Note Title 24/05/2018

TEOREMA Sia $X_1 \ldots X_n$ campione statistico gaussiano di numerosità n, valore atteso μ e varianza σ^2. Allora la media campionaria $\overline{X}$ e la varianza campionaria S^2 sono v.a. indipendenti.

Siano $Z_1 \ldots Z_n$ le standardizzazioni di $X_1 \ldots X_n$ cioè
$$Z_i := \frac{X_i - \mu}{\sigma} \qquad \forall i = 1 \ldots n$$
Allora $\overline{Z} = \dfrac{\overline{X} - \mu}{\sigma}$. Inoltre le v.a.

$\overline{Z}$ e $\sum_{i=1}^{n} (Z_i - \overline{Z})$ sono indipendenti e la v.a.

$\sum_{i=1}^{n} (Z_i - \overline{Z})^2$ ha distribuzione di Pearson a $n-1$ gradi di libertà (χ^2_{n-1}).

DIM
$$(n-1) S_n^2 = (n-2) S_{n-1}^2 + \left(1 - \frac{1}{n}\right)\left(X_n - \overline{X}_{n-1}\right)^2$$

$$\overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i = \frac{1}{n}\sum_{i=1}^{n} \frac{X_i - \mu}{\sigma} = \frac{1}{\sigma}\sum_{i=1}^{n}\left(\frac{1}{n}X_i - \frac{1}{n}\mu\right) =$$
$$= \frac{1}{\sigma}\left(\sum_{i=1}^{n}\frac{1}{n}X_i - \mu\right) = \frac{\overline{X} - \mu}{\sigma}$$

Indico con S_n^2 la varianza campionaria delle Z_i
$$(n-1)\frac{1}{n-1}\sum_{i=1}^{n}(Z_i - \overline{Z}_n)^2 = \cancel{(n-2)}\cancel{\frac{1}{n-2}}\sum_{i=1}^{n-1}(Z_i - \overline{Z}_{n-1})^2 + \left(1 - \frac{1}{n}\right)\left(Z_n - \overline{Z}_{n-1}\right)^2$$

$$\underbrace{\sum_{i=1}^{n}(Z_i - \overline{Z})^2}_{Ts:\ \chi^2_{n-1}} = \underbrace{\sum_{i=1}^{n-1}(Z_i - \overline{Z})^2}_{Hp:\ \chi^2_{n-2}} + \left(1 - \frac{1}{n}\right)\left(Z_n - \overline{Z}_{n-1}\right)^2$$

PASSO INIZIALE $n=2$

$$\left(Z_1 - \frac{Z_1+Z_2}{2}\right)^2 + \left(Z_2 - \frac{Z_1+Z_2}{2}\right)^2 = 2\frac{(Z_1-Z_2)^2}{4} = \frac{1}{2}(Z_1-Z_2)^2$$

$$= \left(\frac{Z_1-Z_2}{\sqrt{2}}\right)^2$$

$\mathbb{P}_{Z_1} = \mathbb{P}_{Z_2} = N(0,1)$ e sono indipendenti.

$$\mathbb{P}_{Z_1-Z_2} = N(0-0, 1+1) = N(0,2)$$

$$\mathbb{P}_{\frac{Z_1-Z_2}{\sqrt{2}}} = N\left(0, \frac{2}{2}\right) = N(0,1)$$

$$\Rightarrow \mathbb{P}_{\left(\frac{Z_1-Z_2}{\sqrt{2}}\right)^2} = \chi^2_1$$

$$\left(1-\frac{1}{n}\right)\left(Z_n - \bar{Z}_{n-1}\right)^2$$

$\mathbb{P}_{Z_n} = N(0,1)$ $\qquad \bar{Z}_{n-1}$ è indipendente da Z_n

$$\mathbb{P}_{\bar{Z}_{n-1}} = N\left(0, \frac{1}{n-1}\right)$$

$$\mathbb{P}_{Z_n - \bar{Z}_{n-1}} = N\left(0, 1+\frac{1}{n-1}\right) = N\left(0, \frac{n}{n-1}\right)$$

$$\left(1-\frac{1}{n}\right)\left(Z_n - \bar{Z}_{n-1}\right)^2 = \frac{n-1}{n}\left(Z_n - \bar{Z}_{n-1}\right)^2 = \left(\sqrt{\frac{n-1}{n}}\left(Z_n - \bar{Z}_{n-1}\right)\right)^2$$

$$\mathbb{P}_{\sqrt{\frac{n-1}{n}}(Z_n - \bar{Z}_{n-1})} = N\left(0, \frac{n}{n-1}\,\frac{n-1}{n}\right) = N(0,1)$$

$$\Rightarrow \mathbb{P}_{\frac{n-1}{n}(Z_n - \bar{Z}_{n-1})^2} = \chi^2_1$$

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Corollario Sia $X_1 \ldots X_n$ campione gaussiano di valore atteso μ e varianza σ^2.
Allora la v.a.

$$V := \frac{(n-1)S^2}{\sigma^2} \quad \text{ha distribuzione } \chi^2_{n-1}$$

DIM. $V = \frac{1}{\sigma^2}(n-1)S^2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \overline{X})^2 =$

$$= \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \mu + \mu - \overline{X})^2 =$$

$$= \frac{1}{\sigma^2} \sum_{i=1}^{n} \left((X_i - \mu) - (\overline{X} - \mu)\right)^2 =$$

$$= \sum_{i=1}^{n} \left(\left(\frac{X_i - \mu}{\sigma}\right) - \left(\frac{\overline{X} - \mu}{\sigma}\right)\right)^2$$

$$Z_i := \frac{X_i - \mu}{\sigma} \qquad \mathbb{P}_{Z_i} = N(0,1) \quad e \quad \overline{Z} = \frac{\overline{X} - \mu}{\sigma}$$

$$\Rightarrow V = \sum_{i=1}^{n} (Z_i - \overline{Z})^2 \Rightarrow \mathbb{P}_V = \chi^2_{n-1}$$

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DISTRIBUZIONE t DI STUDENT A n GRADI DI LIBERTÀ $t(n)$

È una distribuzione A.C. con densità

$$\tau_n(x) := \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}} \left(1 + \frac{x^2}{n}\right)^{-\frac{(n+1)}{2}} \qquad \forall x \in \mathbb{R}$$

$$1 = \int_{\mathbb{R}} \tau_n(x)\,dx$$

$$\mathbb{E}[X] = 0 \quad \forall n \geq 2 \qquad \mathrm{Var}[X] = \frac{n}{n-2} \quad n \geq 3$$

$z_n(x) > 0 \quad \forall x \in \mathbb{R} \qquad z_n(x) = z_n(-x) \leftarrow$

$F_X(x)$ è strettamente crescente

$\forall \alpha \in (0,1) \quad \exists !\, x \quad$ t.c. $\quad F_X(x) = \alpha$

x si indica col simbolo $t_{n,\alpha}$

Inoltre $\quad t_{n,1-\alpha} = -t_{n,\alpha} \quad \forall \alpha \in (0,1)$

TEOREMA Siano Z e Y v.a. indipendenti

t.c. $\quad \mathbb{P}_Z = N(0,1)$

e $\quad \mathbb{P}_Y = \chi^2_n$

Allora

la v.a. $\quad T := \dfrac{Z\sqrt{n}}{\sqrt{Y}} \quad$ ha distribuzione $t(n)$

DIM $\quad T = \varphi \circ (Y, Z) \qquad \varphi : (y,z) \in \mathbb{R}^2 \longmapsto \begin{cases} \dfrac{z\sqrt{n}}{\sqrt{y}} & y > 0 \\ 0 & y \le 0 \end{cases}$

$\psi : \mathbb{R} \to \mathbb{R}$ funzione di Borel non negativa

$$\int_{\mathbb{R}} \psi(t)\, \mathbb{P}_T(dt) = \int_{\mathbb{R}^2} \psi(\varphi(y,z))\, \underbrace{\mathbb{P}_{Y,Z}(dy\,dz)}_{\mathbb{P}_Y(dy)\,\mathbb{P}_Z(dz)}$$

$$= \int_{\mathbb{R}^2} \psi(\varphi(y,z))\, f_Y(y)\, g_Z(z)\, dy\, dz$$

$$f_Y(y) = \begin{cases} \dfrac{\left(\frac{n}{2}\right)^{1/2}}{\Gamma\left(\frac{n}{2}\right)}\, y^{\frac{n}{2}-1} \exp\left(-\dfrac{y}{2}\right) & y > 0 \\ 0 & y \le 0 \end{cases}$$

$$= \int_{(0,+\infty)\times\mathbb{R}} \psi\left(\frac{z\sqrt{n}}{\sqrt{y}}\right) \frac{\left(\frac{n}{2}\right)^{1/2}}{\Gamma\left(\frac{n}{2}\right)}\, y^{\frac{n}{2}-1} \exp\left(-\frac{y}{2}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dy\,dz$$

$$t := \frac{z\sqrt{n}}{\sqrt{y}} \qquad z = \frac{t \cdot \sqrt{y}}{\sqrt{n}} \qquad dz = \frac{\sqrt{y}}{\sqrt{n}}\, dt$$

$$\int_{(0,+\infty)\times\mathbb{R}} \varphi(t)\,\frac{(n/2)^{1/2}}{\Gamma(n/2)}\, y^{n/2-1}\,\frac{1}{\sqrt{2\pi}}\,\frac{y^{1/2}}{\sqrt{n}}\,\exp\left(\frac{-y}{2}-\frac{1}{2}\frac{t^2 y}{n}\right) dy\,dt$$

$$= \int_{(0,+\infty)\times\mathbb{R}} \varphi(t)\,\frac{(n/2)^{1/2}}{\Gamma(n/2)}\,\frac{1}{\sqrt{2\pi n}}\, y^{\frac{n}{2}-\frac{1}{2}}\,\exp\left(-\frac{y}{2}\left(1+\frac{t^2}{n}\right)\right) dy\,dt$$

$$y = 2u\left(1+\frac{t^2}{n}\right)^{-1}$$

$$\int_{\mathbb{R}} \varphi(t)\underbrace{\left(\int_0^{+\infty} \frac{(n/2)^{1/2}}{\Gamma(n/2)}\,\frac{1}{\sqrt{2\pi n}}\, y^{\frac{n}{2}-\frac{1}{2}}\,\exp\left(-\frac{y}{2}\left(1+\frac{t^2}{n}\right)\right) dy\right)} dt$$

$$= \int_{\mathbb{R}} \varphi(t)\, h(t)\, dt$$

$$\int_{\mathbb{R}} \varphi(t)\,\mathbb{P}_T(dt) = \int_{\mathbb{R}} \varphi(t)\, h(t)\, dt \qquad \forall \varphi \text{ di Borel non negative}$$

$\Rightarrow$ $h(t)$ è la densità di $\mathbb{P}_T$

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$X_1 \dots X_n$ campione gaussiano di valore medio μ e varianza σ^2

$$\mathbb{P}_{X_i} = N(\mu, \sigma^2) \quad \forall i = 1 \dots n$$

$$\mathbb{P}_{\bar{X}} = N\left(\mu, \frac{\sigma^2}{n}\right) \qquad \mathbb{P}_{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}} = N(0,1)$$

$$V := \frac{(n-1)S^2}{\sigma^2} \qquad \mathbb{P}_V = \chi^2_{n-1}$$

$$T = \frac{\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right)\sqrt{n-1}}{\sqrt{V}} \qquad \mathbb{P}_T = t(n-1)$$

$$T = \frac{(\bar{X}-\mu)\sqrt{n}}{\cancel{\sigma}} \frac{\cancel{\sqrt{n-1}}}{\cancel{\sqrt{n-1}}\,\frac{S}{\cancel{\sigma}}} = \frac{(\bar{X}-\mu)\sqrt{n}}{S}$$

$x_1 \text{——} x_n$

$X_1 \text{——} X_n$ i.i.d.

Se Y è una statistica del campione $Y = f(X_1, \text{——}, X_n)$ che stima un parametro θ caratterizzante la distribuzione del campione, dico che Y è uno stimatore del parametro θ

$$\mathbb{P}_{X_i} = f(x)\,dx \qquad \forall i = 1 \text{——} n$$

$$\mathbb{P}\left(|X_1 - x_1| < \frac{\delta}{2}, |X_2 - x_2| < \frac{\delta}{2}, \text{——}, |X_n - x_n| < \frac{\delta}{2} \right)$$

$$= \underbrace{\mathbb{P}\left(|X_1 - x_1| < \frac{\delta}{2} \right)}_{\int_{x_1-\delta/2}^{x_1+\delta/2} f(x)dx} \mathbb{P}\left(|X_2 - x_2| < \frac{\delta}{2} \right) \dots \mathbb{P}\left(|X_n - x_n| < \frac{\delta}{2} \right)$$

$$\simeq f(x_1)\delta\, f(x_2)\delta \dots f(x_n)\delta = \delta^n \underbrace{f(x_1) f(x_2) \text{——} f(x_n)}_{g(x_1 \text{——} x_n)}$$

densità congiunta

$$f(x) = f(x|\theta)$$

$$g(x_1 \text{——} x_n | \theta) = \prod_{i=1}^{n} f(x_i | \theta)$$

$$\ln g(x_1 \text{——} x_n | \theta) = \ln \left(\prod_{i=1}^{n} f(x_i|\theta) \right) = \sum_{i=1}^{n} \ln f(x_i|\theta)$$

ESEMPIO $X_1 \text{——} X_n$ campione di Bernoulli

$x_1 \text{——} x_n \in \{0, 1\}$

$$f(x_i|p) = \begin{cases} p & x_i = 1 \\ 1-p & x_i = 0 \end{cases}$$

$$g(x_1 \ldots x_n|p) = \prod_{i=1}^{n} f(x_i|p) = p^k (1-p)^{n-k}$$

$$k := \sum_{i=1}^{n} x_i$$

$$\ln g(x_1 \ldots x_n|p) = \ln\left(p^k (1-p)^{n-k}\right) =$$

$$= k \ln p + (n-k)\ln(1-p) = h(p)$$

$$h'(p) = \frac{k}{p} + \frac{n-k}{1-p}(-1) = \frac{k(1-\cancel{p}) - p(n-\cancel{k})}{p(1-p)}$$

$$= \frac{k - pn}{p(1-p)} \gtrless 0 \qquad k - pn \gtrless 0 \qquad p \lessgtr \frac{k}{n}$$

ho il max quando $p = \frac{k}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$

DISTRIBUZIONE DI POISSON

$$\mathbb{P}(X=k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad k = 0, 1, 2, \ldots$$

$$g(x_1 \ldots x_n|\lambda) = \prod_{i=1}^{n} e^{-\lambda}\frac{\lambda^{x_i}}{x_i!}$$

$$\ln g(x_1 \ldots x_n|\lambda) = \sum_{i=1}^{n} \ln\left(\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right) =$$

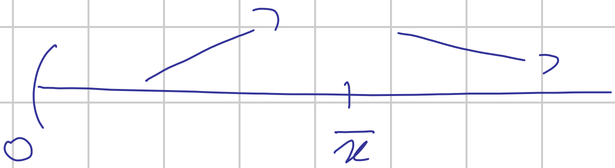
$$= \sum_{i=1}^{n}\left(-\lambda + x_i \ln(\lambda) - \ln(x_i!)\right) =$$

$$= -n\lambda + \ln(\lambda)\left(\sum_{i=1}^{n} x_i\right) - \sum_{i=1}^{n}\ln(x_i!)$$

$$= -n\lambda + n\ln(\lambda)\,\bar{x} - \sum_{i=1}^{n}\ln(x_i!) = h(\lambda)$$

$$h'(\lambda) = -n + \frac{n\bar{x}}{\lambda} = \frac{-n(\lambda - \bar{x})}{\lambda} \gtrless 0$$

$\lambda \underset{>}{\leq} \bar{x}$



ho il max in corrispondenza di $\lambda = \bar{x}$

DISTRIBUZIONE GAUSSIANA

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \quad \forall x \in \mathbb{R}$$

$$g(x_1 \dots x_n|\mu,\sigma) = \prod_{i=1}^{n} f(x_i|\mu,\sigma) =$$

$$= \prod_{i=1}^{n} \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right) =$$

$$= (2\pi\sigma^2)^{-n/2} \cdot \exp\left(\frac{-\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right)$$

$$\ln g(x_1 \dots x_n|\mu,\sigma) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$= \frac{-n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 = h(\mu,\sigma)$$

$$\frac{\partial h}{\partial \mu} = +\frac{1}{\cancel{2}\sigma^2}\sum_{i=1}^{n}\cancel{2}(x_i-\mu) = \frac{1}{\sigma^2}\left(\sum_{i=1}^{n}x_i - n\mu\right) =$$

$$= \frac{1}{\sigma^2}(n\bar{x} - n\mu) = \frac{n}{\sigma^2}(\bar{x}-\mu) = 0 \qquad SS \Leftarrow \boxed{\mu = \bar{x}}$$

$$\frac{\partial h}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\cancel{2}}\cancel{2}\sigma^{-3}\sum_{i=1}^{n}(x_i-\mu)^2 =$$

$$= \frac{1}{\sigma^3}\left(-n\sigma^2 + \sum_{i=1}^{n}(x_i-\mu)^2\right) = 0$$

$$\begin{cases} \frac{\partial h}{\partial \mu} = 0 \\ \frac{\partial h}{\partial \sigma} = 0 \end{cases} \qquad \begin{cases} \mu = \bar{x} \\ -n\sigma^2 + \sum_{i=1}^{n}(x_i-\bar{x})^2 = 0 \end{cases}$$

$$\begin{cases} \mu = \bar{x} \\ \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i-\bar{x})^2 = \frac{n-1}{n}\,\frac{1}{n-1}\sum_{i=1}^{n} (x_i-\bar{x})^2 = \frac{n-1}{n}\, s^2 \end{cases}$$

DISTRIBUZIONE UNIFORME SU UN INTERVALLO

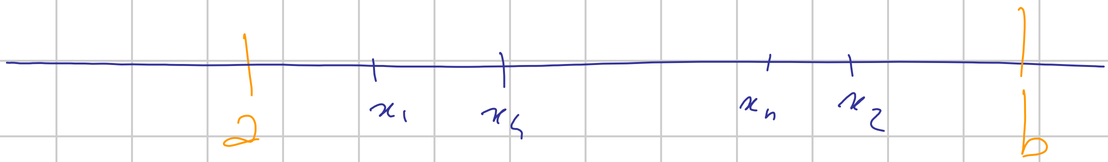
$X_1 \_\_ X_n \qquad \mathbb{P}_{X_i} = U([a,b])$

$$f(x \mid a,b) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{altrimenti} \end{cases}$$

$x_1 \_\_ x_n$

$$g(x_1 \_\_ x_n \mid a,b) = \prod_{i=1}^{n} f(x_i \mid a,b)$$

$$= \begin{cases} 0 & \text{se } \exists\, i = 1 \_\_ n \text{ t.c. } x_i \notin [a,b] \\ \frac{1}{(b-a)^n} & \text{se } x_i \in [a,b] \ \forall i = 1 \_\_ n \end{cases}$$



$b := \max\{x_1 \_\_ x_n\} \qquad a := \min\{x_1 \_\_ x_n\}$

INTERVALLI DI CONFIDENZA

$X_1 \_\_ X_n$ campione statistico

Supponiamo che la distribuzione del campione sia caratterizzata da un qualche parametro incognito $\theta \in \mathbb{R}$.

Siano $L_i = l_i(X_1 \_\_ X_n)$
$L_s = l_s(X_1 \_\_ X_n)$ due statistiche del campione

e sia $\alpha \in (0,1)$.

Considero l'intervallo (L_i, L_s)

Dico che (L_i, L_s) è un INTERVALLO DI CONFIDENZA DI LIVELLO $1-\alpha$ per il parametro θ se

$$\mathbb{P}(\theta \in (L_i, L_s)) \geq 1-\alpha$$

Dico che la semiretta $(L_i, +\infty)$ è un intervallo di confidenza unilaterale superiore per il parametro θ, di livello $1-\alpha$ se

$$\mathbb{P}(\theta \in (L_i, +\infty)) \geq 1-\alpha$$
$$\mathbb{P}(\theta > L_i) \geq 1-\alpha$$

Dico che la semiretta $(-\infty, L_s)$ è un intervallo di confidenza unilaterale inferiore di livello $1-\alpha$ per il parametro θ se

$$\mathbb{P}(\theta \in (-\infty, L_s)) \geq 1-\alpha$$
$$\mathbb{P}(\theta < L_s) \geq 1-\alpha$$

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CHEBYCHEV

$X_1 \_\_ X_n$ valore atteso μ e varianza σ^2

$$\mathbb{P}(|\overline{X}-\mu| \geq t) \leq \frac{\sigma^2}{nt^2} \qquad \forall t > 0$$

$$\mathbb{P}(|\overline{X}-\mu|<t) \geq 1-\frac{\sigma^2}{nt^2}$$

$$\mathbb{P}(-t<\overline{X}-\mu<t) \geq 1-\frac{\sigma^2}{nt^2}$$

SUPPONGO CHE σ SIA NOTA

$$\mathbb{P}(\overline{X}-t<\mu<\overline{X}+t) \geq 1-\frac{\sigma^2}{nt^2}$$

$$\mathbb{P}(\mu\in(\overline{X}-t,\overline{X}+t)) \geq 1-\alpha \qquad \alpha := \frac{\sigma^2}{nt^2}$$

$$t^2=\frac{\sigma^2}{n\alpha} \qquad t=\frac{\sigma}{\sqrt{n\alpha}}$$

$$(L_i,L_s)=\left(\overline{X}-\frac{\sigma}{\sqrt{n\alpha}},\ \overline{X}+\frac{\sigma}{\sqrt{n\alpha}}\right)$$ è un intervallo di confidenza di livello $1-\alpha$ per il valore atteso del campione

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CAMPIONE GAUSSIANO CON VARIANZA σ^2 NOTA E VALORE ATTESO μ IGNOTO

Sia Z v.a. gaussiana standard $\mathbb{P}_Z=N(0,1)$

$\alpha\in(0,1)$

$$\mathbb{P}(|Z|\leq z_{1-\frac{\alpha}{2}})=\mathbb{P}(-z_{1-\frac{\alpha}{2}}\leq Z\leq z_{1-\frac{\alpha}{2}})=$$

$$=\mathbb{P}(Z\leq z_{1-\frac{\alpha}{2}})-\mathbb{P}(Z\leq -z_{1-\frac{\alpha}{2}})=$$

$$=\Phi(z_{1-\frac{\alpha}{2}})-\Phi(\overbrace{-z_{1-\frac{\alpha}{2}}}^{z_{\alpha/2}})=$$

$$=1-\frac{\alpha}{2}-\frac{\alpha}{2}=1-\alpha$$

$X_1 \dots X_n \quad \mathbb{P}_{X_i}=N(\mu,\sigma^2)$ $\quad \mu$ incognito, σ^2 è nota

$\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ è gaussiana standard

$$1-\alpha = \mathbb{P}\left(\left|\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| \leq z_{1-\frac{\alpha}{2}}\right) = \mathbb{P}\left(|\overline{X}-\mu| \leq \frac{\sigma z_{1-\frac{\alpha}{2}}}{\sqrt{n}}\right)$$

$$= \mathbb{P}\left(\frac{-\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \leq \mu - \overline{X} \leq \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}\right) =$$

$$= \mathbb{P}\left(\overline{X} - \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \leq \mu \leq \overline{X} + \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}\right)$$

$\left(\overline{X} - \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}, \overline{X} + \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}\right)$ è un intervallo di confidenza di livello $1-\alpha$ per il parametro μ valore atteso.

L'ampiezza è $\dfrac{2\sigma}{\sqrt{n}} z_{1-\alpha/2}$

Voglio che l'ampiezza sia inferiore a una certa soglia δ

$$\frac{2\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \leq \delta$$

$$\sqrt{n} \geq \frac{2\sigma z_{1-\alpha/2}}{\delta} \qquad n \geq \frac{4\sigma^2 z^2_{1-\alpha/2}}{\delta^2}$$