Note Title 04/06/2018

$Y_1 \ldots Y_{100}$ $\quad Y_i(\Omega) \subseteq \{0, 1, 2\}$

$\alpha = 0.05$ $\quad \mathbb{P}_{Y_i} = Bin(2, 0.5)$

Lo "0" appare 30 volte

L'"1" appare 45 volte

Il "2" compare 25 volte

$$\sum_{j=1}^{k} \frac{(n_j - np_j^0)^2}{np_j^0} < \chi^2_{k-1, 1-\alpha}$$

$n = 100$

$k = 3$

$1 - \alpha = 0.95$

$\chi^2_{2, 0.95} \simeq 5.9915$

$j = 0, 1, 2$

$n_0 = 30$

$n_1 = 45$

$n_2 = 25$

$n = 100$

$$\sum_{j=0}^{2} \frac{(n_j - np_j^0)^2}{np_j^0} < 5.9915 \; ?$$

$p_0^0 = Bin(2, \frac{1}{2})(\{0\}) = \binom{2}{0}\left(\frac{1}{2}\right)^0\left(1 - \frac{1}{2}\right)^{2-0} = \frac{1}{4}$

$p_1^0 = Bin(2, \frac{1}{2})(\{1\}) = \binom{2}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{2-1} = \frac{1}{2}$

$p_2^0 = Bin(2, \frac{1}{2})(\{2\}) = \binom{2}{2}\left(\frac{1}{2}\right)^2\left(1 - \frac{1}{2}\right)^{2-2} = \frac{1}{4}$

$$\frac{\left(30 - 100\frac{1}{4}\right)^2}{100 \cdot \frac{1}{4}} + \frac{\left(45 - 100 \cdot \frac{1}{2}\right)^2}{100 \cdot \frac{1}{2}} + \frac{\left(25 - 100 \cdot \frac{1}{4}\right)^2}{100 \cdot \frac{1}{4}} =$$

$$= \frac{25}{25} + \frac{25}{50} + 0 = 1 + \frac{1}{2} = \frac{3}{2} < 5.9915 \quad \underline{\text{ok}}$$

$n = 11$

12.63	10.33	10.09	9.30	9.61	7.92
9.37	10.40	8.83	9.80	12.75	

$1-\alpha = 0.95 \quad n = 11 \quad \mathbb{P}_{X_i} = N(\mu, \sigma^2)$

$$\left(\bar{x} - \frac{s}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} , \; \bar{x} + \frac{s}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} \right)$$

$\bar{x} = 9.8525 \simeq 9.85$

$s = 1.544186 \simeq 1.54 \qquad s^2 = 2.384511 \simeq 2.38$

$1 - \frac{\alpha}{2} = 0.975 \qquad t_{10, 0.975} \simeq 2.2281$

$$\frac{s}{\sqrt{11}} t_{10, 0.975} \simeq \frac{1.54}{\sqrt{11}} \cdot 2.2281 \simeq 1.03$$

PER I VALORI ATTESI

$$(L_i, L_s) = (9.85 - 1.03, \; 9.85 + 1.03) = (8.82, 10.88)$$

Intervallo di confidenza per la varianza

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1, 1-\frac{\alpha}{2}}} , \; \frac{(n-1)s^2}{\chi^2_{n-1, \frac{\alpha}{2}}} \right)$$

$\chi^2_{10, 0.975} = 20.4832 \qquad \chi^2_{10, 0.025} \simeq 3.2470$

$$\left(\frac{10 \cdot 2.38}{20.4832} , \; \frac{10 \cdot 2.38}{3.247} \right) \simeq (1.16, 7.35)$$

Ex 1

Altezze medie = 170 cm

con s.q.m. = 10 cm

$\mathbb{P}_{H_i} = N(170, 100)$

$$\mathbb{P}(H_i > 175) = \mathbb{P}\left(\underbrace{\frac{H_i - 170}{10}}_{\text{gaussiana standard } Z} > \frac{175-170}{10}\right)$$

$$= \mathbb{P}\left(Z > \frac{1}{2}\right) = 1 - \Phi(0.5) \simeq 1 - 0.69146 \simeq$$

$$\simeq 1 - 0.69 = 0.31$$

$H_1 \dots H_{10}$ $\quad \overline{H}_{10}$ $\quad \mathbb{P}_{H_i} = N(170, 100)$

$$\mathbb{P}(\overline{H}_{10} > 175) = \qquad \mathbb{P}_{\overline{H}_{10}} = N(170, 10)$$

$$= \mathbb{P}\left(\frac{\overline{H}_{10} - 170}{\sqrt{10}} > \frac{175-170}{\sqrt{10}}\right) =$$

$$= \mathbb{P}\left(Z > \frac{5\sqrt{10}}{10}\right) = \mathbb{P}\left(Z > \frac{\sqrt{10}}{2}\right) \simeq \mathbb{P}(Z > 1.58)$$

$$= 1 - \Phi(1.58) \simeq 1 - 0.94295 \simeq 1 - 0.943 = 0.057$$

$$\mathbb{P}(\overline{H}_{100} > 175) \qquad \mathbb{P}_{H_i} = N(170, 100)$$

$$\mathbb{P}_{\overline{H}_{100}} = N(170, 1)$$

$$\mathbb{P}(\overline{H}_{100} > 175) = \mathbb{P}\left(\frac{\overline{H}_{100} - 170}{\sqrt{1}} > \frac{175-170}{\sqrt{1}}\right) =$$

$$= \mathbb{P}(Z > 5) = 1 - \Phi(5) \qquad \Phi(5) \simeq 0.999997$$

$$\simeq 0.000003$$

EX 2

$T \qquad \mathbb{P}_T = \exp(\lambda) \qquad \lambda > 0$ ignoto

$(L_i, L_s) = (0.24, 0.25)$ intervallo di confidenza con livello di fiducia 0.99 per il parametro λ

$$\mathbb{P}(0.24 < \lambda < 0.25) = 0.99$$

$$\mathbb{P}\left(T > \frac{1}{2}\right) = 1 - \mathbb{P}\left(T \leq \frac{1}{2}\right) = 1 - F_T\left(\frac{1}{2}\right) =$$

$$= 1 - \left(1 - e^{-\lambda \frac{1}{2}}\right) = e^{-\frac{\lambda}{2}}$$

$$0.24 < \lambda < 0.25 \iff \frac{-0.25}{2} < -\frac{\lambda}{2} < \frac{-0.24}{2}$$

$$\iff -0.125 < -\frac{\lambda}{2} < -0.12$$

$$\iff e^{-0.125} < e^{-\lambda/2} < e^{-0.12}$$

$$\iff 0.882 < e^{-\lambda/2} < 0.887$$

$(0.882, 0.887)$ è un intervallo di confidenza con livello di fiducia 99% per $e^{-\lambda/2}$

$$\mathbb{P}\left(T > \frac{1}{2}\right) = e^{-\lambda/2}$$

EX 3

1ª metodologia $\quad \bar{x} = \mu_0$

$$s_x^2 = \frac{1}{5} s_y^2$$

$$\text{costo } x_i = 8 \text{ costo } y_j$$

2ª metodologia $\quad \bar{y} = \mu_0$

$$s_y^2 = 5 s_x^2$$

1ª metodologia: faccio n esperimenti

$$\left(\bar{x} - \frac{s_x}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}}, \ \bar{x} + \frac{s_x}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} \right)$$

2ª metodologia: faccio K esperimenti

$$\left(\bar{y} - \frac{s_y}{\sqrt{K}} t_{K-1, 1-\frac{\alpha}{2}}, \ \bar{y} + \frac{s_y}{\sqrt{K}} t_{K-1, 1-\frac{\alpha}{2}} \right)$$

Scelgo la 1ª metodologia sse

$$\frac{s_x}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} < \frac{s_y}{\sqrt{K}} t_{K-1, 1-\frac{\alpha}{2}}$$

$K = 8n$ $\quad$ Scelgo la 1ª metodologia sse

$s_y^2 = 5 s_x^2$

$$\frac{\cancel{s_x}}{\cancel{\sqrt{n}}} t_{n-1, 1-\frac{\alpha}{2}} < \frac{\sqrt{5}\,\cancel{s_x}}{2\sqrt{2\cancel{n}}} t_{8n-1, 1-\frac{\alpha}{2}}$$

SSE $\quad 2\sqrt{2}\, t_{n-1, 1-\frac{\alpha}{2}} < \sqrt{5}\, t_{8n-1, 1-\frac{\alpha}{2}}$

EX 4 $\quad n = 250 \qquad 1-\alpha = 90\%$

$\bar{x} = 65$ (€)

$s_x^2 = 300$ (€^{2})

VALORE ATTESO $\quad \left(\bar{x} - \frac{s_x}{\sqrt{n}} \underbrace{t_{n-1, 1-\frac{\alpha}{2}}}_{\text{}}, \bar{x} + \frac{s_x}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} \right)$ $\quad t_{249, 0.95}$

$\simeq \left(65 - \frac{10\sqrt{3}}{5\sqrt{10}} 1.65, 65 + \frac{10\sqrt{3}}{5\sqrt{10}} 1.65 \right) \simeq (63.185, 66.815)$

VARIANZA $\quad \left(\frac{(n-1)s^2}{\chi^2_{n-1, 1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1, \frac{\alpha}{2}}} \right) =$

$= \left(\frac{249 \cdot 300}{\chi^2_{249, 0.95}}, \frac{249 \cdot 300}{\chi^2_{249, 0.05}} \right) \simeq \left(\frac{249 \cdot 300}{286.81}, \frac{249 \cdot 300}{213.47} \right)$

$\simeq (260.45, 349.90)$

EX 5 $\quad N(1.4, \sigma^2)$ la settimana scorsa

$n = 10 \qquad 1.549 \quad 1.389 \quad . \; . \; . \; . \; . \quad 1.448$

$\bar{x} \simeq 1.342 \qquad n = 10 \qquad s \simeq 0.184 \qquad 1-\alpha = 0.95$

$\left(\bar{x} - \frac{s}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}}, \bar{x} + \frac{s}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}} \right)$

$\simeq \left(1.342 - \frac{0.184}{\sqrt{10}} \underbrace{t_{9, 0.975}}_{\simeq 2.262}, 1.342 + \frac{0.184}{\sqrt{10}} t_{9, 0.975} \right)$

$(1.342 - 0.132, 1.342 + 0.132) = (1.21, 1.474)$

$\alpha = 0.05$

$H_0: \mu = 1.4$ $\qquad H_A: \mu \neq 1.4$

$t_{9,0.975} \simeq 2.262$

Accetto H_0 sse $|\bar{x} - 1.4| < \frac{s}{\sqrt{n}} t_{n-1, 1-\frac{\alpha}{2}}$

Accetto H_0 sse $|1.342 - 1.4| < \frac{0.185}{\sqrt{10}} 2.262$

Accetto H_0 sse $0.068 < 0.132$

— o —

2) $H_0: \mu \leq 1.4$, $\quad H_A: \mu > 1.4$

Accetto H_0 sse $\bar{x} < 1.4 + \frac{s}{\sqrt{n}} t_{n-1, 1-\alpha}$

Accetto H_0 sse $1.342 < 1.4 + \frac{0.185}{\sqrt{10}} t_{9, 0.95}$ $\simeq 1.833$

$\Rightarrow$ $\bar{x}$, accetto H_0

EX 6 $\quad n = 10 \quad N(\mu_x, \sigma^2)$

878 . . . . . . 802 $\qquad \bar{x} = 841.6 \qquad s_x = 66.95$

— o —

$N(\mu_y, \sigma^2) \qquad K = 10$

943, ... 917 $\qquad \bar{y} = 937.1 \qquad s_y = 116.50$

$X_1 \dots X_{10} \qquad N(\mu_x, \sigma^2)$

$Y_1 \dots Y_{10} \qquad N(\mu_y, \sigma^2)$

$\mathbb{P}_{\bar{X}} = N\left(\mu_x, \frac{\sigma^2}{n}\right) \qquad \mathbb{P}_{\bar{Y}} = N\left(\mu_y, \frac{\sigma^2}{k}\right)$

$$\mathbb{P}_{\bar{X}-\bar{Y}} = N\left(\mu_x-\mu_y,\ \frac{\sigma^2}{n}+\frac{\sigma^2}{k}\right) = N\left(\mu_x-\mu_y,\ \sigma^2\left(\frac{1}{n}+\frac{1}{k}\right)\right)$$

$$V_x := \frac{(n-1)S_x^2}{\sigma^2} \qquad \mathbb{P}_{V_x} = \chi^2_{n-1}$$

$$V_y := \frac{(k-1)S_y^2}{\sigma^2} \qquad \mathbb{P}_{V_y} = \chi^2_{k-1}$$

$$\mathbb{P}_{V_x+V_y} = \chi^2_{n+k-2} \qquad V_x+V_y = \frac{(n-1)S_x^2+(k-1)S_y^2}{\sigma^2}$$

$$Z = \frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sigma\sqrt{\frac{1}{n}+\frac{1}{k}}} \qquad \mathbb{P}_Z = N(0,1)$$

$$T := \frac{Z\sqrt{n+k-2}}{\sqrt{V_x+V_y}} \qquad \mathbb{P}_T = t(n+k-2)$$

$$\mathbb{P}\left(|T| < t_{n+k-2,\,1-\frac{\alpha}{2}}\right) = 1-\alpha$$

$$T = \frac{\left((\bar{X}-\bar{Y})-(\mu_x-\mu_y)\right)\sqrt{n+k-2}}{\sqrt{n+k}\ \sqrt{(n-1)S_x^2+(k-1)S_y^2}}$$

$$\mathbb{P}\left(\left|(\bar{X}-\bar{Y})-(\mu_x-\mu_y)\right| \le \frac{t_{n+k-2,1-\frac{\alpha}{2}}\sqrt{n+k}\sqrt{(n-1)S_x^2+(k-1)S_y^2}}{\sqrt{(n+k-2)nk}}\right)$$

$$L_i,\ L_s = \bar{X}-\bar{Y} \pm \frac{t_{n+k-2,1-\frac{\alpha}{2}}\sqrt{n+k}\sqrt{(n-1)S_x^2+(k-1)S_y^2}}{\sqrt{(n+k-2)nk}}$$

$n=k=10$

$\bar{x}-\bar{y} = 841.6-937.1 = -95.5$

$t_{18, 0.975} \simeq 2.101$

$\sqrt{n+k} = \sqrt{20} = 2\sqrt{5}$

$\sqrt{nk} = 10$

$\sqrt{n+k-2} = \sqrt{18} = 3\sqrt{2}$

$S_x = 66.948$

$S_y = 116.5$

$$(L_i, L_s) = \left(-95.5 - \underbrace{77.36}_{89.25}, -95.5 + 77.36\right)$$

$\mu_x - \mu_y < 0$ con livello di confidenza superiore al 95%