POLINOMI (ALGEBRA)

Titolo nota 14/10/2013

$\boxed{\text{P.B1}}$ Cercare $P(x)$ t.c.

$$\boxed{(x-9)\,p(x) = (x-11)\,p(x+2)}$$

$$\Downarrow \quad (x-10)$$

$$p(11) = 0$$

$$\boxed{p(x) = (x-11)\,q(x)} \longleftrightarrow p(x+2) = (x-9)\,q(x+2)$$

$$(x-9)\cancel{(x-11)}\,q(x) = \cancel{(x-11)}(x-9)\,q(x+2)$$

Ruffini

$$p(c) = 0$$

$$p(x) = (x-c)\,q(x)$$

$\mathbb{Z}_3$

$$P_1(x) = x^3 - x$$

$$P_2(x) = 0$$

$$\|$$

$$(x-5)\,q(x) = (x-9)\,q(x+2)$$

$$\boxed{q(x) = (x-9)\,h(x)}$$

$$(x-5)\,h(x) = (x-7)\,h(x+2)$$

$$\boxed{h(x) = (x-7)\,k(x)}$$

$$\cancel{(x-5)}\cancel{(x-7)}\,k(x) = \cancel{(x-7)}\cancel{(x-5)}\,k(x+2)$$

$$k(x) = k(x+2) \qquad k(x_0) = c$$

$$\underbrace{\boxed{k(x) - c}} = \underbrace{k(x+2) - c}$$

$$P(x) = (x-11)(x-9)(x-7)\cdot K \qquad k \in \mathbb{R}$$

Teorema: Dati $P(x)$ e $q(x)$ di grado $\leq n$ tali che $P(x_i) = q(x_i)$ per opportuni $x_1, x_2, \dots, x_{n+1}$ (distinti). Allora $P(x) = q(x)$.

Polinomi in $\mathbb{R}[x]$

In particolare: $\boxed{P(x_i) = 0}$ Per ogni x_i dimostro che $P(x) = 0$

$$\boxed{P(x)} = (x - x_1) P_1(x) = (x - x_1)(x - x_2) P_2(x) =$$

$$= \cdots\cdots = (x - x_1)(x - x_2)\cdots(x - x_n)\,P_n(x)$$

$$0 = P(x_{n+1}) = \cdots = \underbrace{(x_{n+1} - x_1)(x_{n+1} - x_2)\cdots(x_{n+1} - x_n)}_{n}\,\underbrace{P_n(x_{n+1})}_{n+1}$$

$A(x)$ $B(x)$ $q(x)$ $r(x)$

$$A(x) = B(x)\,q(x) + r(x)$$

$\mathbb{Z}[x]$

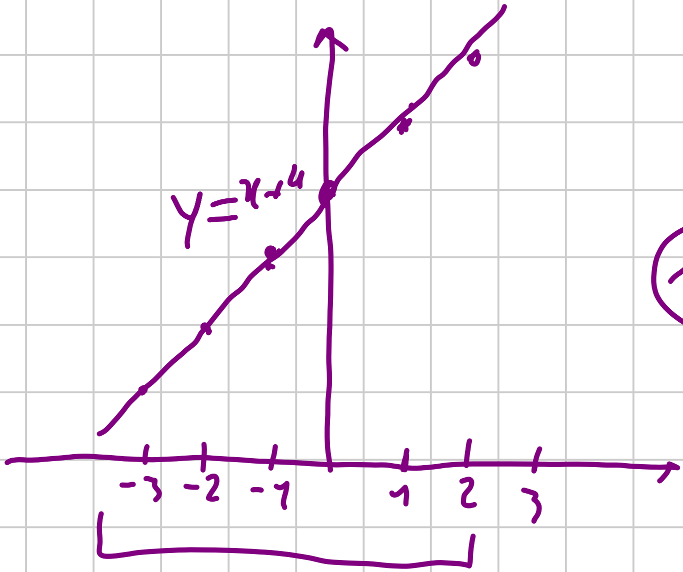
$\boxed{ES\ 3}$ Dato $P(x)$ di grado 1987 t.c. diviso per

$(x+3)$, $(x+2)$, $(x+1)$, $(x-1)$, $(x-2)$ e $(x-3)$

Si ottiene come resti risp. 1 2 3 5 6 1987.

Trovare il resto $r(x)$ della divisione tra $P(x)$ e $A(x)$

$$A(x) = (x+3)(x+2)(x+1)(x-1)(x-2)(x-3)$$



$$q(x) = p(x) - (x+4)$$

$$1982 \rightarrow p(x) = \underbrace{q(x)} + \underbrace{(x+4)}\ 7$$

$$* \ q(i) = 0 \quad i = -3, -2, -1, 1, 2$$

$$\boxed{q(3) = 1980}$$

$$\underbrace{q(x) = (x+3)(x+2)(x+1)(x-1)(x-2)\,h(x)}$$

$$\overset{33}{\cancel{1980}} = \cancel{6} \cdot \cancel{5} \cdot 4 \cdot \cancel{2} \cdot 1 \cdot h(3) \qquad h(3) = \frac{33}{4}$$

$$\rightarrow \boxed{h(x) = (x-3)\,k(x) + \frac{33}{4}}$$

$$q(x) = \underbrace{(x+3)(x+2)(x+1)(x-1)(x-2)}\underbrace{\left((x-3)\,K(x) + \frac{33}{4}\right)}$$

$$q(x) = \underbrace{(x+3)(x+2)(x+1)(x-1)(x-2)(x-3)}\underbrace{K(x)} + \underbrace{\boxed{}\cdot\frac{33}{4}}$$

$$p(x) = q(x) + (x+4)$$

$$\frac{33}{4}\left(3\cdot 2\cdot 1\cdot(-1)\cdot(-2)\right) + 4$$

$$p(x) = A(x)\cdot K(x) + \boxed{\frac{33}{4}\cdot(x+3)(x+2)(x+1)(x-1)(x-2) + (x+4)}$$

$$\uparrow \quad r(x)$$

$\boxed{\text{P.B. 4}}$ Trovare il resto della divisione:

$$\left(x^{120}+5x^{67}+8x^{2}+2\right):\left(x^{16}-x^{4}+1\right)$$

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Trovare resto di

$$\boxed{\left(x^{120}+5x^{67}+8x^2+2\right)} : \left(x^{16}-x^4+1\right)$$

x^{16} x^4-1

$$\boxed{x^{64}} \equiv (x^4-1)^4 = \underbrace{x^{16}}_{\equiv x^4 - 1} - 4x^{12} + 6x^8 - 4x^4 + 1 \equiv$$

$$\equiv \boxed{-4x^{12} + 6x^8 - 3x^4}$$

$$x^{67} \equiv \underbrace{-4x^{15} + 6x^{11} - 3x^7}$$

$$\underbrace{\cancel{(1-x^{12})x^4}} \cdot x^{60} \equiv \underbrace{\cancel{(1-x^{12})x^4}}(-4x^8 + 6x^4 - 3)$$

NO $\quad \cancel{3}a \equiv \cancel{3}b \pmod 6$

$$5 \cdot 3a \equiv 5 \cdot 3b \pmod 6$$

$$a \equiv b$$

$$x^{16} - x^4 + 1 \equiv 0 \qquad x^4$$

$$x^4(x^{12}-1) \equiv -1$$

$$x^4\boxed{(1-x^{12})} \equiv 1$$

$$x^{60} \equiv -4x^8 + 6x^4 - 3$$

$$x^{120} \equiv \left(-4x^8 + 6x^4 - 3\right)^2$$

$$\equiv \left(\qquad \right)$$

m, n primi tra loro

$$mx + ny = 1$$

Dati $A(x)$ e $B(x)$ polinomi "primi tra loro"

$\exists$ $\underline{\text{no}}$ $K(x)$ e $H(x)$ t.c.

$$\underbrace{A(x)}_{\uparrow} \cdot K(x) + \underbrace{B(x)}_{\uparrow} \cdot H(x) = 1$$

multiplo

pol. di cui si cerca l'inversa

$$B(x) \cdot H(x) = 1 - A(x) \cdot K(x)$$

$$= 1$$