Titolo nota 16/10/2013

$$P_n(K) = \binom{n}{K} p^K (1-p)^{n-K}$$

$$P_n(K) = \ldots$$

$$|A_1 \cup A_2 \cup A_3 \dots \cup A_n| = \sum_k |A_k| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| \dots + (-1)^{n-1} |A_1 \cap A_2 \dots A_n|$$

Se $x \in A_1\ A_2\ A_3\ A_4\ A_5$

viene contato nue volte per ognuno $\longrightarrow 5$

" " " " $A_i \cap A_j \longrightarrow \binom{5}{2} = 10$

" " " $A_i \cap A_j \cap A_k \longrightarrow \dots = 10$

" " " intersez. a 4. $\longrightarrow \dots 5$

" " l'intersez. 5 $\longrightarrow 1$

$$|A_1 \cup A_2 \cup \dots| = 5 - 10 + 10 - 5 + 1 = 1$$

$a \in A_1 A_2 \dots A_K$

a viene contato una volta per ogni $A_i \longrightarrow K$

" " int. doppie $\longrightarrow \binom{K}{2}$

" " int. triple $\longrightarrow \binom{K}{3}$

" " int. K-1 $\longrightarrow \binom{K}{1} = K$

" " int. di tutti $\longrightarrow 1$

$$\longrightarrow K - \binom{K}{2} + \binom{K}{3} - \binom{K}{4} \dots - \dots + (-1)^{K-1} = 1 \,?$$

$$1 - K + \binom{K}{2} - \binom{K}{3} \dots - \quad + (-1)^{K} = 0$$

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 + 4 + 6 + 4 + 1 \to 2^3 \\
1 - 5 + 10 - 10 + 5 - 1 = 0
\end{array}
$$

$$\varphi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)(\ldots$$

$$\varphi(3000) = 3000\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 800$$

$$|A_2 \cup A_3 \cup A_5| = 1500 + 1000 + 600 - 500 - 300 - 200 + 100 = 2200$$

$N = n! - ($n° perm. con almeno un punto fisso)

$A = \{$ perm. con almeno un punto fisso $\}$

$A = A_1 \cup A_2 \cup A_3 \cdots \cup A_n$

$A_1 = \{ f \mid f(1) = 1 \} \longrightarrow |A_1| = (n-1)!$

$A_2 = \{ f \mid f(2) = 2 \} \longrightarrow |A_2| = (n-1)!$

$\vdots$

$A_n = \{ f : f(n) = n \}$

$\vdots$

$|A| = |A_1| + |A_2| + \dots$

$\times \times \times \cdot \cdot \cdot \cdot$

$$|A_i \cap A_j| = (n-2)!$$

$$|A_i \cap A_j \cap A_k| = (n-3)!$$

$$|A| = (n-1)! \cdot n - \binom{n}{2}(n-2)! + \binom{n}{3}(n-3)! + - \cdots - + (-1)^{n-1}$$

$$= n! - \frac{n!}{2!\cancel{(n-2)!}}\cancel{(n-2)!} + \frac{n!}{3!\cancel{(n-3)!}}\cancel{(n-3)!}$$

$$= n! - \frac{n!}{2!} + \frac{n!}{3!} - \frac{n!}{4!} + \cdots + (-1)^{n-1}$$

$$N = \cancel{n!} - \cancel{n!} + \frac{n!}{2!} - \frac{n!}{3!} + \frac{n!}{4!} + \cdots + (-1)^n$$

$$N = n!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \cdots + \frac{(-1)^n}{n!}\right)$$

$$N_7(0) = \cancel{7!}\,\frac{3\cdot4\cdot5\cdot6\cdot7 - 4\cdot5\cdot6\cdot7 + 5\cdot6\cdot7 - 6\cdot7 + 7 - 1}{\cancel{7!}}$$

$$P_7(0) = \frac{\text{n° casi f.}}{\text{n° casi p.}} = \frac{\cancel{7!}\left(\frac{1}{2!} - \frac{1}{3!} \cdots - \frac{1}{7!}\right)}{\cancel{7!}} = \underline{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!}}$$

$$P_n(n) = \frac{1}{n!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{-1} = \frac{1}{e} = \underline{1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots}$$

$$\frac{1}{e} = 0,\underline{3678}79\ldots$$

$$P_8(0) = \frac{1}{2!} - \frac{1}{3!} \ldots + \frac{1}{8!} = 0,\underline{3678}81\ldots$$

$$P_n(0) \quad P_n(1) \quad P_n(2) \quad \ldots \quad P_n(K) =$$

$n = 8 \qquad K = 3 \qquad P(1\,1\,1 - - - - -) = \frac{1}{8} \cdot \frac{1}{7} \cdot \frac{1}{6} \cdot P_5(0)$

$$1\,1\,1\,0\,0\,0\,0\,0 = \frac{8!}{3!\,5!} \overset{\text{def}}{=} \binom{8}{3}$$

$$P_8(3) = \binom{8}{3} \cdot \frac{1}{8} \cdot \frac{1}{7} \cdot \frac{1}{6} \cdot P_5(0) = \frac{(8)_3}{3!} \cdot \frac{1}{(8)_3} \cdot P_5(0) = \frac{1}{3!} \cdot P_5(0)$$

$$\boxed{P_n(K) = \frac{1}{K!} \cdot P_{n-K}(0)}$$

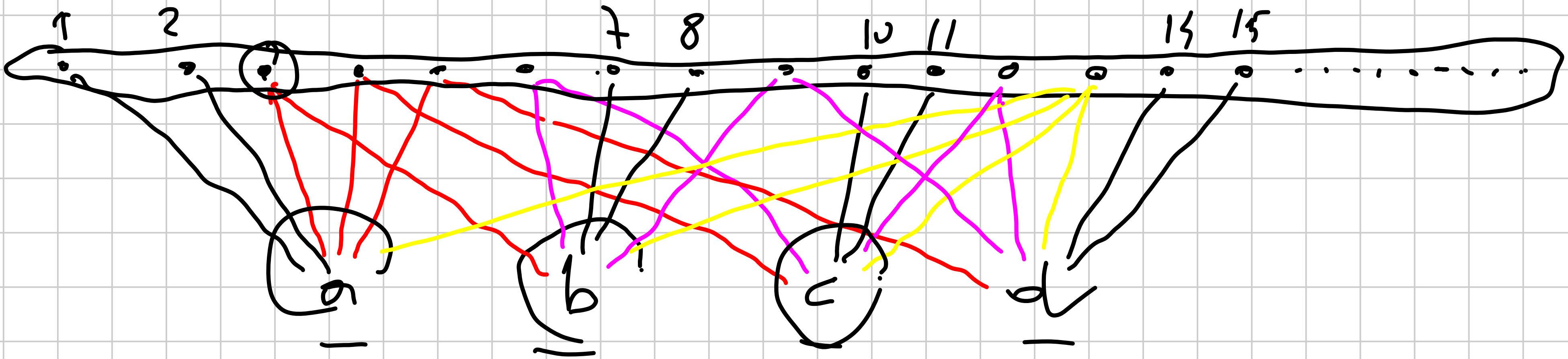
X_8	0	1	2	3	4	5	6	7	8
P	$P_8(0)$	$P_7(0)$	$\frac{1}{2!}P_6(0)$	$\frac{1}{3!}P_5(0)$	$\frac{1}{4!}P_4(0)$	$\frac{1}{5!}P_3(0)$	$\frac{1}{6!}P_2(0)$	0	$\frac{1}{8!}$

$$M(X_8) = 0 + P_7(0) + P_6(0) + \frac{1}{2!}P_5(0) + \frac{1}{3!}P_4(0) + \frac{1}{4!}P_3(0) + \frac{1}{5!}P_2(0) + \frac{1}{7!} = 1$$

X_7	0	1	2	3	4	5	7

$$\boxed{M(X_n) = \sum_{k=0}^{n-1} P_{n-1}(k)} = 1$$

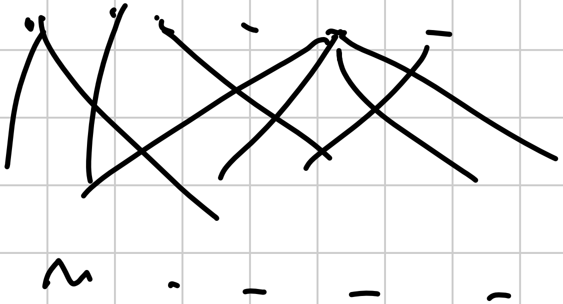
$n = 4$ $\qquad n! \Rightarrow 4!$ permutazioni $\qquad N_4(0) = 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9$



a c d b
a d b c

$$M = \frac{\text{n° tot. punti fissi}}{\text{n° tot. perm.}} = \frac{\text{n° tot. archi}}{4!} = \frac{4 \cdot 3!}{4!}$$

arco — punto fisso



n° tot. di fini

||

$$\frac{\text{n° tot. archi}}{n!} = \frac{n\cdot(n-1)!}{n!} = \frac{n!}{n!} = 1$$

$$\varphi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)(\cdot\ \cdot$$

$n = 3000$ $\qquad \varphi(3000) = 800$

$A\{1, 2, 3, \ldots\ 3000\}$ E "il n° sia coprimo con 3000"

$$P(E) = \frac{n° \text{ cas. f.}}{n° \text{ cas. p.}} = \frac{\varphi(3000)}{3000}$$

in generale $P(E) = \dfrac{\varphi(n)}{n}$

$$P(E) = \frac{3000\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)}{3000} = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$$

$P(\overline{E}_2) = \underbrace{1-\frac{1}{2}}_{1/2}$ $\qquad P(\overline{E}_3) = \underbrace{1-\frac{1}{3}}_{=\frac{2}{3}} = \frac{2}{3}$ $\qquad P(\overline{E}_5) = 1-\frac{1}{5} = \frac{4}{5}$

$$P(E) = P(\bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_5) = P(\bar{E}_2) \cdot P(\bar{E}_3) \cdot P(\bar{E}_5)$$

$$= P(\bar{E}_2) \cdot P(\bar{E}_3 / \bar{E}_2) \cdot P(\bar{E}_5 / \bar{E}_2 \cap \bar{E}_3)$$

$p(E_2) \downarrow \frac{1}{2}$ $\quad p(E_3) \downarrow \frac{1}{3}$ $\quad p(E_5) \downarrow \frac{1}{5}$