Note Title 09/05/2019

$X_1 \_\_\_ X_n$ i.i.d.

Supponiamo che la distribuzione abbia
- valore atteso finito μ
- varianza finita σ^2

$x_1 \ldots x_n$ $\quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$\bar{X} = f_0(X_1, \ldots, X_n) \qquad S^2 = g_0(X_1 \_\_ X_n)$$

$X_1 \_\_ X_n$ campione statistico con valore atteso μ e varianza σ^2

$$\mathbb{E}[\bar{X}] = \mathbb{E}\left[\frac{1}{n} \sum_{i=1}^{n} X_i\right] = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[X_i] = \frac{1}{\cancel{n}} \cancel{n} \mu = \mu$$

$$Var[\bar{X}] = Var\left[\frac{1}{n} \sum_{i=1}^{n} X_i\right] = \frac{1}{n^2} Var\left[\sum_{i=1}^{n} X_i\right] =$$

$$= \frac{1}{n^2} \left(\sum_{i=1}^{n} Var[X_i] + \cancel{2 \sum_{1 \le i < j \le n} Cov(X_i, X_j)} \right) =$$

$= 0$

$$= \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2} n \sigma^2 = \frac{\sigma^2}{n}$$

$$\mathbb{P}(|\bar{X} - \mu| > \varepsilon) \le \frac{Var[\bar{X}]}{\varepsilon^2} = \frac{\sigma^2}{\varepsilon^2} \cdot \frac{1}{n}$$

Proposizione Se la distribuzione del campione statistico $X_1 \_\_ X_n$ ha valore atteso e varianza finiti, allora, la media campionaria è uno stimatore corretto del valore atteso.

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - 2\overline{X}\underbrace{\sum_{i=1}^{n} X_i}_{n\overline{X}} + \underbrace{\sum_{i=1}^{n}\overline{X}^2}_{n\overline{X}^2}\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - 2n\overline{X}^2 + n\overline{X}^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right)$$

$$\mathbb{E}[S^2] = \frac{1}{n-1}\mathbb{E}\left[\left(\sum_{i=1}^{n} X_i^2\right) - n\overline{X}^2\right] =$$

$$= \frac{1}{n-1}\mathbb{E}\left[\sum_{i=1}^{n}(X_i - \mu + \mu)^2 - n(\overline{X} - \mu + \mu)^2\right]$$

$$= \frac{1}{n-1}\left\{\sum_{i=1}^{n}\mathbb{E}\left[(X_i-\mu)^2 + 2\mu(X_i-\mu) + \mu^2\right] - n\,\mathbb{E}\left[(\overline{X}-\mu)^2 + 2\mu(\overline{X}-\mu) + \mu^2\right]\right\}$$

$$= \frac{1}{n-1}\left\{\sum_{i=1}^{n}\left(\mathbb{E}[(X_i-\mu)^2] + 2\mu\,\cancel{\mathbb{E}[X_i-\mu]} + \mathbb{E}[\mu^2]\right) - n\left(\mathbb{E}[(\overline{X}-\mu)^2] + 2\mu\,\cancel{\mathbb{E}[\overline{X}-\mu]} + \mathbb{E}[\mu^2]\right)\right\}$$

$$= \frac{1}{n-1}\left\{\left(\sum_{i=1}^{n}\mathrm{Var}[X_i]\right) \cancel{+ n\mu^2} - n\,\mathrm{Var}[\overline{X}] \cancel{- n\mu^2}\right\}$$

$$= \frac{1}{n-1}\left\{n\sigma^2 - \cancel{n}\cdot\frac{\sigma^2}{\cancel{n}}\right\} = \frac{1}{\cancel{n-1}}\cancel{(n-1)}\sigma^2$$

$N(\mu, \sigma^2)$ $\qquad f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad \forall x \in \mathbb{R}$

$F_X(t) = \int_{-\infty}^{t} f(x)\,dx$ $\qquad$ funzione strettamente crescente

$F_X'(t) = f(t) > 0 \quad \forall t \in \mathbb{R}$

$\mathbb{P}_{X_0} = N(0,1)$ $\qquad f_0(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad \forall x \in \mathbb{R}$

allora $\forall \mu \in \mathbb{R}$ $\forall \sigma^2 > 0$ la v.a $X := \mu + \sigma X_0$ ha distribuzione gaussiana $N(\mu, \sigma^2)$

— o —

Sia $\mathbb{P}_X = N(\mu, \sigma^2)$ - Voglio calcolare $F_X(t)$

So che $\mathbb{P}_{X_0} = N(0,1)$ $\Rightarrow$ $\mu + \sigma X_0$ ha distribuzione $N(\mu, \sigma^2) = \mathbb{P}_X$

$F_X(t) = \mathbb{P}(X \leq t) = \mathbb{P}(\mu + \sigma X_0 \leq t)$ con $\mathbb{P}_{X_0} = N(0,1)$

$= \mathbb{P}\left(X_0 \leq \frac{t-\mu}{\sigma}\right) = F_{X_0}\left(\frac{t-\mu}{\sigma}\right) = \alpha$

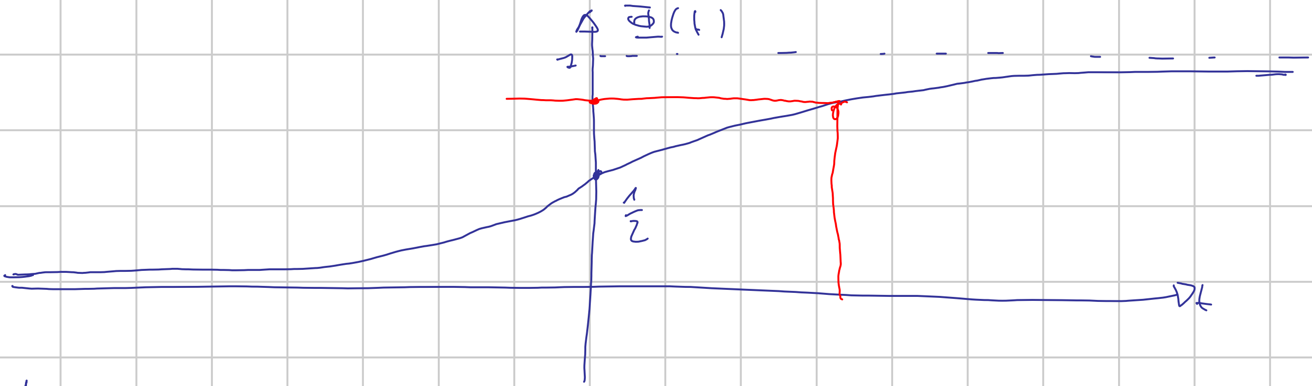
$\Phi(t)$ legge di $N(0,1)$

$$\Phi(t) := \int_{-\infty}^{t} \underbrace{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)}_{f_0(x) \text{ è una funzione pari}} dx \qquad \forall t \in \mathbb{R}$$

$\Phi(t) + \Phi(-t) = 1$

$\Phi(t)$ strettamente crescente $\qquad \lim_{t \to -\infty} \Phi(t) = 0 \qquad \lim_{t \to +\infty} \Phi(t) = 1$

$\Rightarrow \forall \alpha \in (0,1) \qquad \exists!\, t_\alpha \in \mathbb{R} : \quad \Phi(t_\alpha) = \alpha$



t_α si chiama quantile di livello α,

Nel caso della distribuzione $N(0,1)$ si indica col simbolo z_α

$$\Phi\left(z_{\frac{1}{2}}\right) = \frac{1}{2} \Rightarrow z_{\frac{1}{2}} = 0$$

$$z_\alpha \gtrless 0 \quad \text{SSE} \quad \alpha \gtrless \frac{1}{2}$$

$$\longrightarrow \Phi(t) + \Phi(-t) = 1$$

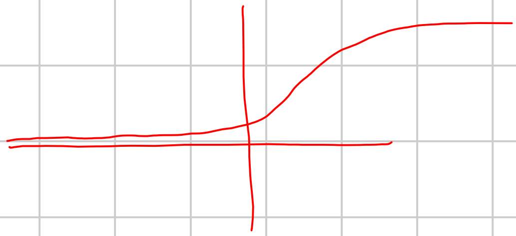
$$t = z_\alpha \qquad \alpha + \Phi(-t) = 1$$

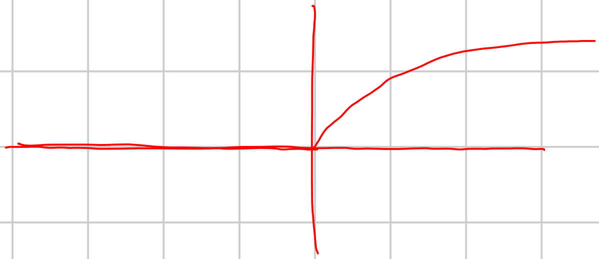
$$\Phi(-t) = 1 - \alpha$$

$$-t = z_{1-\alpha}$$

$$-z_\alpha = z_{1-\alpha}$$

P_X distribuzione t.c. la legge è strettamente crescente





DISTRIBUZIONI $\Gamma(\alpha, \lambda)$

Sono distribuzioni AC associate alle densità

$$f(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x > 0 \end{cases}$$

$\Gamma(\alpha)$:= funzione Γ di Eulero

$$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x}\, dx \qquad \alpha - 1 > -1 \quad \alpha > 0$$

vicino a 0 $\quad x^{\alpha-1} e^{-x} \sim x^{\alpha-1}$

$$\int_0^1 x^{\beta}\, dx < +\infty \quad \text{sse} \quad \beta > -1$$

$$\int_0^1 x^{\beta}\, dx = \lim_{\varepsilon \downarrow 0} \int_{\varepsilon}^1 x^{\beta}\, dx$$

$$f_n(x) = \begin{cases} x^{\beta} & x > \frac{1}{n} \\ 0 & 0 < x \le \frac{1}{n} \end{cases}$$

$$\int_{\varepsilon}^1 x^{\beta}\, dx = \begin{cases} \ln(x) \Big|_{x=\varepsilon}^{x=1} & \beta = -1 \\ \dfrac{1}{\beta+1} x^{\beta+1} \Big|_{x=\varepsilon}^{x=1} & \beta \neq -1 \end{cases}$$

$$= \begin{cases} \beta = -1 & -\ln(\varepsilon) \longrightarrow +\infty \\ \beta \neq -1 & \dfrac{1}{\beta+1}\left(1 - \varepsilon^{\beta+1}\right) \longrightarrow \begin{cases} \beta+1 > 0 & \dfrac{1}{\beta+1} \\ \beta+1 < 0 & +\infty \end{cases} \end{cases}$$

$$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x}\, dx$$

$$\Gamma(1) = \int_0^{+\infty} e^{-x}\, dx = -e^{-x} \Big|_{x=0}^{x \to +\infty} = 1$$

$$\Gamma(2) = \int_0^{+\infty} \underset{f}{x}\, \underset{g'}{e^{-x}}\, dx = x(-e^{-x}) \Big|_{x=0}^{x \to +\infty} + \int_0^{+\infty} e^{-x}\, dx$$

$$= \int_0^{+\infty} e^{-x}\,dx = \Gamma(1)$$

$$\Gamma(\alpha+1) = \int_0^{+\infty} x^{\alpha+1-1} e^{-x}\,dx = \int_0^{+\infty} \underset{f}{x^{\alpha}}\ \underset{g'}{e^{-x}}\,dx =$$

$$= x^{\alpha}(-e^{-x})\Big|_{x=0}^{x\to+\infty} + \int_0^{+\infty} \alpha x^{\alpha-1} e^{-x}\,dx$$

$$= \int_0^{+\infty} \alpha x^{\alpha-1} e^{-x}\,dx = \alpha \int_0^{+\infty} x^{\alpha-1} e^{-x}\,dx = \alpha\Gamma(\alpha)$$

$$\forall \alpha>0 \qquad \Gamma(\alpha+1) = \alpha\Gamma(\alpha)$$

$$\Gamma(1) = 1 \qquad \Gamma(2) = 1\cdot\Gamma(1) = 1$$

$$\Gamma(3) = 2\Gamma(2) = 2\cdot 1$$

$$\Gamma(4) = 3\Gamma(3) = 3\cdot 2\cdot 1$$

$\vdots$ INDUZIONE

$$\Gamma(n) = (n-1)!$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{+\infty} x^{1/2-1} e^{-x}\,dx = \int_0^{+\infty} x^{-1/2} e^{-x}\,dx$$

$$x = y^2 \qquad dx = 2y\,dx \qquad x=0 \quad y=0$$

$$x\to+\infty \quad y\to+\infty$$

$$= \int_0^{+\infty} y^{-1} e^{-y^2}\, 2y\,dy = 2\int_0^{+\infty} e^{-y^2}\,dy = \int_{\mathbb{R}} e^{-y^2}\,dy = \sqrt{\pi}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \qquad \Gamma(\alpha+1) = \alpha\Gamma(\alpha)$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}$$

$$\Gamma\left(\frac{7}{2}\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right) = \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}$$

$\vdots$

$$\Gamma\left(\frac{2n+1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

$n! = n(n-1)\cdots 1$

$n!! = n(n-2)(n-4)\cdots$

$\cdots$

$$\Gamma(n) = (n-1)! \qquad \Gamma\left(\frac{2n}{2}\right) = (n-1)!$$

La distribuzione $\Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$ si chiama DISTRIBUZIONE DI PEARSON A n GRADI DI LIBERTÀ o anche DISTRIBUZIONE χ^2 A n GRADI DI LIBERTÀ

Si indica col simbolo χ^2_n.

Sia X_0 una v.a. con $\mathbb{P}_{X_0} = N(0,1)$

$$f_0(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) \qquad \forall x \in \mathbb{R}$$

$$Y := X_0^2 \qquad \mathbb{P}_Y = g(x)\,dx$$

$$g(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{2\sqrt{x}}\left(f_0(\sqrt{x}) + f_0(-\sqrt{x})\right) & x > 0 \end{cases}$$

$x > 0$

$$g(x) = \frac{1}{2\sqrt{x}}\, 2f_0(\sqrt{x}) = \frac{1}{\sqrt{x}} f_0(\sqrt{x}) = \frac{1}{\sqrt{x}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x}{2}\right)$$

$$= \frac{1}{\sqrt{\pi}}\left(\frac{1}{2}\right)^{1/2} x^{-1/2}\exp\left(-\frac{x}{2}\right) = \frac{\left(\frac{1}{2}\right)^{1/2}}{\Gamma\left(\frac{1}{2}\right)} x^{-1/2}\exp\left(-\frac{x}{2}\right)$$

cioè $\mathbb{P}_{X_0} = N(0,1) \Rightarrow \mathbb{P}_{X_0^2} = \chi^2_1 = \Gamma\left(\frac{1}{2}, \frac{1}{2}\right)$

$\Gamma(\alpha, \lambda)$ $\qquad \frac{1}{\Gamma(\alpha)} \lambda^\alpha x^{\alpha-1} e^{-\lambda x}$ $\qquad \frac{1}{\Gamma(\frac{1}{2})} \left(\frac{1}{2}\right)^{1/2} x^{\frac{1}{2}-1} e^{-\frac{x}{2}}$

Siano X e Y due v.a. indipendenti

$\mathbb{P}_X = \Gamma(\alpha, \lambda)$

$\mathbb{P}_Y = \Gamma(\beta, \lambda)$ $\Rightarrow \mathbb{P}_{X+Y} = \Gamma(\alpha + \beta, \lambda)$

Dim X e Y v.a. indipendenti e AC, $\mathbb{P}_X = f(x)\,dx$

$\mathbb{P}_Y = g(x)\,dx$

$\Rightarrow \mathbb{P}_{X+Y} = h(x)\,dx \qquad h(x) = \int_{\mathbb{R}} f(y)\, g(x-y)\,dy$

$$f(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

$$g(x) = \begin{cases} \frac{\lambda^\beta}{\Gamma(\beta)} x^{\beta-1} e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

$$h(x) = \int_{\mathbb{R}} f(y)\, g(x-y)\,dy = \int_0^{+\infty} f(y)\, g(x-y)\,dy = 0 \quad \forall x \le 0$$

$x > 0$

$$h(x) = \int_0^{+\infty} f(y)\, g(x-y)\,dy \qquad g(x-y) \neq 0 \quad x-y > 0$$

$y < x$

$$= \int_0^x \frac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \cancel{e^{-\lambda y}} \frac{\lambda^\beta}{\Gamma(\beta)} (x-y)^{\beta-1} e^{-\lambda(x-\cancel{y})}\,dy$$

$$= \frac{\lambda^{\alpha+\beta} e^{-\lambda x}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^x y^{\alpha-1} (x-y)^{\beta-1}\,dy \qquad y = tx$$

$x - y = x - tx = x(1-t) \qquad dy = x\,dt$

$$= \frac{\lambda^{\beta+\alpha} e^{-\lambda x}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 x^{\alpha-1} t^{\alpha-1} (1-t)^{\beta-1} x^{\beta-\cancel{1}} \cancel{x}\, dt$$

$$= \frac{\lambda^{\alpha+\beta} e^{-\lambda x}}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha+\beta-1} \int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt$$

$$C = \frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt$$

$$= C\, x^{\alpha+\beta-1} e^{-\lambda x}$$

$$\Gamma(\alpha+\beta, \lambda) \qquad \boxed{\frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha+\beta)}} x^{\alpha+\beta-1} e^{-\lambda x}$$

$$\frac{\cancel{\lambda^{\alpha+\beta}}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt = \frac{\cancel{\lambda^{\alpha+\beta}}}{\Gamma(\alpha+\beta)}$$

$$\int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

— o —

$\chi^2_n = \Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$ $\qquad$ $\mathbb{P}_X = \chi^2_n$, $\mathbb{P}_Y = \chi^2_k$ $\qquad$ X e Y indipendenti.

$\Rightarrow \mathbb{P}_{X+Y} = \chi^2_{n+k}$

— o —

$X_1 — X_n$ campione (statistico) gaussiano

$\mathbb{P}_{X_i} = N(\mu, \sigma^2)$ $\qquad$ $X_1 — X_n$ indipendenti

$X_i = \mu + \sigma Z_i$ $\qquad$ Z_i gaussiane standard

$Z_i := \frac{X_i - \mu}{\sigma}$ $\Rightarrow$ $Z_1 — Z_n$ è un campione gaussiano standard cioè $Z_1 \ldots Z_n$ sono v.a. indipendenti e $\mathbb{P}_{Z_i} = N(0,1)$

$\forall i = 1 \ldots n \qquad \mathbb{P}_{Z_i^2} = \chi_1^2$

$$\mathbb{P}_{Z_1^2 + Z_2^2} = \chi_2^2$$

$$\vdots$$

$$\mathbb{P}_{Z_1^2 + Z_2^2 + \ldots Z_n^2} = \chi_n^2$$

$$\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2$$

— o —

Sia $X_1 \ldots X_n$ campione gaussiano, $\mathbb{P}_{X_i} = N(\mu, \sigma^2)$
allora $\overline{X}$ e S^2 sono v.a. indipendenti.

Considero $Z_i := \frac{X_i - \mu}{\sigma}$ standardizzazione del campione gaussiano $X_1 \ldots X_n$

Sia $\overline{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_i$ e sia $\sum_{i=1}^{n} (Z_i - \overline{Z})^2$

Si ha

$$\mathbb{P}_{\sum_{i=1}^{n} (Z_i - \overline{Z})^2} = \chi_{n-1}^2$$

$n = 2 \qquad \mathbb{P}_{Z_1} = \mathbb{P}_{Z_2} = N(0,1)$

$$\overline{Z} = \frac{Z_1 + Z_2}{2} \qquad \mathbb{P}_{Z_1 + Z_2} = N\left(0, \frac{1}{2}\right)$$

$$(Z_1 - \overline{Z})^2 + (Z_2 - \overline{Z})^2 = \left(Z_1 - \frac{Z_1 + Z_2}{2} \right)^2 + \left(Z_2 - \frac{Z_1 + Z_2}{2} \right)^2$$

$$= \left(\frac{Z_1 - Z_2}{2} \right)^2 + \left(\frac{Z_2 - Z_1}{2} \right)^2 = 2 \left(\frac{Z_1 - Z_2}{2} \right)^2 = \left(\frac{Z_1 - Z_2}{\sqrt{2}} \right)^2$$

$$\mathbb{P}_{Z_i} = N(0,1) \qquad Z_1 - Z_2 = Z_1 + (-Z_2)$$

$$\mathbb{P}_X = f(x)\,dx \qquad \mathbb{P}_{aX+b} = \frac{1}{|a|} f\left(\frac{x-b}{a}\right) \qquad a \neq 0 \quad b \in \mathbb{R}$$

$$\mathbb{P}_{-Z_2} = g(x)\,dx \qquad g(x) = \frac{1}{|-1|} f_0\left(\frac{x}{-1}\right) = f_0(x)$$

$$\mathbb{P}_{-Z_2} = N(0,1) \qquad Z_1 - Z_2 = Z_1 + (-Z_2)$$

$$\mathbb{P}_{Z_1 - Z_2} = N(0,2) \qquad \mathbb{P}_{\frac{Z_1 - Z_2}{\sqrt{2}}} = N(0,1)$$

$$\mathbb{P}_{\left(\frac{Z_1 - Z_2}{\sqrt{2}}\right)^2} = \chi^2_1$$

— o —

X_1 ___ X_n campione gaussiano $\mathbb{P}_{X_i} = N(\mu, \sigma^2)$

$\to$ $Z_i = \dfrac{X_i - \mu}{\sigma}$ campione gaussiano standard

$$\chi^2_{n-1} \simeq \sum_{i=1}^{n} (Z_i - \bar{Z})^2$$

$$X_i = \mu + \sigma Z_i \qquad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{n}\sum_{i=1}^{n}(\mu + \sigma Z_i)$$

$$\bar{X} = \frac{1}{n}\left(n\mu + \sigma \sum_{i=1}^{n} Z_i\right) = \mu + \sigma \bar{Z} \Rightarrow \bar{Z} = \frac{\bar{X} - \mu}{\sigma}$$

$$\chi^2_{n-1} \simeq \sum_{i=1}^{n} (Z_i - \bar{Z})^2 = \sum_{i=1}^{n} \left(\frac{X_i - \cancel{\mu}}{\sigma} - \frac{\bar{X} - \cancel{\mu}}{\sigma}\right)^2 =$$

$$= \frac{1}{\sigma^2}\sum_{i=1}^{n} (X_i - \bar{X})^2 = \frac{n-1}{\sigma^2} S^2$$

Pongo $V := \dfrac{n-1}{\sigma^2} S^2 \Rightarrow \mathbb{P}_V = \chi^2_{n-1}$

DISTRIBUZIONE t DI STUDENT A n GRADI LIBERTÀ $t(n)$

È la distribuzione AC associata alla densità

$$z_n(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}} \left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}} \qquad \forall x \in \mathbb{R}$$

Se $\mathbb{P}_X = z_n(x)\,dx \Rightarrow \begin{cases} \mathbb{E}[X] = 0 \\ Var[X] = \begin{cases} \frac{n}{n-2} & n \geq 3 \\ +\infty & n = 1,2 \end{cases} \end{cases}$

PROPOSIZIONE Siano X e Y v.a. indipendenti t.c.

$\mathbb{P}_X = N(0,1)$

$\mathbb{P}_Y = \chi^2_n$

Sia $T := \frac{X\sqrt{n}}{\sqrt{Y}}$

Allora $\mathbb{P}_T = t(n)$

— o —

$X_1 \ldots X_n$ campione gaussiano

$\mathbb{P}_{X_i} = N(\mu, \sigma^2)$

$Z_i := \frac{X_i - \mu}{\sigma} \qquad \mathbb{P}_{Z_i} = N(0,1)$

$\mathbb{P}_{\bar{Z}} = N\left(0, \frac{1}{n}\right) \qquad \mathbb{P}_{\bar{Z}\sqrt{n}} = N(0,1) \qquad \boxed{\bar{Z} = \frac{\bar{X}-\mu}{\sigma}}$

$V := \frac{(n-1)S^2}{\sigma^2} \qquad \mathbb{P}_V = \chi^2_{n-1}$

$$T = \frac{\bar{Z}\sqrt{n}\,\cancel{\sqrt{n-1}} \cdot \sigma}{\cancel{\sqrt{n-1}}\; S} = \frac{\frac{\bar{X}-\mu}{\cancel{\sigma}}\sqrt{n}\,\cancel{\sigma}}{S}$$

$$T := \frac{(\overline{X} - \mu)\sqrt{n}}{S} \quad \text{è v.c.} \quad \mathbb{P}_T = t(n-1)$$