$$\lim_{x\to+\infty}\left(1+\frac{r}{x}\right)^x = e^r \qquad \forall r\in\mathbb{R}$$

$$\lim_{x\to-\infty}\left(1+\frac{r}{x}\right)^x = e^r$$

$y=-x \Rightarrow x\to-\infty$ SSE $y\to+\infty$

$$\left(1+\frac{r}{x}\right)^x = \left(1+\frac{(-r)}{y}\right)^{-y} = \Bigg(\underbrace{\left(1+\frac{-r}{y}\right)^y}_{\hookrightarrow e^{-r}}\Bigg)^{-1} \to \left(e^{-r}\right)^{-1} = e^r$$

$$\lim_{x\to 0}(1+x)^{1/x}$$

$$\lim_{x\to 0^+}(1+x)^{1/x}$$

$y=\frac{1}{x} \Rightarrow x\to 0^+$ SSE $y\to+\infty$

$$(1+x)^{\frac{1}{x}} = \left(1+\frac{1}{y}\right)^y \to e$$

$$\lim_{x\to 0^-}(1+x)^{\frac{1}{x}}$$

$y=\frac{1}{x}$ $x\to 0^-$ SSE $y\to-\infty$

$(1+x)^{\frac{1}{x}} = \left(1+\frac{1}{y}\right)^y \to e$ quando $y\to-\infty$

$\Rightarrow$ limite destro e limite sinistro sono uguali

$\Rightarrow \exists \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

——— o ———

FUNZIONI ESPONENZIALI

$a\in\mathbb{R}$, $p>0$ $\qquad f(x) = a p^x \qquad f: x\in\mathbb{R} \mapsto a p^x \in\mathbb{R}$

1) $a=0 \Rightarrow f(x)=0$

2) $p=1 \Rightarrow f(x)=a$

$a>0 \quad p>1$ — Se $x_1<x_2 \Rightarrow p^{x_1}<p^{x_2}$ ←

$\Rightarrow a p^{x_1} < a p^{x_2}$

$\Rightarrow f$ è strettamente monotona crescente.

$a>0 \quad p<1$ Se $x_1<x_2$ $p^{x_1} > p^{x_2}$

$\Rightarrow a p^{x_1} > a p^{x_2}$

$\Rightarrow f$ è strettamente monotona decrescente

La situazione si rovescia se $a<0$.

$$\lim_{x\to+\infty} a p^x = \begin{cases} +\infty & a>0 \quad p>1 \\ -\infty & a<0 \quad p>1 \\ 0 & \forall a \quad p\in(0,1) \end{cases}$$

$$\lim_{x\to-\infty} a p^x = \begin{cases} 0 & \forall a \quad p>1 \\ +\infty & a>0 \quad p\in(0,1) \\ -\infty & a<0 \quad p\in(0,1) \end{cases}$$

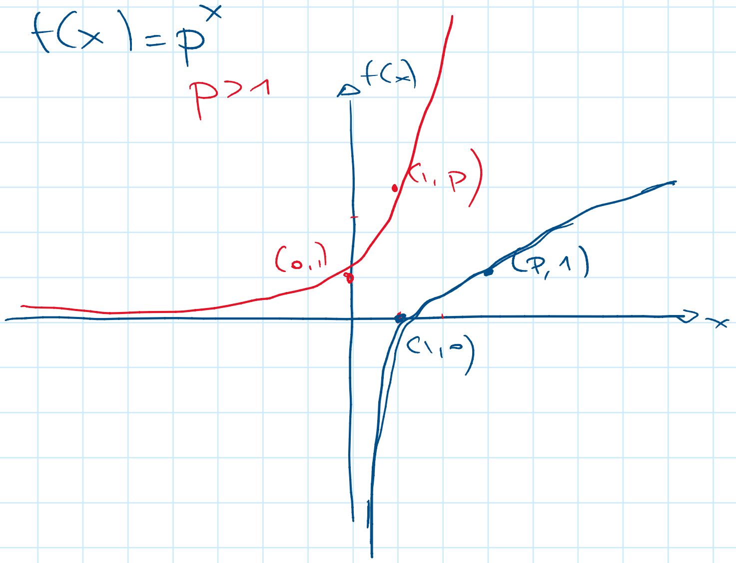
$x\to-\infty \quad y=-x\to+\infty$

$a p^x = a p^{-y} = \dfrac{a}{p^y}$

— o —

$f(x)=p^x$

$p>1$



$f: \mathbb{R} \to (0,+\infty)$

è biunivoca e dunque invertibile.

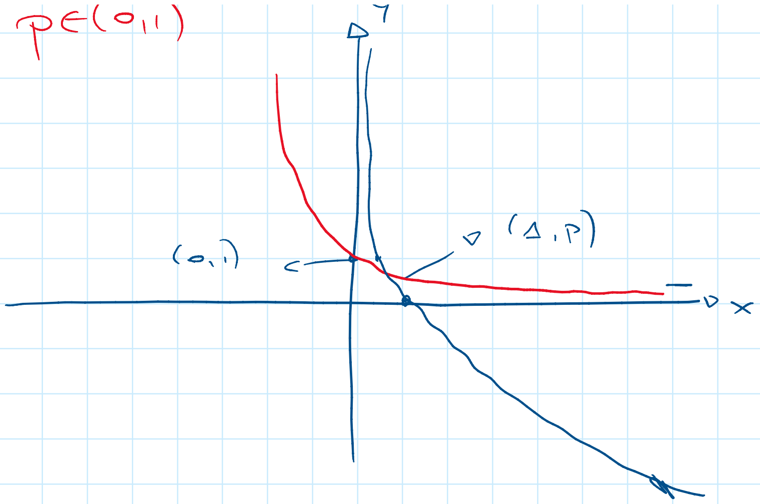
$f^{-1}: (0,+\infty) \to \mathbb{R}$

f^{-1} si dice LOGARITMO IN BASE p e si indica $\log_p$

$p\in(0,1)$

$f(x)=p^x$

$p\in(0,1)$



$$f(x)=p^x$$

$$f:\mathbb{R}\to(0,+\infty)$$

$$f^{-1}:(0,+\infty)\to\mathbb{R}$$

$$\log_p$$

$p\in(0,1)\cup(1,+\infty)$

$$\log_p(p^x)=x \quad \forall x\in\mathbb{R}$$

$$p^{\log_p(x)}=x \quad \forall x>0$$

$$\log_p(1)=0 \qquad \log_p(p)=1$$

$a,b>0 \qquad \exists!\; x>0 \quad \exists!\; y>0$ T.c. $a=p^x \quad b=p^y$

$$\left(x=\log_p(a) \qquad y=\log_p(b)\right)$$

$$ab=p^x\cdot p^y=p^{x+y}$$

$$\to \log_p(ab)=\log_p\left(p^{x+y}\right)=x+y=\log_p(a)+\log_p(b) \leftarrow$$

$b=a \qquad \log_p(a^2)=2\log_p(a)$

$b=a^2 \qquad \log_p(a^3)=\log_p(a)+2\log_p(a)=3\log_p(a)$

$$\log_p(a^n)=n\log_p(a)$$

$b=\frac{1}{a} \qquad 0=\log_p(1)=\log_p(a)+\log_p\left(\frac{1}{a}\right)$

$$\log_p\left(\frac{1}{a}\right)=-\log_p(a)$$

$$\log_p\left(\frac{a}{b}\right)=\log_p\left(a\cdot\frac{1}{b}\right)=\log_p(a)+\log_p\left(\frac{1}{b}\right)$$

$$\log_p\left(\frac{a}{b}\right) = \log_p\left(a \cdot \frac{1}{b}\right) = \log_p(a) + \log_p\left(\frac{1}{b}\right)$$
$$= \log_p(a) - \log_p(b)$$

$q > 0 \ , q \neq 1$ $\qquad p = q^{\log_q(p)}$

$$p^x = \left(q^{\log_q(p)}\right)^x = q^{x \log_q(p)} \Leftarrow$$

$$\log_q(p^x) = \log_q\left(q^{x \log_q(p)}\right) = x \cdot \log_q(p) \Leftarrow$$

$$x = \frac{\log_q(y)}{\log_q(p)}$$

$y > 0$

$\log_q(p^x) = \log_q(y)$

$p^x = y$

$$p^{\frac{\log_q(y)}{\log_q(p)}} = q^{\log_q(y)} = y$$

$$\log_p\left(p^{\frac{\log_q(y)}{\log_q(p)}}\right) = \log_p(y)$$

$$\frac{\log_q(y)}{\log_q(p)} = \log_p(y)$$

$q = \frac{1}{p}$ $\qquad \log_p(y) = \frac{\log_{\frac{1}{p}}(y)}{\log_{\frac{1}{p}}(p)}$

$x = \log_{\frac{1}{p}}(p)$

$\left(\frac{1}{p}\right)^x = p \Rightarrow x = -1$

$$\log_p(y) = -\log_{\frac{1}{p}}(y)$$

p^x $\qquad p = e^{\log_e(p)}$

$$p^x = \left(e^{\log_e(p)}\right)^x = e^{x \log_e(p)}$$

$p^x = e^{cx}$ $\qquad c := \log_e(p) \begin{cases} > 0 & p > 1 \\ < 0 & p \in (0,1) \end{cases}$

$| < 0$ $\quad p \in (0,1)$

e $\quad e^x = \exp(x)$

$\log_e$ LOGARITMO NEPERIANO $\quad \ln$ $\quad \log$ $\quad | \; \log_{10} = \log \quad \log_{10}$

$$\lim_{x \to 1} \frac{x^4 - 7x^3 + 17x^2 - 17x + 6}{x^4 - 2x^3 - 15x^2 + 32x - 16}$$

numeratore $|_{x=1} = 1 - 7 + 17 - 17 + 6 = 0$

denominatore $|_{x=1} = 1 - 2 - 15 + 32 - 16 = 0$

$\cancel{\frac{0}{0}}$

Se $p(x)$ è un polinomio e $p(x_0) = 0$

Allora $x - x_0$ fattorizza $p(x)$ cioè $p(x)$ si

può scrivere come $p(x) = (x - x_0)\, q(x)$

dove $\deg(q(x)) = \deg(p(x)) - 1$ $\qquad \deg = \text{grado}$

$$
\begin{array}{rrrrr|l}
\cancel{x^4} & -7x^3 & +17x^2 & -17x & +6 & x-1 \\
\cancel{-x^4} & +x^3 & & & & x^3 - 6x^2 + 11x - 6 \\
\hline
& \cancel{-6x^3} & +17x^2 & -17x & +6 & \\
& \cancel{6x^3} & -6x^2 & & & \\
\hline
& & 11x^2 & -17x & +6 & \\
& & -11x^2 & 11x & & \\
\hline
& & & \cancel{-6x} & \cancel{+6} & \\
& & & \cancel{6x} & \cancel{-6} & \\
\hline
& & & & // &
\end{array}
$$

$$x^4 - 7x^3 + 17x^2 - 17x + 6 = (x-1)\underbrace{(x^3 - 6x^2 + 11x - 6)}$$

in $x = 1$: $1 - 6 + 11 - 6 = 0$

$\cancel{x^3} - 6x^2 + 11x - 6 \quad | \quad x - 1$

$$
\begin{array}{l|l}
\cancel{x^3}-6x^2+11x-6 & x-1 \\
\hline
-\cancel{x^3}+x^2 & x^2-5x+6 \\
\hline
-\cancel{5x^2}+11x-6 & \\
\cancel{5x^2}-5x & \\
\hline
\cancel{6x-6} & \\
\cancel{-6x+6} & \\
\hline
// &
\end{array}
$$

in $x=1$: $1-6+11-6=0$

$$=(x-1)(x-1)(x^2-5x+6)$$

$$=\boxed{(x-1)^2\underbrace{(x^2-5x+6)}_{x=1:\ 1-5+6=2}}$$

$$
\begin{array}{l|l}
\cancel{x^4}-2x^3-15x^2+32x-16 & x-1 \\
\hline
-\cancel{x^4}+x^3 & x^3-x^2-16(x-1) \\
\hline
-\cancel{x^3}-15x^2+32x-16 & \\
+\cancel{x^3}-x^2 & \\
\hline
-16x^2+32x-16 & \\
16(x-1)(x-1) & \\
\hline
// &
\end{array}
$$

$$-16x^2+32x-16 \longrightarrow -16(x^2-2x+1)=-16(x-1)^2$$

$$x^4-2x^3-15x^2+32x-16=(x-1)\underbrace{(x^3-x^2-16x+16)}_{x=1\ :\ 1-1-16+16=0}$$

$$x^3-x^2-16x+16 = x^2(x-1)-16(x-1)$$

$$=(x-1)(x^2-16)$$

$$x^4-2x^3-15x^2+32x-16=(x-1)(x-1)\underbrace{(x^2-16)}_{x=1\ \ 1-16=-15}=\boxed{(x-1)^2(x^2-16)}$$

$$f(x)\ \frac{x^4-7x^3+17x^2-17x+6}{x^4-2x^3-15x^2+32x-16}=\frac{\cancel{(x-1)^2}(x^2-5x+6)}{\cancel{(x-1)^2}(x^2-16)}$$

$\lim\limits_{x\to 1}$ $\qquad x\neq 1 \qquad x-1\neq 0$

$\forall x\neq 1 \qquad f(x)=\dfrac{x^2-5x+6}{x^2-16}$

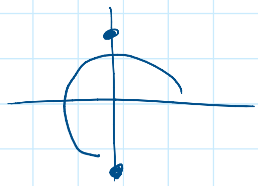
$$x^2-5x+6 \longrightarrow 2 \qquad -2$$

$x^2 - 16$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - 5x + 6}{x^2 - 16} = -\frac{2}{15}$$

$\to 2$, $\to -15$

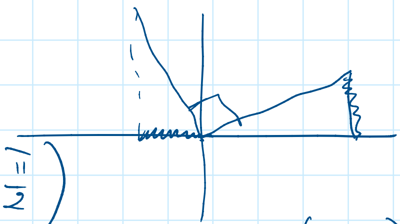
$$\lim_{x \to \frac{\pi}{2}} \frac{\cos(x)}{\cos(3x)} \qquad \cancel{\frac{0}{0}}$$



$\sin(x)$ $\qquad \lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$y = x - \frac{\pi}{2}$ $\qquad x \to \frac{\pi}{2}$ SSE $y \to 0$ $\qquad x = y + \frac{\pi}{2}$

$$\frac{\cos(x)}{\cos(3x)} = \frac{\cos\left(y + \frac{\pi}{2}\right)}{\cos\left(3y + \frac{3}{2}\pi\right)} = \frac{\sin(y)}{-\cos\left(3y + \frac{\pi}{2}\right)}$$



$\cos(\alpha + \pi) = -\cos(\alpha)$

$$3y + \frac{3}{2}\pi = 3y + \frac{\pi}{2} + \pi$$

$$= \frac{-\sin(y)}{-\sin(3y)} = \frac{\sin(y)}{\sin(3y)} \qquad \lim_{y \to 0} \frac{\sin(y)}{y} = 1$$

$$= \boxed{\frac{\sin(y)}{y}} \cdot \cancel{y} \, \frac{1}{\boxed{\frac{\sin(3y)}{3y}} \, 3\cancel{y}} \to \frac{1}{3}$$

$\to 1$ $\qquad \to 1$

$$\lim_{z \to 0} \frac{\sin(z)}{z} = 1$$

$$\lim_{x \to 0^+} \frac{1 - \cos(\sqrt{x})}{\sin(x)\cos(x)} \qquad \cancel{\frac{0}{0}}$$

$$\lim_{y \to 0} \frac{1 - \cos(y)}{y^2} = \frac{1}{2} \qquad y = \sqrt{x}$$

$\to \frac{1}{2}$

$$\frac{1-\cos(\sqrt{x})}{\sin(x)\cos(x)} = \underbrace{\boxed{\frac{1-\cos(\sqrt{x})}{(\sqrt{x})^2}}}_{\to \frac{1}{2}} \cancel{(\sqrt{x})^2} \frac{1}{\underbrace{\boxed{\frac{\sin(x)}{x}}}_{\to 1} \underbrace{\cos(x)}_{\to 1} \cdot \cancel{x}} \to \frac{1}{2}$$