha + \beta)$$

$$\bar{y}_1'(x) = e^x \cdot \alpha + e^x(\alpha x+\beta) = e^x(\alpha x+\alpha+\beta)$$

$$\bar{y}_1''(x) = e^x \cdot \alpha + e^x(\alpha x+\alpha+\beta) = e^x(\alpha x + 2\alpha + \beta)$$

$$e^x(\alpha x+2\alpha+\beta) + e^x(\alpha x+\beta) = x \cdot e^x$$

$$e^x(\underbrace{2\alpha}x + \underbrace{2\alpha+2\beta}) = x \cdot e^x$$

$$\begin{cases} 2\alpha = 1 \\ 2\alpha + 2\beta = 0 \end{cases}$$

$$\alpha = \frac{1}{2} \qquad \beta = -\alpha = -\frac{1}{2}$$

$$\bar{y}_1(x) = e^x\left(\frac{1}{2}x - \frac{1}{2}\right) = \frac{1}{2}e^x(x-1)$$

$$\boxed{y'' + y = \cos(x)}$$

$$g_2(x) = \cos(x) = e^{0x} \cdot 1 \cos(1 \cdot x)$$

$$\lambda = 0 + i1 = i$$

è radice del polinomio caratteristico

$$\bar{y}_2(x) = x(\gamma\cos(x) + \delta\sin(x))$$

$$\bar{y}_2'(x) = \gamma\cos(x) + \delta\sin(x) + x(-\gamma\sin(x) + \delta\cos(x))$$

$$\bar{y}_2''(x) = -\gamma\sin(x) + \delta\cos(x) - \gamma\sin(x) + \delta\cos(x) +$$
$$+ x(-\gamma\cos(x) - \delta\sin(x))$$
$$= -2\gamma\sin(x) + 2\delta\cos(x) - \gamma x\cos(x) - \delta x\sin(x)$$

$$\bar{y}_2'' + \bar{y}_2 = \cos(x)$$

$$-\underbrace{2\gamma}\sin(x) + \underbrace{2\delta}\cos(x) \cancel{-\gamma x\cos(x)} \cancel{-\delta x\sin(x)}$$
$$+ x(\cancel{\gamma\cos(x)} + \cancel{\delta\sin(x)}) = \cos(x)$$

$$\cos(x)\, 2\delta - 2\gamma\sin(x) = \cos(x)$$

$$\begin{cases} 2\delta = 1 \\ -2\gamma = 0 \end{cases} \qquad \delta = 1/2 \qquad \gamma = 0$$

$$\bar{y}_2(x) = x\left(0\cos(x) + \frac{1}{2}\sin(x)\right) = \frac{1}{2}x\sin(x)$$

$$y(x) = c_1\cos(x) + c_2\sin(x) + \frac{1}{2}e^x(x-1) + \frac{1}{2}x\sin(x)$$

$$y'' - 3y' + 2y = (x-1)(e^x - 1)$$

$$g(x) = (x-1)(e^x-1) = (x-1)e^x - (x-1) = g_1(x) - g_2(x)$$

$g_1(x) = (x-1)e^x$

$\bar{y}_2$ risolve

$y'' - 3y' + 2y = (x-1)e^x$

$g_2(x) = x-1$

$\bar{y}_2$ risolve

$y'' - 3y' + 2y = x-1$

$\Rightarrow$ So che $\bar{y}(x) := \bar{y}_1(x) - \bar{y}_2(x)$ risolve la mia eq.

OMOGENEA: $\quad y'' - 3y' + 2y = 0$

Polinomio $\quad \lambda^2 - 3\lambda + 2 = 0 \qquad \Delta = 9 - 8 = 1$

$$\lambda_{1,2} = \frac{3 \pm 1}{2} \begin{cases} 2 \\ 1 \end{cases}$$

$$y_0(x) = c_1 e^{2x} + c_2 e^x$$

$$y'' - 3y' + 2y = (x-1)e^x \qquad g_1(x) = (x-1)e^{1\cdot x}$$

$\gamma = 1$

$$\bar{y}_1(x) = x(\alpha x + \beta)e^x = (\alpha x^2 + \beta x)e^x$$

$$\bar{y}_1'(x) = (2\alpha x + \beta)e^x + (\alpha x^2 + \beta x)e^x$$

$$= (\alpha x^2 + (\beta + 2\alpha)x + \beta)e^x$$

$$\bar{y}_1''(x) = (\underline{2\alpha x} + \underline{\beta + 2\alpha})e^x + (\alpha x^2 + \underline{(\beta+2\alpha)}x + \underline{\beta})e^x$$

$$= e^x(\alpha x^2 + (\beta + 4\alpha)x + 2\beta + 2\alpha)$$

$$e^x(\cancel{\alpha x^2} + (\beta+4\alpha)x + 2\beta + 2\alpha)$$

$$-3e^x(\cancel{\alpha x^2} + (\beta+2\alpha)x + \beta)$$

$$+2e^x(\cancel{\alpha x^2} + \beta x) \quad = \quad (x-1)e^x$$

$$e^x\left(x(\cancel{\beta} + 4\alpha - \cancel{3\beta} - 6\alpha + \cancel{2\beta}) + 2\beta + 2\alpha - 3\beta\right) = (x-1)e^x$$

$$e^x(-2\alpha x + 2\alpha - \beta) = (x-1)e^x$$

$$\begin{cases} -2\alpha = 1 & \alpha = -\frac{1}{2} \\ 2\alpha - \beta = -1 & \beta = 2\alpha + 1 = 2 \cdot \frac{-1}{2} + 1 = 0 \end{cases}$$

$$\bar{y}_1(x) = x\left(-\frac{1}{2}x + 0\right)e^x = -\frac{1}{2}x^2 e^x$$

— o —

$$y'' - 3y' + 2y = x - 1$$

$g_2(x) = e^{0x} P_1(x)$ $\qquad \lambda = 0$ non è radice del pol. caratteristico

$$\bar{y}_2(x) = (\gamma x + \delta)e^{0x} = \gamma x + \delta$$

$$\bar{y}_2'(x) = \gamma$$

$$\bar{y}_2''(x) = 0$$

$$0 - 3\gamma + 2(\gamma x + \delta) = x - 1$$

$$\underline{2\gamma}\, x + (\underline{2\delta - 3\gamma}) = \underline{x} - \underline{1} \qquad \begin{cases} 2\gamma = 1 \\ 2\delta \ldots \end{cases}$$

$$2\gamma x + (2\delta - 3\gamma) = x - 1 \qquad \begin{cases} 2\gamma = 1 \\ 2\delta - 3\gamma = -1 \end{cases}$$

$$\gamma = \frac{1}{2} \qquad 2\delta = 3\gamma - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\delta = \frac{1}{4}$$

$$\overline{y}_2(x) = \frac{1}{2}x + \frac{1}{4}$$

$$y(x) = C_1 e^{x} + C_2 e^{2x} + \left(-\frac{1}{2}x^2 e^{x}\right) - \left(\frac{1}{2}x + \frac{1}{4}\right)$$

$$= \left(-\frac{1}{2}x^2 + C_1\right)e^{x} + C_2 e^{2x} - \frac{1}{2}x - \frac{1}{4}$$