Un po' di conti con R

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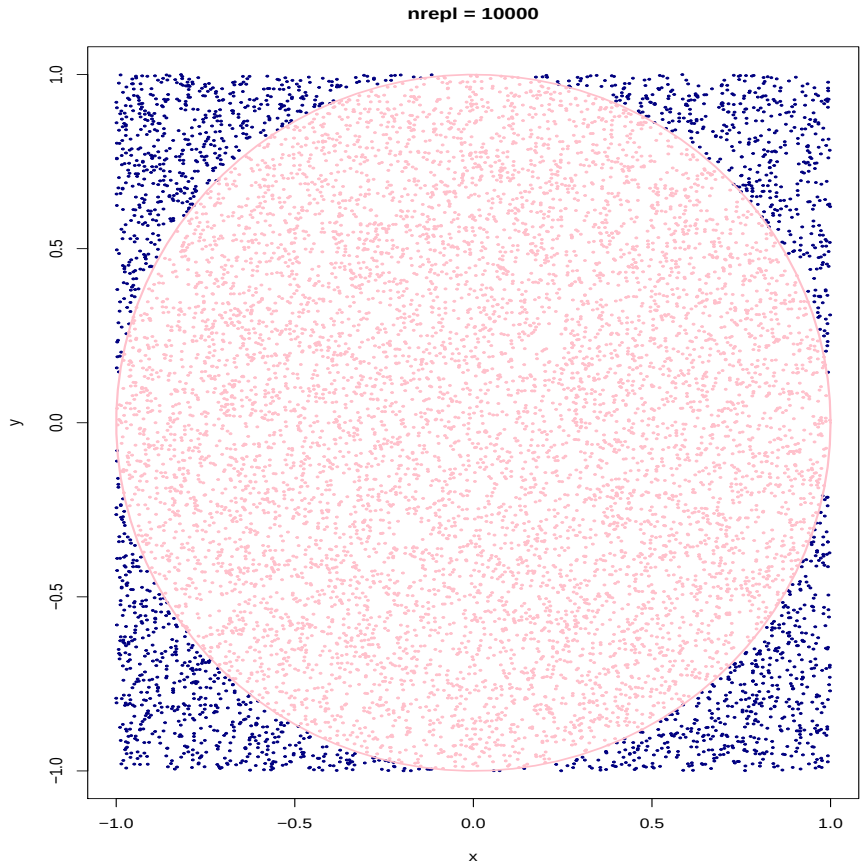
```
pi.greco=function(nrepl=10000){
+      xy=runif(nrepl*2, min=-1, max=1)
+      dim(xy)=c(nrepl,2)
+      inside=apply(xy^2,1,sum)<1
+      plot(xy, pch=20, cex=.5, col=c("navy","pink")
+           [inside+1],
+           xlab="x", ylab="y",
+           main=paste("nrepl =",nrepl))
+      a=seq(0,2*pi,length.out=100)
+      xy.circle=cbind(cos(a),sin(a))
+      lines(xy.circle,col="pink",lwd=2)
+      return( sum(inside)/nrepl *4)
+ }
```

```
> pi.greco()
[1] 3.1408
> pi.greco()
[1] 3.1556
> pi.greco()
[1] 3.1396
> pi.greco()
[1] 3.14
> pi.greco()
[1] 3.1288
> pi.greco()
[1] 3.1188
```



```
> pi.greco6 = function(nrepl=10000){
+      x=runif(nrepl, min=0, max=0.5)
+      dim(x) = c(nrepl,1)
+      somma= sum(1/sqrt(1 -x^2),1)
+      print(somma*3/nrepl)
+ }
```

```
> pi.greco6()
[1] 3.141353
> pi.greco6()
[1] 3.141853
> pi.greco6()
[1] 3.142525
> pi.greco6()
[1] 3.14333
```

```
> pi.greco6()
[1] 3.142465
> pi.greco6()
[1] 3.140517
> pi.greco6()
[1] 3.144073
> pi.greco6()
[1] 3.142422
```

```
> pi.greco6()
[1] 3.14008
> pi.greco6()
[1] 3.14226
> pi.greco6()
[1] 3.141166
> pi.greco6()
[1] 3.143552
```

Consideriamo la matrice stocastica P

	1	2	3	4	5
1	1	0	0	0	0
2	0.2	0.8	0	0	0
3	0	0	0.2	0.3	0.5
4	0	0	0	0.6	0.4
5	0	0.2	0.4	0.4	0

```
> library(foreign)
> library(xtable)
> Ide5=matrix(c(1,0,0,0,0,
+               0,1,0,0,0,
+               0,0,1,0,0,
+               0,0,0,1,0,
+               0,0,0,0,1), nrow=5, ncol = 5, byrow=TRUE)
> P=matrix(c(1,0,0,0,0,
+            .2,.8,0,0,0,
+            0,0,.2,.3,.5,
+            0,0,0,.6,.4,
+            0,.2,.4,.4,0), nrow=5, ncol = 5, byrow=TRUE)
> P2 <- P %*% P
> P3 <- P2 %*%P
> P4 <- P3 %*%P
> B <- Ide5 + P + P2 + P3 + P4
> B
```

```
        [,1]   [,2]   [,3]   [,4]   [,5]
[1,] 5.0000 0.0000 0.0000 0.0000 0.0000
[2,] 1.0214 1.4251 0.0000 0.0000 0.0000
[3,] 0.0548 0.2554 1.7216 1.5776 1.2536
[4,] 0.0464 0.2312 0.4288 3.0032 1.1744
[5,] 0.1620 0.4138 0.7456 1.4960 1.7776
```

```
> xtable(B, type = "latex", file = "B.tex")
```

	1	2	3	4	5
1	5.00	0	0	0	0
2	1.64	3.36	0	0	0
3	0.06	0.38	1.72	1.58	1.25
4	0.05	0.34	0.43	3.00	1.17
5	0.22	0.76	0.75	1.50	1.78

$$C_1 = \{1\}, \quad T = \{2, 3, 4, 5\},$$

P matrice stocastica 8×8

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0.5	0.5	0	0	0	0	0
4	0	0	0	0.6	0.4	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.53	1.97	2.40	0.79	0.25	0.07	0	0
2	0.18	2.57	3.46	1.23	0.42	0.14	0	0
3	0.08	2.10	4.60	0.86	0.27	0.08	0	0
4	0.90	0.64	0.96	3.64	1.36	0.50	0	0
5	1.26	1.14	1.72	1.68	1.59	0.61	0	0
6	2.47	1.72	1.97	0.62	0.18	1.05	0	0
7	0.73	0.45	0.69	3.41	1.26	0.46	1.00	0
8	0.55	0.28	0.46	3.15	1.15	0.40	1.00	1.00

$$C_1 = \{1, 2, 3, 4, 5, 6\} \quad T = \{7, 8\}$$

P matrice stocastica 8×8

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.80	0.20	0	0	0	0
3	0	0.5	0.5	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.46	1.94	2.33	1.27	0	0	0	0
2	0	2.47	3.29	2.24	0	0	0	0
3	0	2.06	4.52	1.42	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	0.95	1.43	3.27	1.00	0.40	0	0
6	2.43	1.71	1.93	0.93	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

$$C_1 = \{4\}, \qquad T = \{1, 2, 3, 5, 6, 7, 8\}$$

P

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	1	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	1	0

B

	1	2	3	4	5	6	7	8
1	2.46	2.42	1.65	1.48	0	0	0	0
2	0	2.95	2.36	2.69	0	0	0	0
3	0	2.95	2.95	2.10	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	1.21	1.03	3.40	1.00	0.40	0	0
6	2.43	2.06	1.44	1.07	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

Anche qui $C_1 = \{4\}$ e tutti gli altri stati sono transienti.

P

	1	2	3
1	0.60	0.40	0.00
2	0.80	0.20	0.00
3	0.00	0.30	0.70

```
CalcolaB=function(P){
      X = diag(nrow(P))
      Y = X
      massima = nrow(P)-1
      for(potenza in 1:massima)
      {
          X <- X %*% P
          Y <- Y + X
      }
      return(Y)
}
```

```
      [,1] [,2] [,3]
[1,] 2.28 0.72 0.00
[2,] 1.44 1.56 0.00
[3,] 0.24 0.57 2.19
```

```
> xtable(CalcolaB(Q))
```

B

	1	2	3
1	2.28	0.72	0.00
2	1.44	1.56	0.00
3	0.24	0.57	2.19

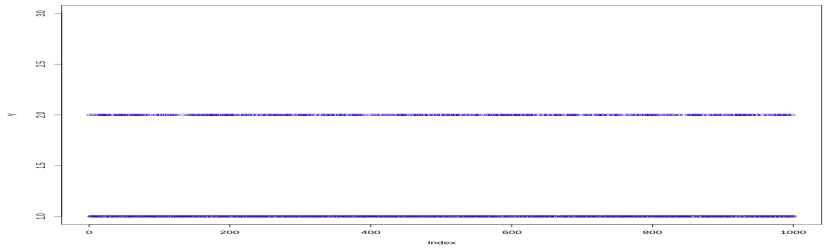
$$C_1 = \{1, 2\}, \quad T = \{3\}$$

```
Markov=function(P, nrepl, passi){
    prova =1
    Y = vector( length = passi)
    for(prova in 1:nrepl)
    {
        xi=runif(passi, min=0, max=1)
        dim(xi)=c(passi,1)
        passo = 0
        i <- sample(1:nrow(P), size = 1)
        print(paste("Sorteggio X[0](omega)", i))
        X = matrix(, ncol = passi, nrow = nrepl)
        arresto = 0
        k=1
        j=1
```

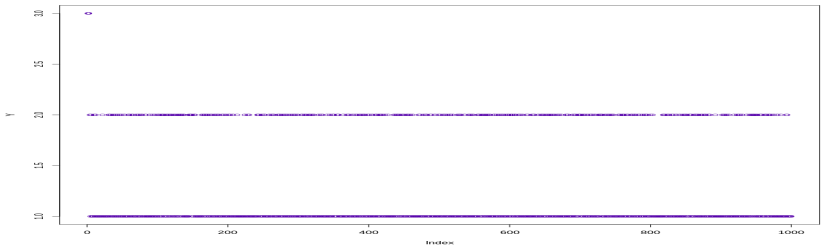
```
while(arresto < xi[1])
{
    arresto = arresto + P[i,j]
    X[prova,1] = j
    Y[1] = X[prova,1]
    j= j+1
}
for(k in 2:passi)
{
```

```
            i=X[prova,k-1]
            arresto = 0
            j=1
            while(arresto < xi[k])
            {
                arresto = arresto + P[i,j]
                X[prova,k] = j
                j=j+1
            }
            Y[k] <- X[prova,k]
        }
        print(Y)
        rosso = prova/nrepl
        blu = 1 - rosso
        plot(Y, col = rgb(rosso,0,blu))
    }
}
```

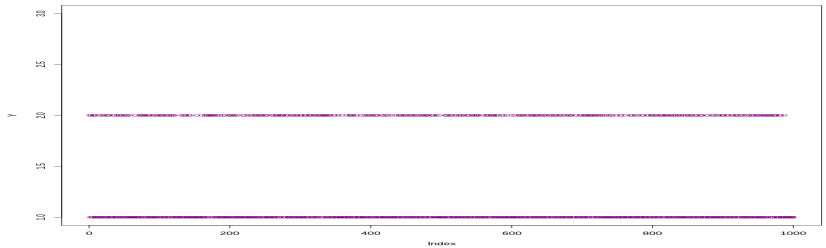
Simulazioni



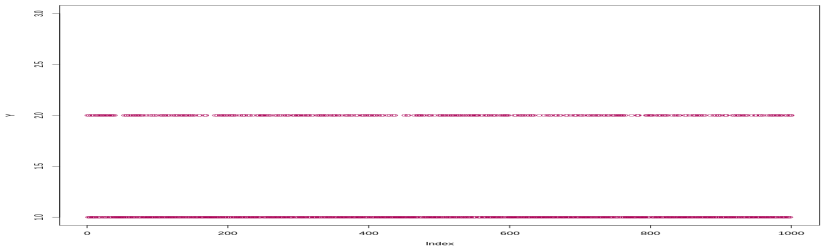
(a) $X_0(\omega) = 3$



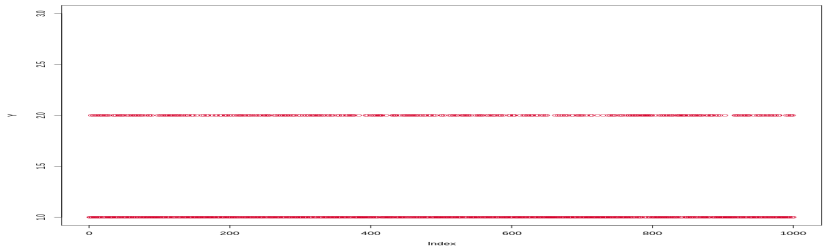
(b) $X_0(\omega) = 3$



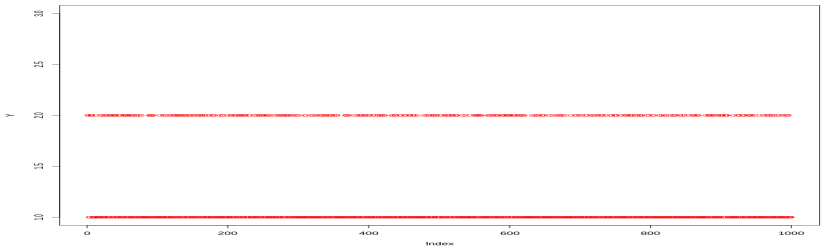
(c) $X_0(\omega) = 3$



(d) $X_0(\omega) = 3$



(e) $X_0(\omega) = 2$



(f) $X_0(\omega) = 1$

P

	1	2	3	4	5
1	1.00	0.00	0.00	0.00	0.00
2	0.20	0.30	0.00	0.00	0.00
3	0.00	0.00	0.20	0.30	0.50
4	0.00	0.00	0.00	0.60	0.40
5	0.00	0.20	0.40	0.40	0.00

```
        [,1]   [,2]   [,3]   [,4]   [,5]
[1,] 5.0000 0.0000 0.0000 0.0000 0.0000
[2,] 1.6384 3.3616 0.0000 0.0000 0.0000
[3,] 0.0648 0.3824 1.7216 1.5776 1.2536
[4,] 0.0544 0.3392 0.4288 3.0032 1.1744
[5,] 0.2240 0.7568 0.7456 1.4960 1.7776
```

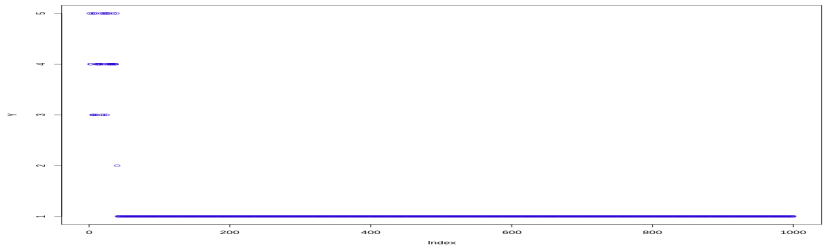
```
> xtable(CalcolaB(R))
```

B

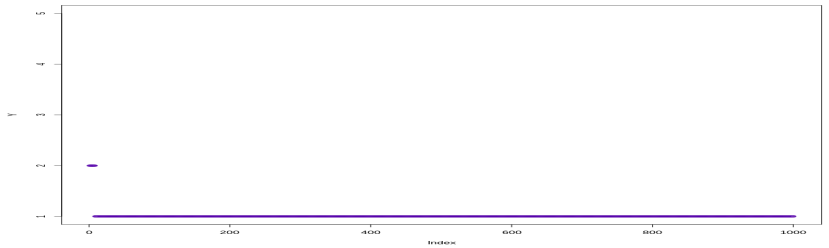
	1	2	3	4	5
1	5.00	0	0	0	0
2	1.64	3.36	0	0	0
3	0.06	0.38	1.72	1.58	1.25
4	0.05	0.34	0.43	3.00	1.17
5	0.22	0.76	0.75	1.50	1.78

$$C_1 = \{1\}, \quad T = \{2, 3, 4, 5\}$$

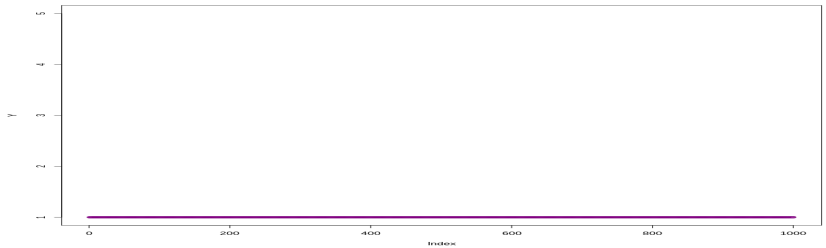
Simulazioni



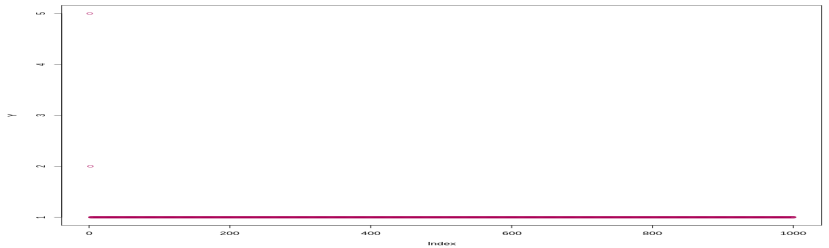
(a) $X_0(\omega) = 4$



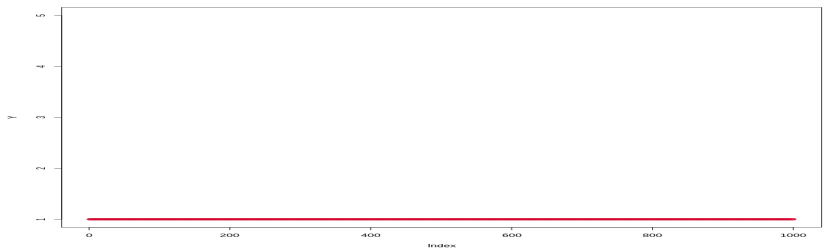
(b) $X_0(\omega) = 2$



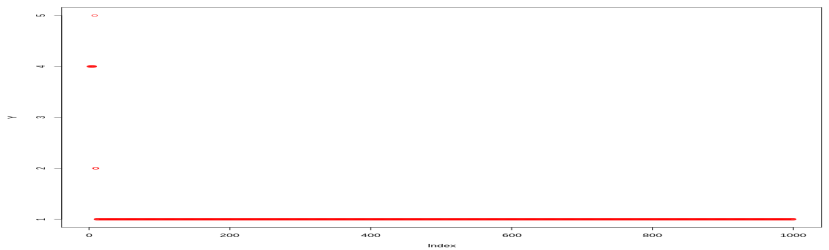
(c) $X_0(\omega) = 1$



(d) $X_0(\omega) = 4$



(e) $X_0(\omega) = 1$



(f) $X_0(\omega) = 3$

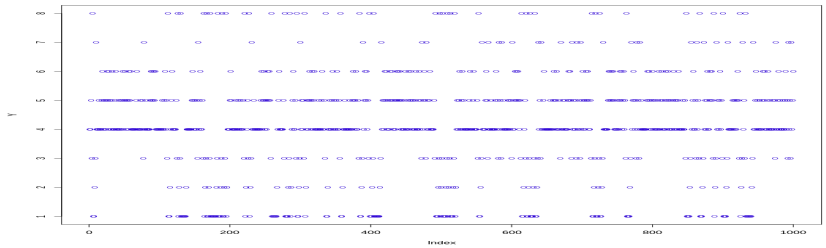
P

	1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0	0.8	0.2	0	0	0	0
3	0	0	0	0	0	0	0.5	0.5
4	0	0	0	0.6	0.4	0	0	0
5	0	0	0.2	0.4	0	0.4	0	0
6	0	0	0	0.5	0.5	0	0	0
7	0	0	0	1	0	0	0	0
8	1	0	0	0	0	0	0	0

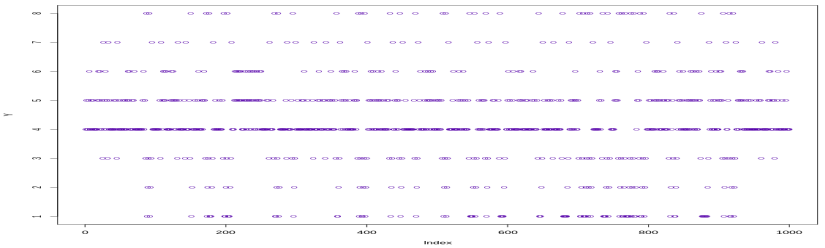
```
> CalcolaB(P)
[,1]     [,2]      [,3]     [,4]      [,5]      [,6]     [,7]     [,8]
[1,] 3.130010 1.183206 0.9361920 1.353344 0.4504064 0.1217536 0.412544 0.412544
[2,] 1.031360 1.355200 1.1873280 2.225562 0.8299264 0.2677760 0.551424 0.551424
[3,] 1.411680 0.501120 1.4723200 2.153120 0.8038400 0.2483200 0.704800 0.704800
[4,] 0.204480 0.058240 0.3614464 4.505818 1.9052032 0.6665728 0.149120 0.149120
[5,] 0.372800 0.117120 0.5269120 3.140339 2.4812288 0.9053440 0.228128 0.228128
[6,] 0.219200 0.064000 0.3772480 3.369920 1.9648960 1.6904960 0.157120 0.157120
[7,] 0.145600 0.035200 0.2982400 4.049408 1.6664320 0.5708800 1.117120 0.117120
[8,] 2.958016 1.094144 0.8250880 1.035776 0.3043840 0.0721920 0.355200 1.355200
```

Tutti gli elementi sono positivi quindi S è l'unica classe minimale. Tutti gli stati sono ricorrenti.

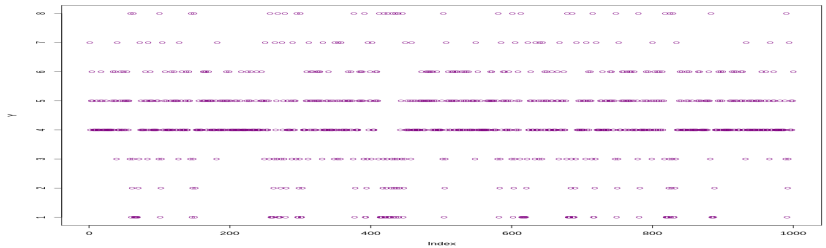
Simulazioni



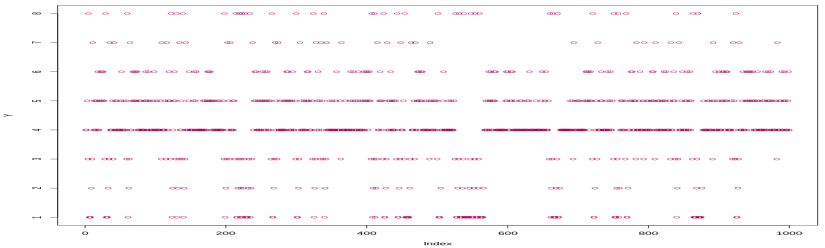
(a) $X_0(\omega) = 5$



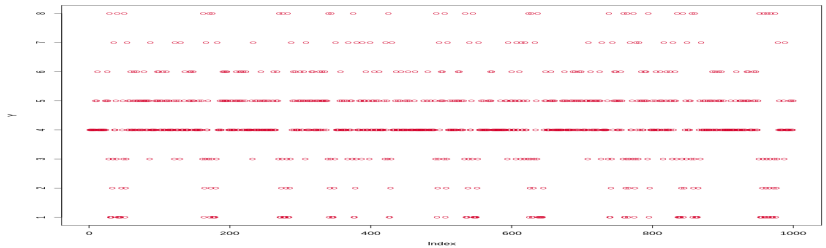
(b) $X_0(\omega) = 4$



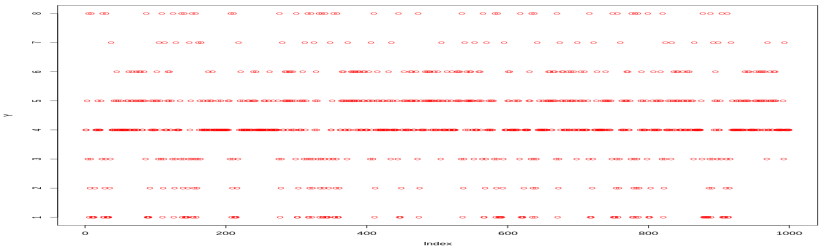
(c) $X_0(\omega) = 3$



(d) $X_0(\omega) = 7$



(e) $X_0(\omega) = 4$



(f) $X_0(\omega) = 4$

P

	1	2	3	4	5	6	7	8
1	0.6	0.40	0.00	0.00	0.00	0.00	0.00	0.00
2	0.0	0.00	0.80	0.20	0.00	0.00	0.00	0.00
3	0.0	0.50	0.50	0.00	0.00	0.00	0.00	0.00
4	0.0	0.00	0.00	0.60	0.40	0.00	0.00	0.00
5	0.00	0.00	0.20	0.40	0.00	0.40	0.00	0.00
6	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

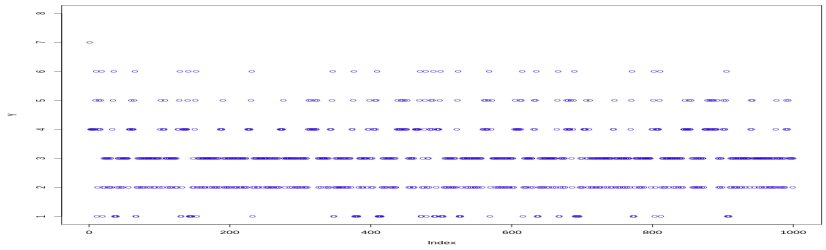
```
> CalcolaB(P)
[1] 7
[,1]     [,2]      [,3]     [,4]      [,5]      [,6] [,7] [,8]
[1,] 2.528666 1.969046 2.3969760 0.786464 0.2462464 0.0726016    0    0
[2,] 0.181504 2.571316 3.4619720 1.230770 0.4170944 0.1373440    0    0
[3,] 0.083040 2.095047 4.5999315 0.864793 0.2747400 0.0824480    0    0
[4,] 0.899072 0.637748 0.9575624 3.638642 1.3625792 0.5043968    0    0
[5,] 1.260992 1.139403 1.7173950 1.681453 1.5947728 0.6059840    0    0
[6,] 2.465856 1.722464 1.9654080 0.615616 0.1815040 1.0491520    0    0
[7,] 0.734720 0.451880 0.6877200 3.406448 1.2609920 0.4582400    1    0
[8,] 0.550400 0.283600 0.4634400 3.152480 1.1456000 0.4044800    1    1
> xtable(CalcolaB(P))
```

B

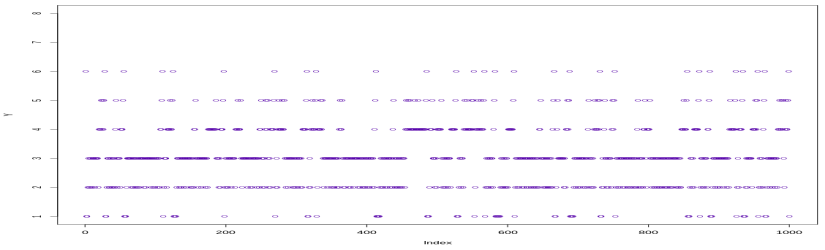
	1	2	3	4	5	6	7	8
1	2.53	1.97	2.40	0.79	0.25	0.07	0	0
2	0.18	2.57	3.46	1.23	0.42	0.14	0	0
3	0.08	2.10	4.60	0.86	0.27	0.08	0	0
4	0.90	0.64	0.96	3.64	1.36	0.50	0	0
5	1.26	1.14	1.72	1.68	1.59	0.61	0	0
6	2.47	1.72	1.97	0.62	0.18	1.05	0	0
7	0.73	0.45	0.69	3.41	1.26	0.46	1.00	0
8	0.55	0.28	0.46	3.15	1.15	0.40	1.00	1.00

$$C_1 = \{1, 2, 3, 4, 5, 6\}, \qquad T = \{7, 8\}$$

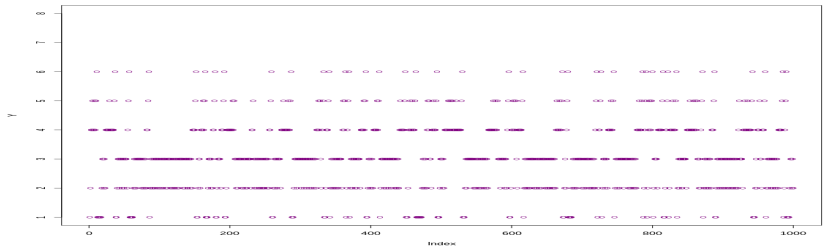
Simulazioni



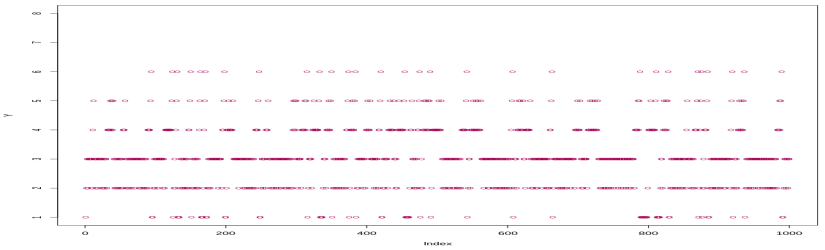
(a) $X_0(\omega) = 8$



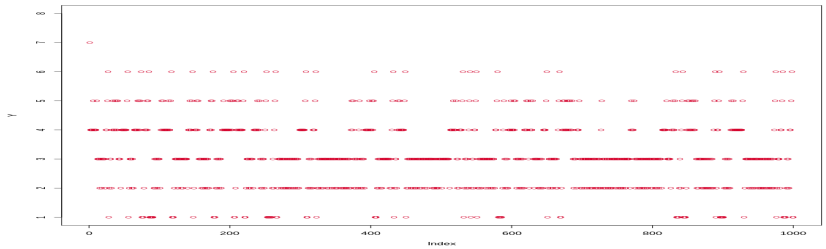
(b) $X_0(\omega) = 5$



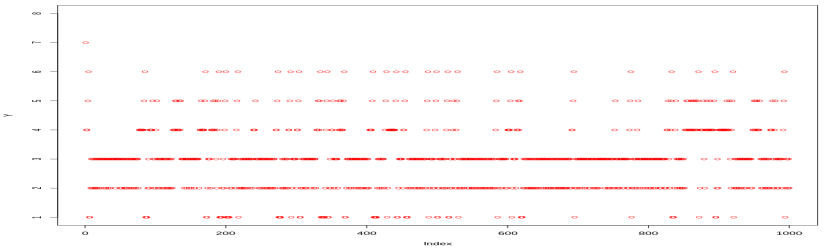
(c) $X_0(\omega) = 1$



(d) $X_0(\omega) = 1$



(e) $X_0(\omega) = 8$



(f) $X_0(\omega) = 8$

Pter

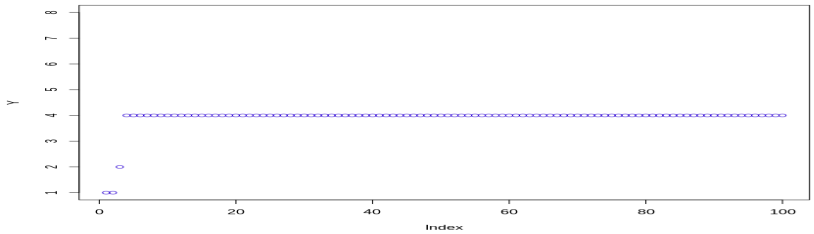
	1	2	3	4	5	6	7	8
1	0.60	0.40	0	0	0	0	0	0
2	0	0	0.80	0.20	0	0	0	0
3	0	1.00	0	0	0	0	0	0
4	0	0	0	1.00	0	0	0	0
5	0	0	0.20	0.40	0	0.40	0	0
6	1.00	0	0	0	0	0	0	0
7	0	0	0	1.00	0	0	0	0
8	0	0	0	0	0	0	1.00	0

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]
[1,]	2.458010	2.416358	1.647411	1.478221	0	0.0	0	0
[2,]	0.000000	2.952000	2.361600	2.686400	0	0.0	0	0
[3,]	0.000000	2.952000	2.952000	2.096000	0	0.0	0	0
[4,]	0.000000	0.000000	0.000000	8.000000	0	0.0	0	0
[5,]	0.953344	1.210176	1.032768	3.403712	1	0.4	0	0
[6,]	2.430016	2.059264	1.444352	1.066368	0	1.0	0	0
[7,]	0.000000	0.000000	0.000000	7.000000	0	0.0	1	0
[8,]	0.000000	0.000000	0.000000	6.000000	0	0.0	1	1

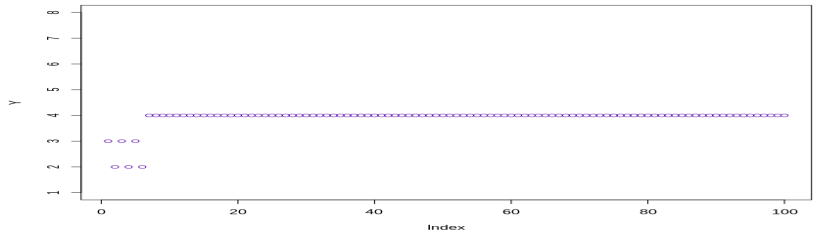
B di *Pter*

	1	2	3	4	5	6	7	8
1	2.46	2.42	1.65	1.48	0	0	0	0
2	0	2.95	2.36	2.69	0	0	0	0
3	0	2.95	2.95	2.10	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	1.21	1.03	3.40	1.00	0.40	0	0
6	2.43	2.06	1.44	1.07	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

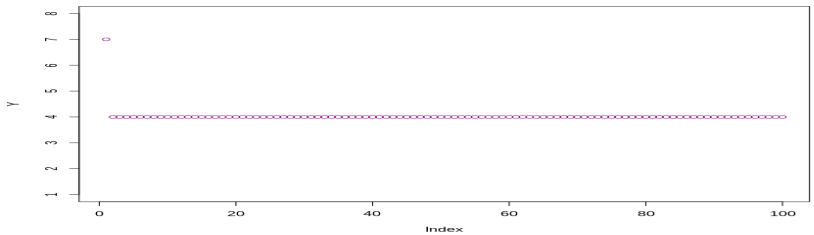
Simulazioni



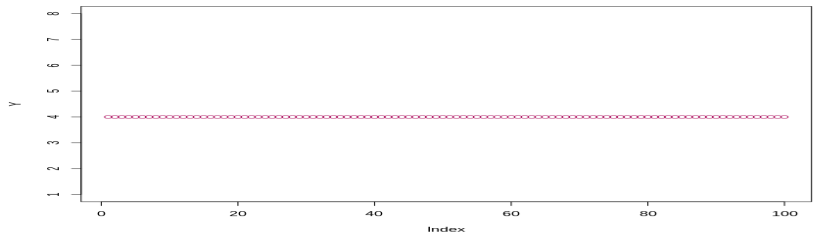
(a) $X_0(\omega) = 6$



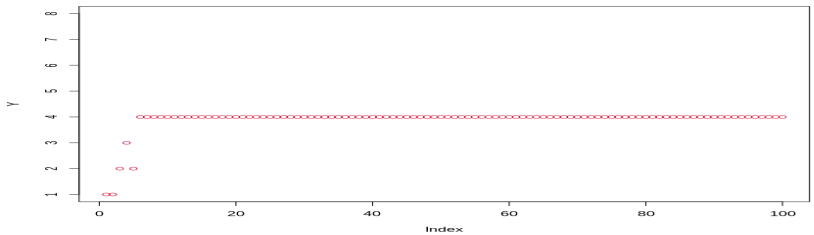
(b) $X_0(\omega) = 2$



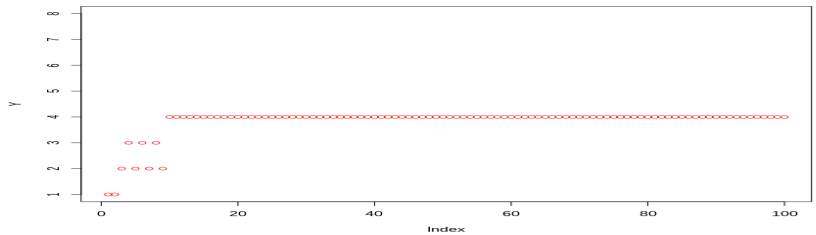
(c) $X_0(\omega) = 8$



(d) $X_0(\omega) = 5$



(e) $X_0(\omega) = 6$



(f) $X_0(\omega) = 1$

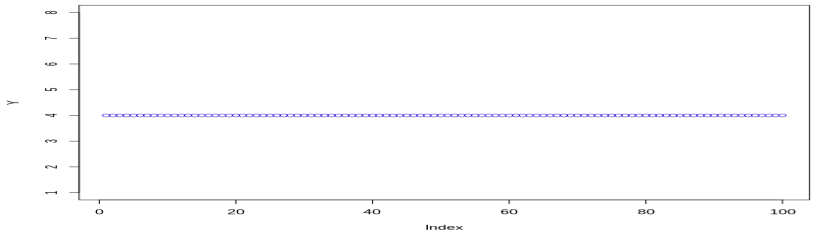
Pquater

	1	2	3	4	5	6	7	8
1	0.60	0.40	0	0	0	0	0	0
2	0	0	0.80	0.20	0	0	0	0
3	0	0.50	0.50	0	0	0	0	0
4	0	0	0	1.00	0	0	0	0
5	0	0	0.20	0.40	0	0.40	0	0
6	1.00	0	0	0	0	0	0	0
7	0	0	0	1.00	0	0	0	0
8	0	0	0	0	0	0	1.00	0

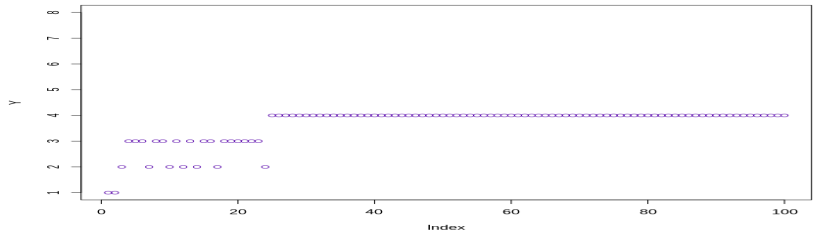
B di *Pquater*

	1	2	3	4	5	6	7	8
1	2.46	1.94	2.33	1.27	0	0	0	0
2	0	2.47	3.29	2.24	0	0	0	0
3	0	2.06	4.52	1.42	0	0	0	0
4	0	0	0	8.00	0	0	0	0
5	0.95	0.95	1.43	3.27	1.00	0.40	0	0
6	2.43	1.71	1.93	0.93	0	1.00	0	0
7	0	0	0	7.00	0	0	1.00	0
8	0	0	0	6.00	0	0	1.00	1.00

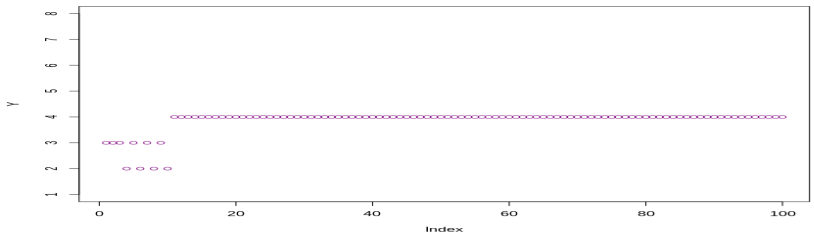
Simulazioni



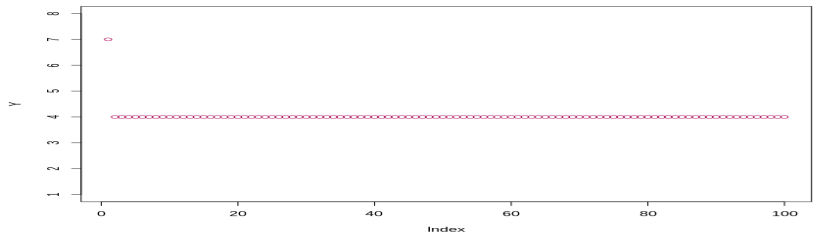
(a) $X_0(\omega) = 7$



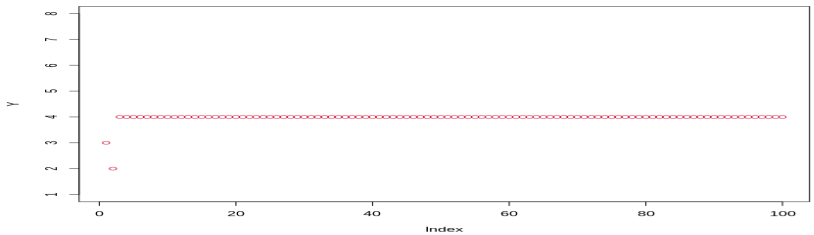
(b) $X_0(\omega) = 1$



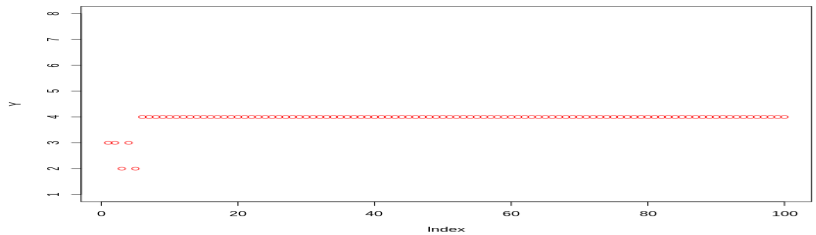
(c) $X_0(\omega) = 2$



(d) $X_0(\omega) = 8$



(e) $X_0(\omega) = 2$



(f) $X_0(\omega) = 2$