DISUGUAGLIANZE ALGEBRICHE

Titolo nota 14/10/2013

$$\sqrt[n+1]{ab^n} \le \frac{a+nb}{n+1} \qquad c.\ a, b > 0$$

$$\frac{a^3+b^3+c^3}{a^2+b^2+c^2} \geq \frac{a+b+c}{3}$$

a

a b $a, b > 0$

$$\frac{a+b}{2}, \quad \sqrt{ab}, \quad \frac{2}{\frac{1}{a}+\frac{1}{b}}, \quad \sqrt{\frac{a^2+b^2}{2}}$$

AM GM AM QM

$$x^2 \geq 0$$

$$(a-b)^2 \geq 0 \qquad a^2+b^2 \geq 2ab$$

$$\boxed{\frac{a}{b}+\frac{b}{a} \geq 2}$$

$$2(a^2+b^2) \geq (a+b)^2$$

$$\boxed{x+\frac{1}{x} \geq 2}$$

$$\frac{a^2+b^2}{2} \geq \left(\frac{a+b}{2}\right)^2$$

$$\sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2}$$

$$a=b \iff QM = AM$$

$$QM \geq AM$$

$$a^2+b^2 \geq 2ab$$

$$a \to \sqrt{a}$$
$$b \to \sqrt{b}$$

$$a+b \geq 2\sqrt{a}\sqrt{b}$$

$$\frac{a+b}{2} \geq \sqrt{ab} \qquad AM \geq GM$$

$$\frac{a+b}{2} \sqrt{ab} \geq ab \qquad \sqrt{ab} \geq \frac{2ab}{a+b} = \frac{2}{\frac{1}{b}+\frac{1}{a}}$$

$$GM \geq HM$$

max

$$(a,b) \geq \quad QM \geq AM \geq GM \geq HM \qquad \geq \min(a,b)$$

$$M_p = \sqrt[p]{\frac{a^p+b^p}{2}} \qquad \lim_{p\to 0} M_p = GM$$

$$p < q \implies M_p \leq M_q$$

$$a_1, a_2 \ldots a_n > 0$$

$$AM = \frac{\sum_{i=1}^{n} a_i}{n} \qquad GM = \sqrt[n]{\prod a_i}$$

$$HM = \frac{n}{\sum_i \frac{1}{a_i}}$$

ANCHE IN n DIM.

$$QM \geq \boxed{AM \geq GM} \geq HM$$

RIARRANGIAMENTO

$$a_1 \geq a_2 \geq \ldots \geq a_n$$
$$b_1 \geq b_2 \geq \ldots \geq b_n$$

$$\underline{\underline{\sum a_i b_i}} \geq \sum a_i b_{\sigma(i)} \geq \underline{\underline{\sum a_i b_{n+1-i}}}$$

100€ 50€ 10€

20, 10, 5

$a_1 \geq a_2$

$b_1 \geq b_2$

$(a_1 - a_2)(b_1 - b_2) \geq 0$

$(a_1 - a_2) \geq 0$

$(b_1 - b_2) \geq 0$

$a_1 b_1 + a_2 b_2 \geq a_1 b_2 + a_2 b_1$

$\sum a_i b_{\sigma(i)}$

$i < j \quad a_i \geq a_j$

$k < m \quad b_k \geq b_m$

① $\ldots + a_i b_m + a_j b_k + \ldots$

PER ASSURDO SIA IL MASSIMO

① $\leq \ldots a_i b_k + a_j b_m \ldots$

$(a_i - a_j)(b_k - b_m) \geq 0$

b_i, a_i ORDINATI

$$a_1 b_1 + \dots a_n b_n \geq a_1 b_1 + \dots + a_n b_n$$

$$\text{———} \geq a_1 b_2 + a_2 b_3 + \dots a_n b_1$$

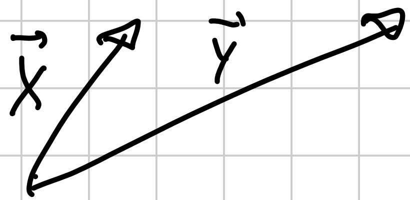
$$\text{———} \geq a_1 b_3 \dots$$

$$\vdots$$

$$\text{———} \geq a_1 b_n + \dots a_n b_1$$

$$n \sum_i a_i b_i \geq \sum_i a_i \sum_j b_j \qquad \frac{\sum a_i b_i}{n} \geq \frac{\sum a_i}{n} \frac{\sum b_i}{n}$$

$$\frac{\sum a_i}{n} \frac{\sum b_i}{n} \geq \frac{\sum a_i b_{n+1-i}}{n}$$



$$|\vec{x}\cdot\vec{y}| \leq \|\vec{x}\|\,\|\vec{y}\|$$

$$\sum a_i b_i \leq \sqrt{\sum a_i^2}\sqrt{\sum b_i^2}$$

$\vec{x} = (a_1, a_2 \ldots a_n)$

$\vec{y} = (b_1 \; --- \; b_n)$

$$\left(\sum a_i b_i\right)^2 \leq \sum a_i^2 \sum b_i^2$$

$$a_i = \lambda b_i$$

$$0 \leq (a_1 + t b_1)^2 + (a_2 + t b_2)^2 \ldots + (a_n + t b_n)^2$$

$$A t^2 + B t + C \geq 0 \qquad \Delta \leq 0$$

$$a_1^2 + t^2 b_1^2 + 2a_1 t b_1 + \ldots$$

$$t^2 \sum b_i^2 + 2t \sum a_i b_i + \sum a_i^2 \geq 0$$

$$\Delta = \left(\sum a_i b_i\right)^2 - \sum a_i^2 \sum b_i^2 \leq 0$$

ENGEL : PROBLEM SOLVING STRATEGIES

$$\sqrt[n+1]{ab^n} \leq \frac{a+nb}{n+1}$$

$a, b > 0$

$$\sqrt[n]{\prod a_i} \leq \frac{\sum a_i}{n}$$

$$a, \underbrace{b, b, \ldots b}_{n}$$

$$\sqrt[n+1]{ab^n} \leq \frac{a+nb}{n+1}$$

$a > 0$
$b > 0$
$c > 0$

$$\underbrace{\frac{a+b+c}{3} \geqslant \underbrace{\sqrt{\frac{ab+bc+ca}{3}}}} \geqslant \sqrt[3]{abc}$$

$ab \quad bc \quad ca$

$$\frac{ab+bc+ca}{3} \geqslant \sqrt[3]{abbcca} = \sqrt[3]{a^2b^2c^2}$$

$$\sqrt{\frac{ab+bc+ca}{3}} \geqslant \sqrt{abc}$$

$$\frac{(a+b+c)^2}{9} \geqslant \frac{ab+bc+ca}{3}$$

$$\frac{a^2+b^2+c^2+2ab+2bc+2ca}{3} \geqslant ab+bc+ca$$

$a^2+b^2+c^2 \geq ab+bc+ca$

$a \cdot a + b \cdot b + c \cdot c \geq a \cdot b + b \cdot c + c \cdot a$

POLINOMI SIMMETRICI

$a \geq b \geq c$

$2a^2+2b^2+2c^2-2ab-2bc-2ca \geq 0$

$(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

a, b, c

a, b, c

$\vec{x} = (a, b, c)$

$\vec{y} = (b, c, a)$

$|ab+bc+ca| \leq \sqrt{a^2+b^2+c^2}\sqrt{a^2+b^2+c^2}$

$$\frac{a+b+c}{abc} \le \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

$a, b, c > 0$

$$\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} \le \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

$a \ge b \ge c$

$\frac{1}{a} \le \frac{1}{b} \le \frac{1}{c}$

$$\frac{1}{b}\frac{1}{c} + \frac{1}{c}\frac{1}{a} + \frac{1}{a}\frac{1}{b} \le \frac{1}{a}\frac{1}{a} + \frac{1}{b}\frac{1}{b} + \frac{1}{c}\frac{1}{c}$$

$$\frac{(a+b)(b+c)(c+a)}{abc} \ge 8$$

$a, b, c > 0$

$$\frac{(a+b)(b+c)(c+a)}{8} \ge abc$$

$$\frac{a+b}{2} \geq \sqrt{ab} \qquad \frac{b+c}{2} \geq \sqrt{bc} \qquad \frac{c+a}{2} \geq \sqrt{ac}$$

$$\frac{1}{n-1}\left(\sum_{i=1}^{n} a_i\right)^2 - \sum_{i=1}^{n} a_i^2 \leq \boxed{2a_1a_2} \qquad a_i > 0$$

AM ≤ QM

$(a_1+a_2),\ a_3,\ a_4\ \ldots\ a_n$

$$\frac{(a_1+a_2)+a_3 \ldots + a_n}{n-1} \leq \sqrt{\frac{(a_1+a_2)^2+a_3^2 \ldots + a_n^2}{n-1}}$$

$$\frac{\left(\sum a_i\right)^2}{(n-1)^2} \leq \frac{\sum a_i^2 + 2a_1a_2}{(n-1)}$$

$$\sum_{i=1}^{n} \frac{1}{\sqrt{1-x_i}} \geq n\sqrt{\frac{n}{n-1}}$$

$x_i > 0$

$\sum x_i = 1$

HM ≤ GM ≤ AM ≤ Q

CS

$\{\sqrt{1-x_i}\}$

$$\frac{n}{\sum_i \frac{1}{\sqrt{1-x_i}}} \leq \frac{\sqrt{1-x_1}+\sqrt{1-x_2} \dots + \sqrt{1-x_n}}{n}$$

RIARR.

CHEB.

$$\sqrt{\frac{\frac{1-x_1+1-x_2 \dots + 1-x_n}{n}}{}}$$

$$= \sqrt{\frac{n-1}{n}}$$

$$\sum \frac{1}{\sqrt{}} \geq n\sqrt{\frac{n}{n-1}}$$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3 \qquad a, b, c > 0$$

$$\frac{\frac{a}{b} + \frac{b}{c} + \frac{c}{a}}{3} \geq \sqrt[3]{\frac{a}{b} \frac{b}{c} \frac{c}{a}}$$

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \geq 3$$

$$(n+1)^n \geq 2^n n! \qquad n \geq 1$$

$$\frac{1+2 \ldots + n}{n} \geq \sqrt[n]{n!} \qquad 1, 2, \ldots n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \Rightarrow \frac{\cancel{n}(n+1)}{2} \geq \sqrt[n]{n!}$$

$$(n+1)^n \geq 2^n n!$$

$$(a+b)(a^4+b^4) \geq (a^2+b^2)(a^3+b^3)$$

$$ab^4 + a^4 b \geq a^2 b^3 + a^3 b^2 \qquad a, b > 0$$

$$\boxed{b^3 + a^3} \geq \boxed{a^2 b + a b^2} \qquad a \geq b$$

$$a^2 \geq b^2$$

$$a \geq b \qquad a^3 \geq b^3$$

$$\cancel{(a+b)}(a^2+b^2-ab) \quad a^2 \geq b^2$$

$$\geq ab\,\cancel{(a+b)}$$