Rette, parabole. Eq e sistemi di 1° grado. Eq. di 2° grado

DIPARTIMENTO DI MATEMATICA E INFORMATICA

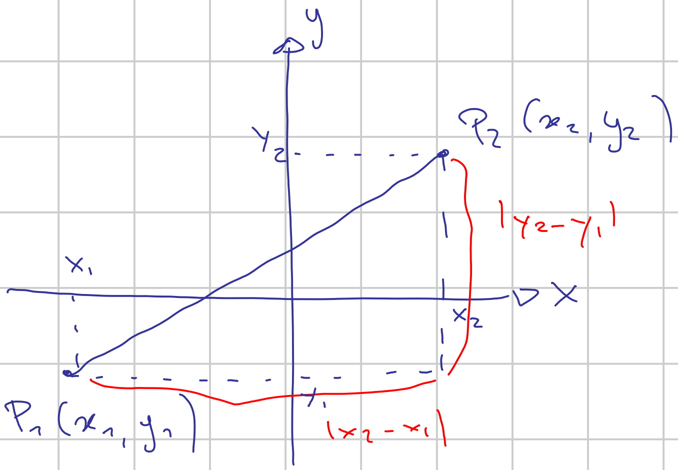
VIA DI SANTA MARTA 3

dentro Scuola di Ingegneria

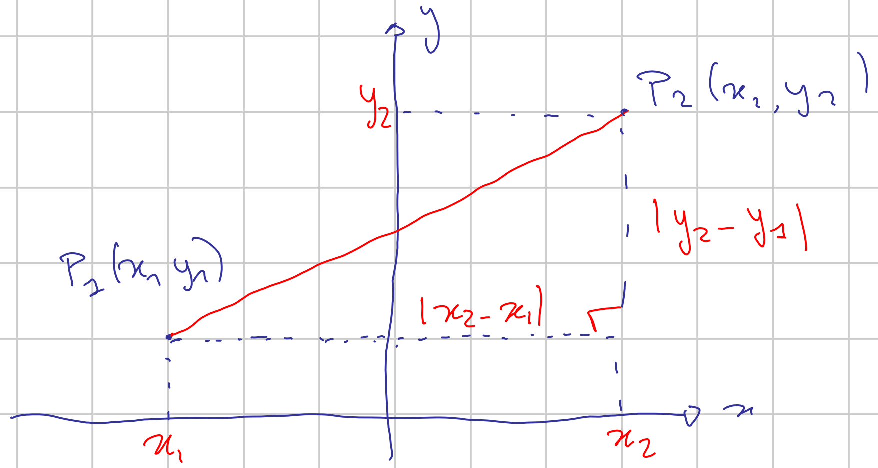
055-2758938

laura.poggiolini@unifi.it

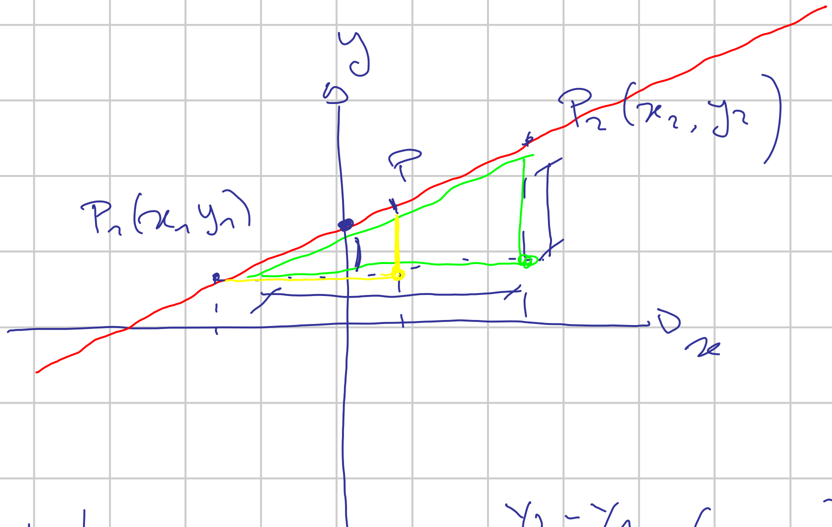
MARCO ABATE Matematica e Statistica - Le basi per le scienze della vita. McGraw Hill



$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$



$$y_2 - y_1 : x_2 - x_1 = y - y_1 : x - x_1$$

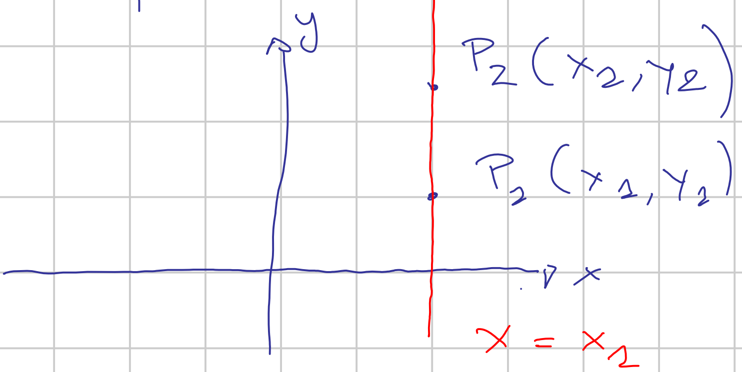
$$(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$$

se $x_2 \neq x_1$ $\quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y = \underbrace{\frac{y_2 - y_1}{x_2 - x_1}}_{m} x + \underbrace{y_1 - x_1 \frac{y_2 - y_1}{x_2 - x_1}}_{q}$$

$x = 0 \Rightarrow y = q$

$x_2 = x_1$



Dati tre numeri a, b, c, quali sono i pti del piano le cui coordinate soddisfano l'eq. $ax + by + c = 0$?

Se $b \neq 0$ $\quad by = -ax - c \quad y = \frac{-a}{b}x + \frac{-c}{b}$

$y = mx + q$ T.c. $m = \frac{-a}{b} \quad q = \frac{-c}{b}$

Se $b = 0$ $\quad ax + c = 0$

Se $a \neq 0$ $\quad x = \frac{-c}{a}$

Se $a = 0$ $\quad c = 0$ → $c = 0$ tutto il piano

→ $c \neq 0$ $\nexists$ pto del piano che soddisfi l'eq.

$$\begin{cases} y = ax + b \\ y = 0 \end{cases}$$

$ax + b = 0 \quad a \neq 0 \quad x = \frac{-b}{a}$

$a = 0 \quad b = 0 — b = 0 \quad \forall x$

$b \neq 0 \quad b = 0 \quad \nexists x$

$a, b, c, d, \alpha, \beta \qquad (x, y)$

$$* \begin{cases} ax + by = \alpha \\ cx + dy = \beta \end{cases}$$

$$\begin{cases} cax + cby = c\alpha \\ acx + ady = a\beta \end{cases}$$

$(\cancel{cax} + cby) - (\cancel{acx} + ady) = c\alpha - a\beta$

$$\boxed{(cb - ad)y = c\alpha - a\beta}$$

$$\begin{cases} dax + dby = d\alpha \\ bcx + bdy = b\beta \end{cases}$$

$$(dax + \cancel{bdy}) - (bcx + \cancel{bdy}) = d\alpha - b\beta$$

$$\boxed{(da - bc)x = d\alpha - b\beta}$$

$$\begin{cases} (ad - bc)x = d\alpha - b\beta \\ (ad - bc)y = a\beta - c\alpha \end{cases}$$

$ad - bc \neq 0$

$$x = \frac{d\alpha - b\beta}{ad - bc}$$

$$y = \frac{a\beta - c\alpha}{ad - bc}$$

$(\bar{x}, \bar{y})$

$ad - bc = 0$

$$ax + by = \alpha$$
$$cx + dy = \beta$$

$a \neq 0 \qquad d = \dfrac{bc}{a}$

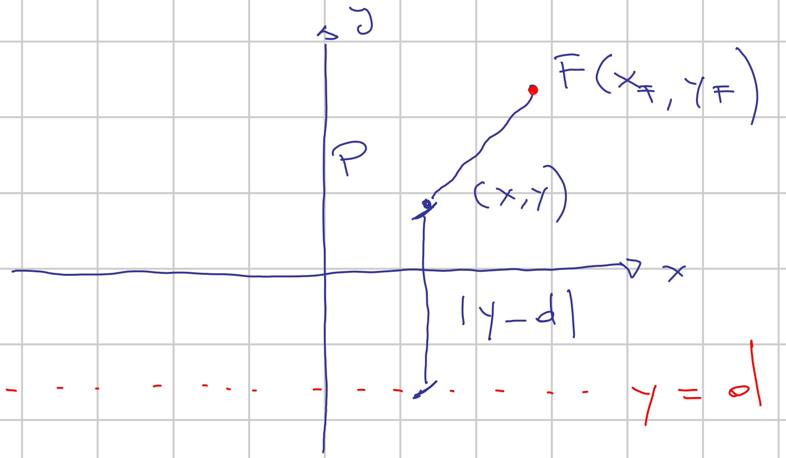
$$ax + by = \alpha$$
$$\frac{a}{c}\left(cx + \frac{bc}{a}y\right) = \beta \cdot \frac{a}{c}$$

$$\longrightarrow \begin{cases} ax + by = \alpha \\ ax + by = \beta \dfrac{a}{c} \end{cases}$$

Ho le stesse rette SSE $\alpha = \beta \dfrac{a}{c}$

SSE $\beta a - c\alpha = 0$

$ad - bc = 0$



fuoco

direttrice

$F \notin$ direttrice

$y_F \neq d$

$$d(P,F) = \sqrt{(x-x_F)^2 + (y-y_F)^2}$$

$$d(P, \text{direttrice}) = |y-d|$$

$$\sqrt{(x-x_F)^2 + (y-y_F)^2} = |y-d|$$

$$(x-x_F)^2 + (y-y_F)^2 = (y-d)^2$$

$$x^2 - 2x_F x + x_F^2 + \cancel{y^2} - 2y_F y + y_F^2 = \cancel{y^2} - 2dy + d^2$$

$$x^2 - 2x_F x + x_F^2 + y_F^2 - d^2 = 2y_F y - 2dy$$

$$x^2 - 2x_F x + x_F^2 + y_F^2 - d^2 = \underbrace{2(y_F - d)}_{\neq 0} y$$

$$\to y = \frac{1}{2(y_F-d)} \left(x^2 - 2x_F x + x_F^2 + y_F^2 - d^2 \right)$$

$$y = \underbrace{\frac{1}{2(y_F-d)}}_{a} x^2 \underbrace{- 2\frac{x_F}{2(y_F-d)}}_{b} x + \underbrace{\frac{x_F^2 + y_F^2 - d^2}{2(y_F-d)}}_{c}$$

$$y = ax^2 + bx + c \qquad a \neq 0$$

$$\frac{1}{2(y_F-d)} = a \qquad y = ax^2 - 2ax_Fx + a(x_F^2 + y_F^2 - d^2)$$

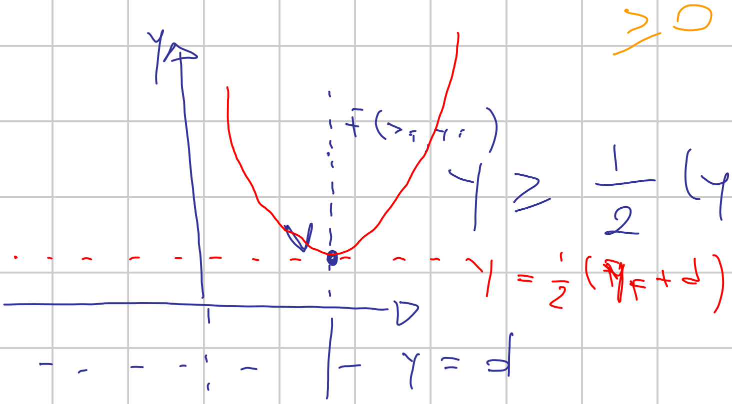
$$y = a\left(\underline{x^2 - 2x_Fx + x_F^2} + y_F^2 - d^2\right)$$

$$y = a\left((x-x_F)^2 + y_F^2 - d^2\right)$$

1° CASO $a>0$ (cioè $y_F > d$)

$$y = \underbrace{a(x-x_F)^2}_{\geq 0} + a(y_F^2-d^2) \geq a(y_F^2-d^2)$$

$$= \frac{1}{2(y_F-d)}(y_F^2-d^2)$$



$$y \geq \frac{1}{2}(y_F+d)$$

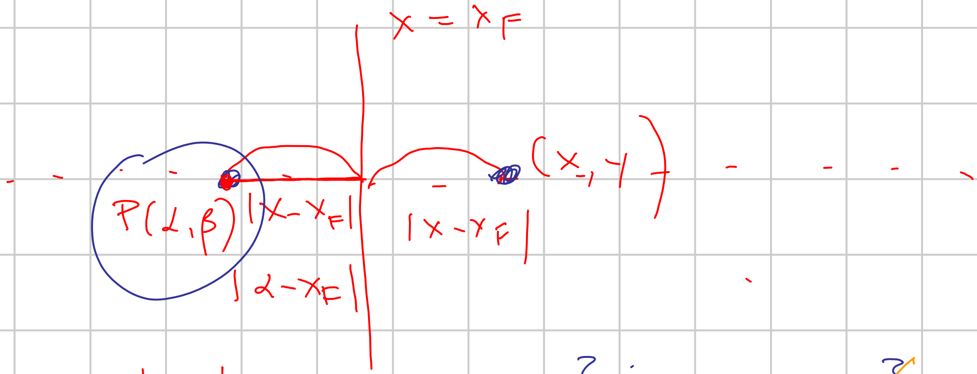
$$y = \frac{1}{2}(y_F+d)$$

SSE

$$x - x_F = 0$$

SSE

$$x = x_F$$



$$|x-x_F| = |\alpha - x_F|$$

$$x^2 - 2x_Fx + \cancel{x_F^2} = \alpha^2 - 2x_F\alpha + \cancel{x_F^2}$$

$$\alpha^2 - 2x_F\alpha - x^2 + 2x_Fx = 0$$

$$\alpha^2 - x^2 = 2x_F\alpha - 2x_Fx$$

$$(\alpha-x)(\alpha+x) = 2x_F(\alpha-x)$$

$$(\alpha-x)(\alpha+x-2x_F) = 0$$

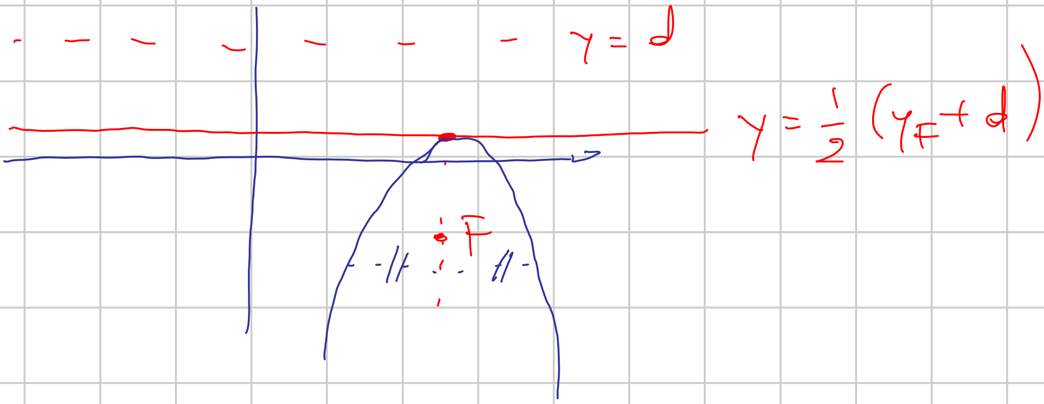
$\alpha = x$ il p.to di partenza

$\alpha + x - 2x_F = 0 \qquad \alpha = 2x_F - x$

2° CASO $a<0$ $\qquad y = a\left((x-x_F)^2 + y_F^2 - d^2\right)$ $\qquad a = \dfrac{1}{2(y_F-d)}$

$$y = a(x-x_F)^2 + a(y_F^2-d^2) = \underbrace{a(x-x_F)^2}_{\leq 0} + \frac{1}{2}(y_F+d)$$

$y \leq \frac{1}{2}(y_F+d)$ vale l'$=$ SSE $x = x_F$



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Dati tre numeri a, b, c con $a \neq 0$, chi sono i pti del piano le cui coordinate (x, y) soddisfano l'eq $y = ax^2+bx+c$?

$a \neq 0$ $\qquad y = ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) =$

$$= a\left(x^2+2\frac{b}{2a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\frac{-b^2+4ac}{4a^2}\right)$$

$$\boxed{a = \frac{1}{2(y_F-d)} \qquad \frac{b}{2a} = -x_F \qquad \frac{-b^2+4ac}{4a^2} = y_F^2-d^2}$$

$x_F = -\dfrac{b}{2a} = x_V$

$2(y_F-d) = \dfrac{1}{a}$ $\qquad y_F - d = \dfrac{1}{2a}$ $\qquad \boxed{d = y_F - \dfrac{1}{2a}}$

$$\frac{-b^2+4ac}{4a^2} = (y_F - d)(y_F + d)$$

$$\cancel{4a^2}\,\frac{-b^2+4ac}{\cancel{4a^2}} = \frac{1}{\cancel{2a}}\left(y_F + y_F - \frac{1}{2a}\right)\cancel{4a^2}\;2a$$

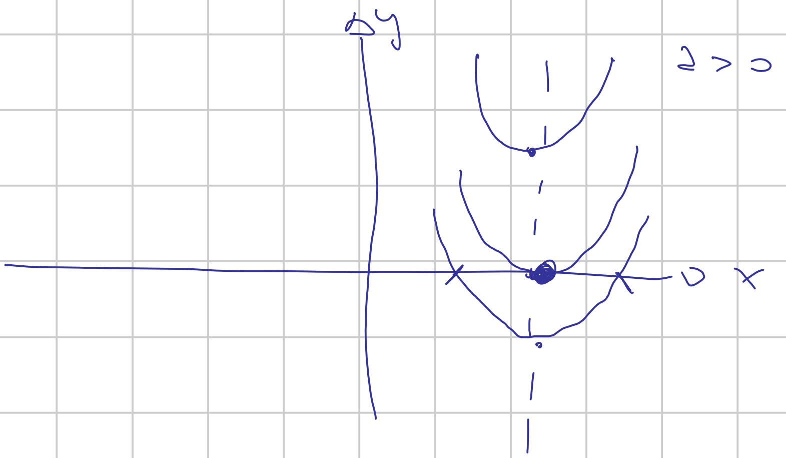
$$-b^2+4ac = \left(2y_F - \frac{1}{2a}\right)2a$$

$$-b^2+4ac = 4ay_F - 1 \qquad y_F = \frac{-b^2+4ac+1}{4a}$$

— o —

$a \neq 0 \qquad ax^2+bx+c=0$

$$\begin{cases} y = ax^2+bx+c \\ y = 0 \end{cases}$$



$$y = ax^2+bx+c =$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) =$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right) =$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2}\right)$$

$$x_v = \frac{-b}{2a} \qquad y_v = \frac{-b^2+4ac}{4a} \gtrless 0$$

$-b^2+4ac > 0$ cioè $b^2-4ac < 0$ $\quad\nexists$ soluzione

$-b^2+4ac = 0$ cioè $b^2-4ac = 0$ $\quad\exists!$ soluzione

$$x_v = \frac{-b}{2a}$$

$-b^2+4ac < 0$ cioè $b^2-4ac > 0$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

1° $$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a} \qquad x_1=\frac{-b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}$$

2° $$x+\frac{b}{2a}=\frac{-\sqrt{b^2-4ac}}{2a} \qquad x_2=\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}$$