GEOMETRIA EUCLIDEA PIANA

Titolo nota — 13/10/2013

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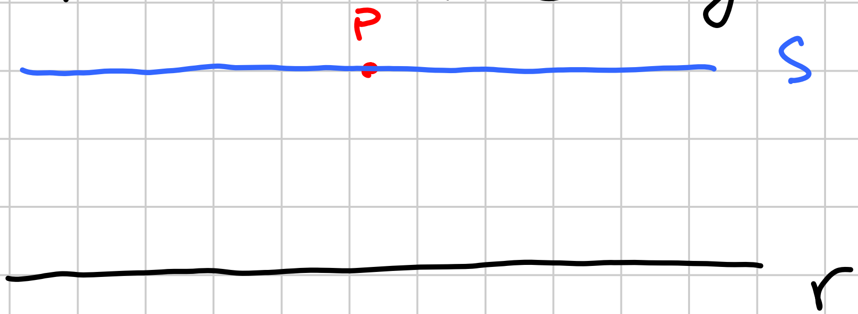
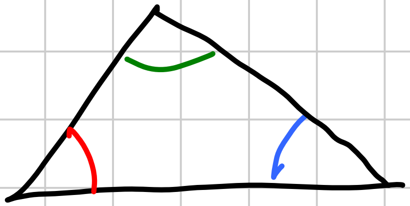
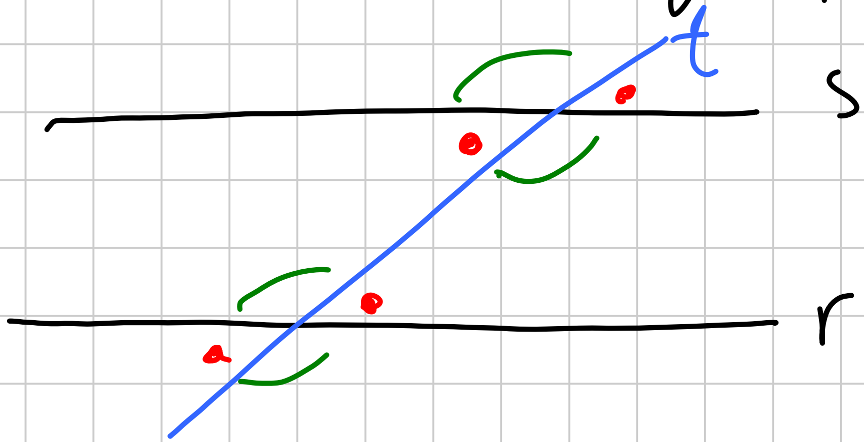


$\exists!\ s \parallel r$ passante per P

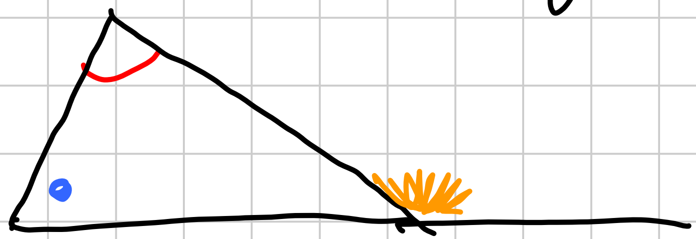


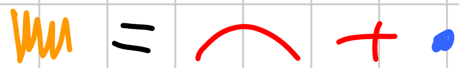
angolo + angolo + angolo = angolo piatto = 180°

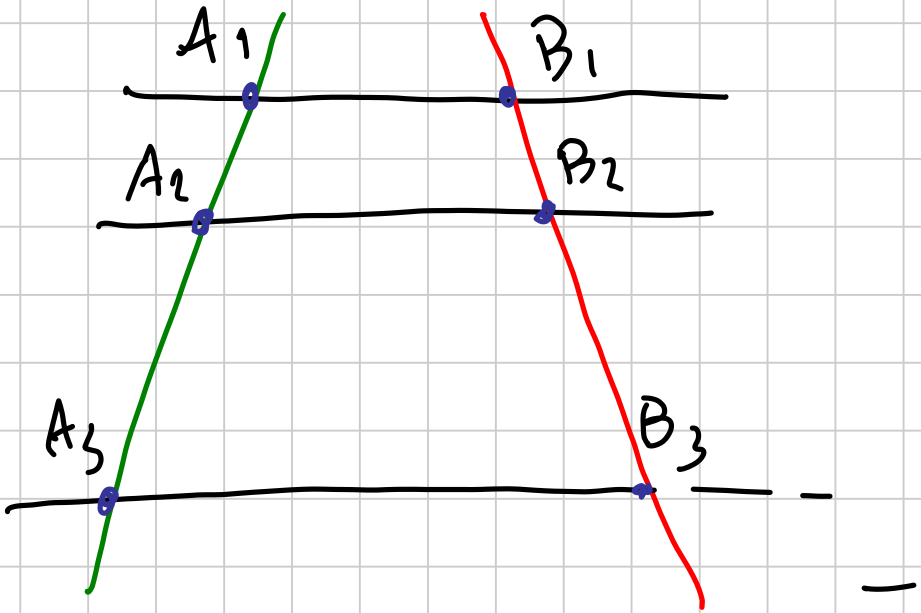


• + ⌒ = 180°

Teorema dell'angolo esterno



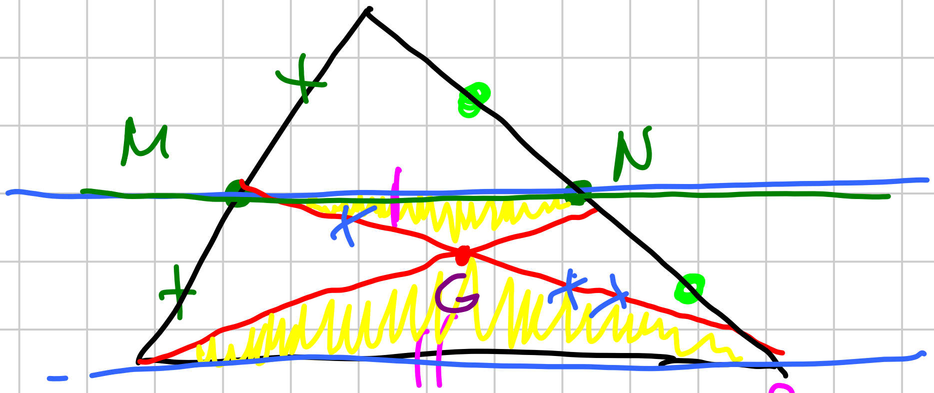


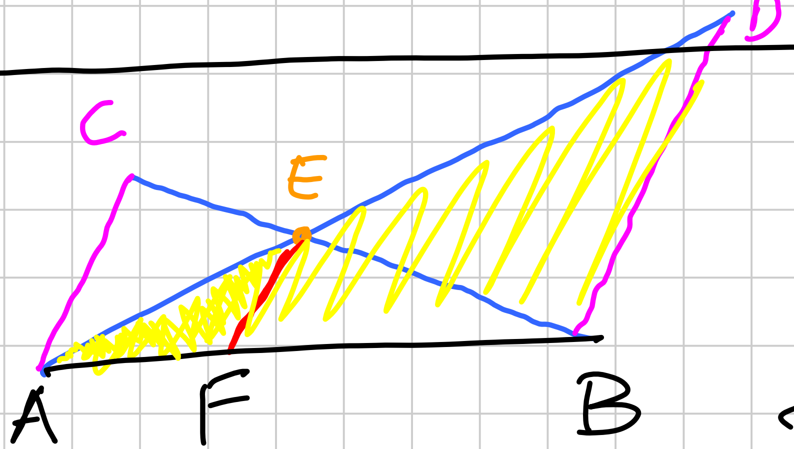


$$\frac{A_1A_2}{A_2A_3} = \frac{B_1B_2}{B_2B_3}$$

TALETE

SIMILITUDINE → criteri di similitudine x triangoli





AC
BD paralleli fra loro

E = ∩ fra BC e AD

tracciamo EF // a AC e BD

$$\frac{1}{AC} + \frac{1}{BD} = \frac{1}{EF}$$

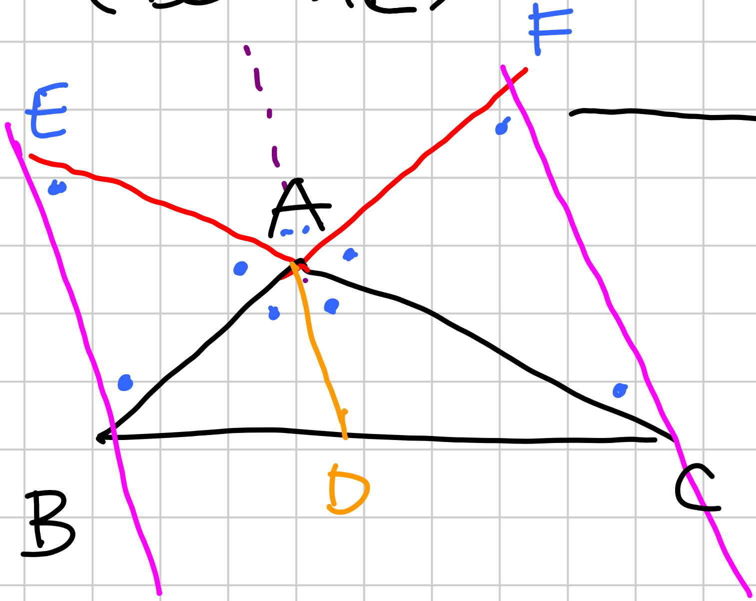
$\overset{\triangle}{ABD}$ è simile ad $\overset{\triangle}{AFE}$ $\overset{\triangle}{ABC}$ è simile a $\overset{\triangle}{FBE}$

$$\frac{EF}{BD} = \frac{AF}{AB} \qquad \frac{EF}{AC} = \frac{FB}{AB}$$

$$\frac{EF}{BD} + \frac{EF}{AC} = \frac{AF}{AB} + \frac{FB}{AB}$$

$$= \frac{AF+FB}{AB} = \frac{AB}{AB} = 1$$

$$EF\left(\frac{1}{BD}+\frac{1}{AC}\right)=1 \qquad \frac{1}{BD}+\frac{1}{AC}=\frac{1}{EF}$$



$\hat{BAC} = 120°$ AD biseca $\hat{BAC}$

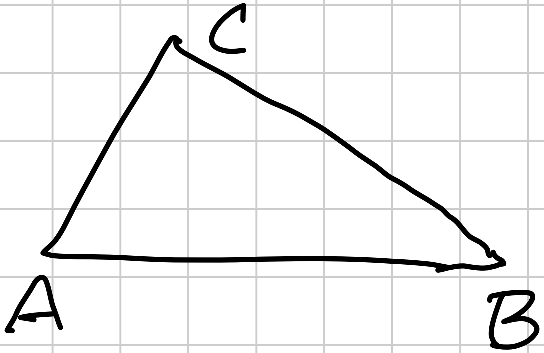
$$\frac{1}{AD}=\frac{1}{AB}+\frac{1}{AC}$$

• $= 60°$

$\overset{\triangle}{ABE}$ e $\overset{\triangle}{CAF}$ sono equilateri

$$\frac{1}{AD}=\frac{1}{\underset{=AB}{EB}}+\frac{1}{CF} \leftarrow AC \rightarrow \frac{1}{AD}=\frac{1}{AB}+\frac{1}{AC}$$

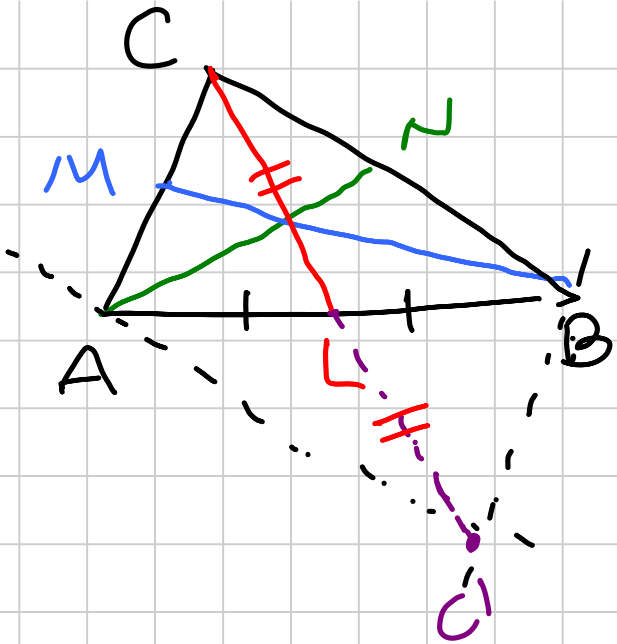
DISUGUAGLIANZA TRIANGOLARE



$AB \le AC + CB$ e cicliche

$AB \ge |AC - CB|$

La somma delle lunghezze delle mediane è minore del perimetro



$$AN + CL + BM < AB + BC + AC$$

$$2CL = CC' < AC + CB \;=\; AC'$$

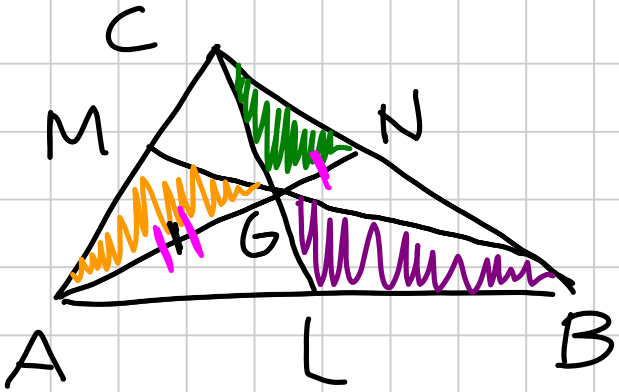
$$2AN = AA' < AC + AB$$

$$2BM = BB' < AB + CB$$

$$\cancel{2}(AN + CL + BM) < \cancel{2}(AB + BC + AC)$$

(Senza usare questo risultato), mostrare che

$$AB + BC + AC > \frac{2}{3}(AN + CL + BM)$$



$$AG < AM + GM = \frac{1}{2} AC + GM$$

$$BG < LB + GL = \frac{1}{2} AB + GL$$

$$CG < CN + GN = \frac{1}{2} CB + GN$$

$$AG + BG + CG < \frac{1}{2}(AC + AB + CB) + GM + GL + GN$$

$$(AG - GN) + (BG - GM) + (CG - GL) < \frac{1}{2}\text{perimetro}$$

$$\frac{1}{3} AN + \frac{1}{3} BM + \frac{1}{3} CL$$

$$\frac{1}{3}(AN+BM+CL) < \frac{1}{2}(AB+BC+CA)$$

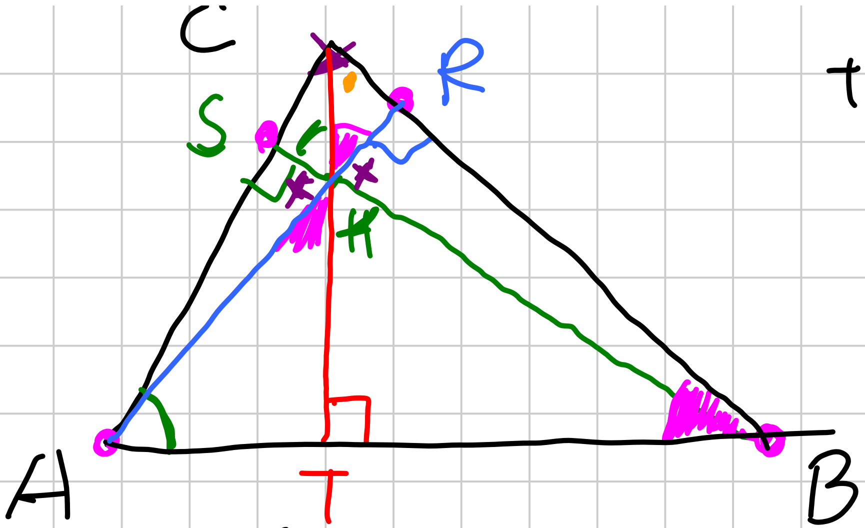
$$AN+BM+CL < \frac{3}{2}(AB+BC+CA)$$

Si usa una proprietà delle mediane

prima abbiamo usato la simmetria

PUNTI NOTEVOLI DI UN TRIANGOLO

ORTOCENTRO

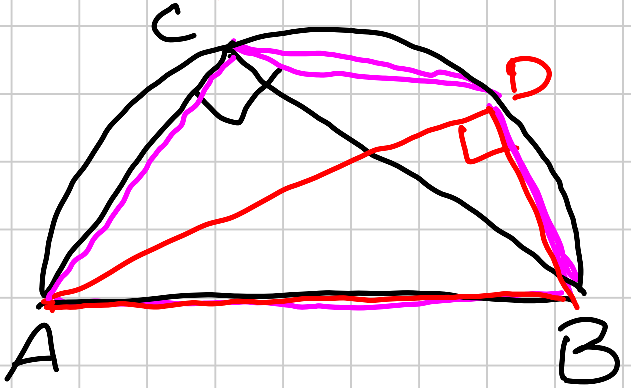


$T\hat{C}B$

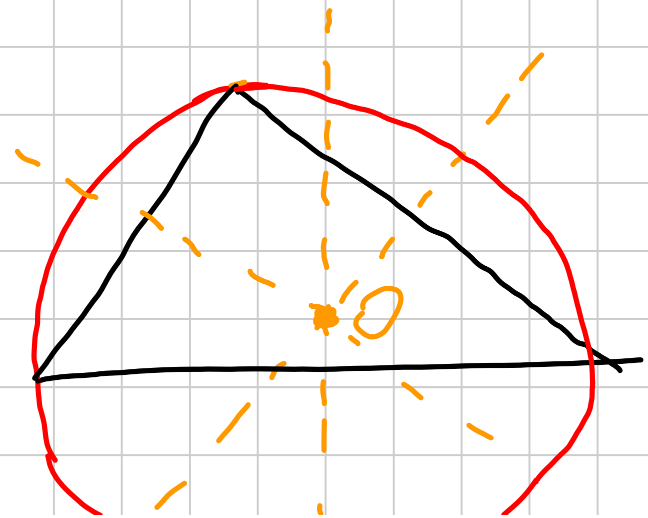
$\overset{\triangle}{TCB}$ $\overset{\triangle}{CHR}$

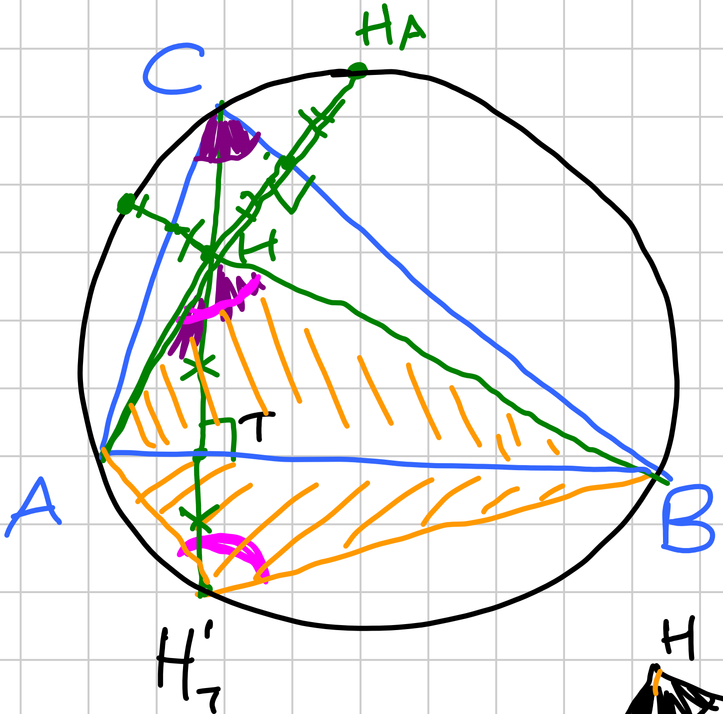
entrambi rettangoli

$R\hat{C}T$ in comune



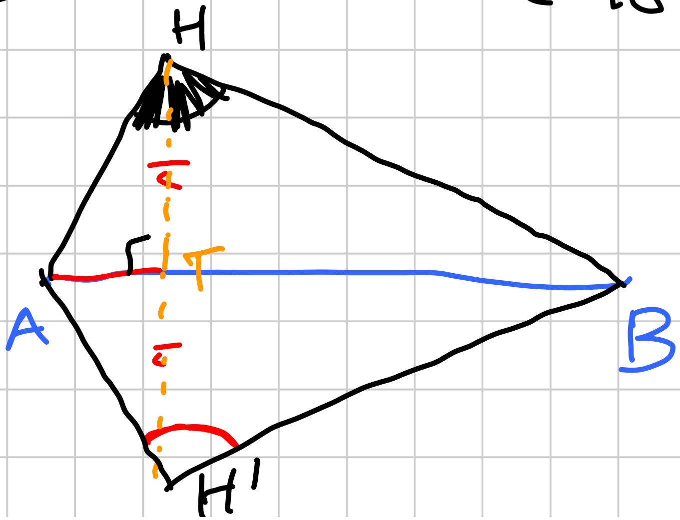
CIRCOCENTRO





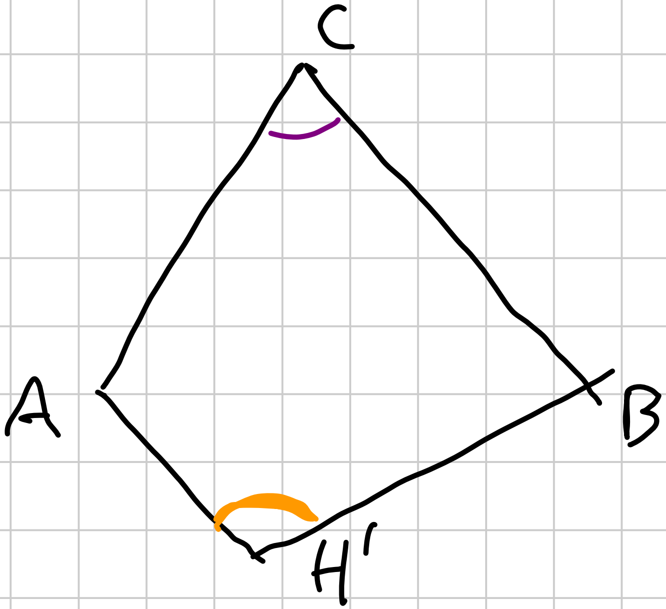
$+ \hat{C} = 180^\circ$

$AH'_T BC$ ha 2 angoli opposti che hanno somma $= 180^\circ$



$A\hat{H'}B = A\hat{H}B$

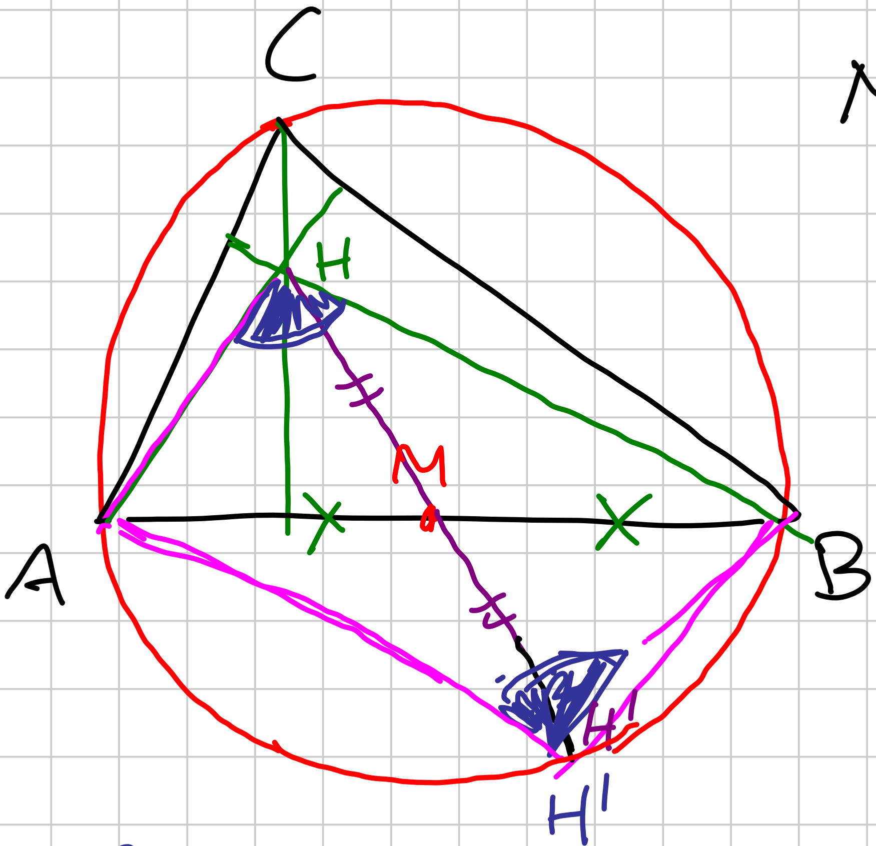
$= C\hat{A}B + C\hat{B}A$



$$A\hat{H'}B + A\hat{C}B = C\hat{A}B + C\hat{B}A + A\hat{C}B$$

$$= 180°$$

ALTRA PROPRIETÀ DELL'ORTOCENTRO



AM = MB perché ho scelto M = punto medio di AB

HM = MH' per costruzione

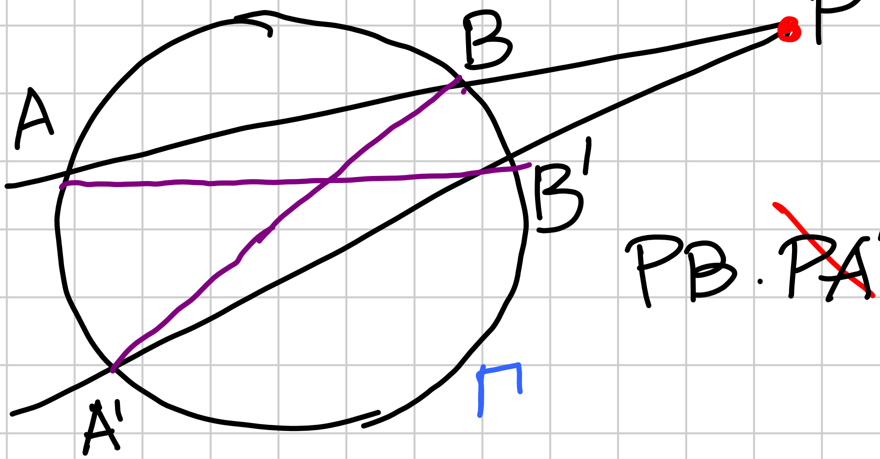
AH'BH è un parallelogramma

$A\hat{H'}B = A\hat{H}B$ esattamente come prima $\Rightarrow$

$H' \in$ circonferenza circoscritta ad ABC

POTENZA DI UN PUNTO RISPETTO A UNA CIRCONFERENZA $\to$ ASSE RADICALE

etc.....



$P\hat{A'}B$ è simile a $P\hat{A}B'$

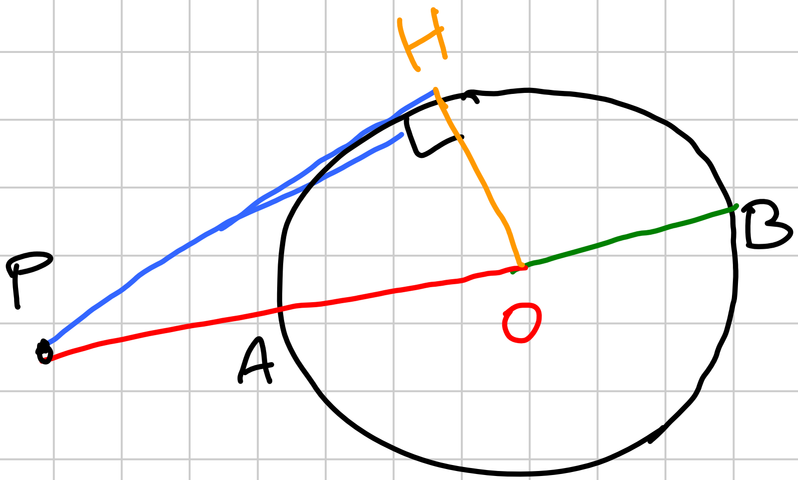
$$PB \cdot \cancel{PA'} \cdot \frac{PA}{\cancel{PA'}} = \frac{PB'}{\cancel{PB}} \cdot \cancel{PB} \cdot PA'$$

$$PA \cdot PB = PA' \cdot PB'$$

$$\mathrm{pow}_\Gamma(P) = PA \cdot PB \qquad \begin{cases} <0 & \text{se } P \text{ è interno a } \Gamma \\ =0 & \text{se } P \in \Gamma \\ >0 & \text{se } P \text{ è esterno a } \Gamma \end{cases}$$

Teorema delle 2 corde

Teorema secante/tangente



$$PH^2 = PA \cdot PB$$

$$PB = PA + 2R$$

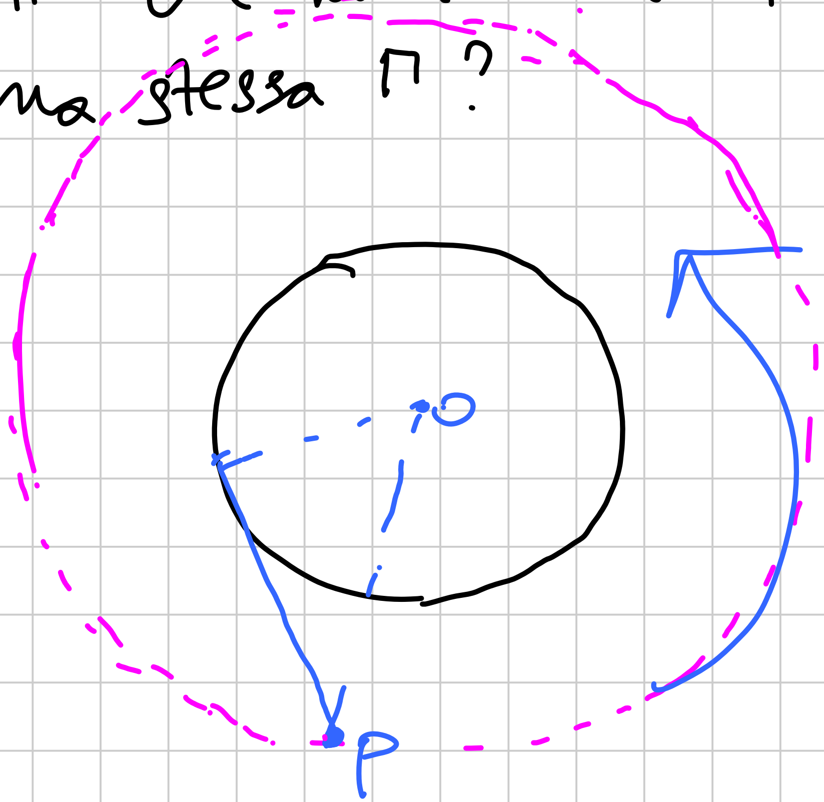
$$(PA+R)^2 = PH^2 + R^2$$

distanza di P da O

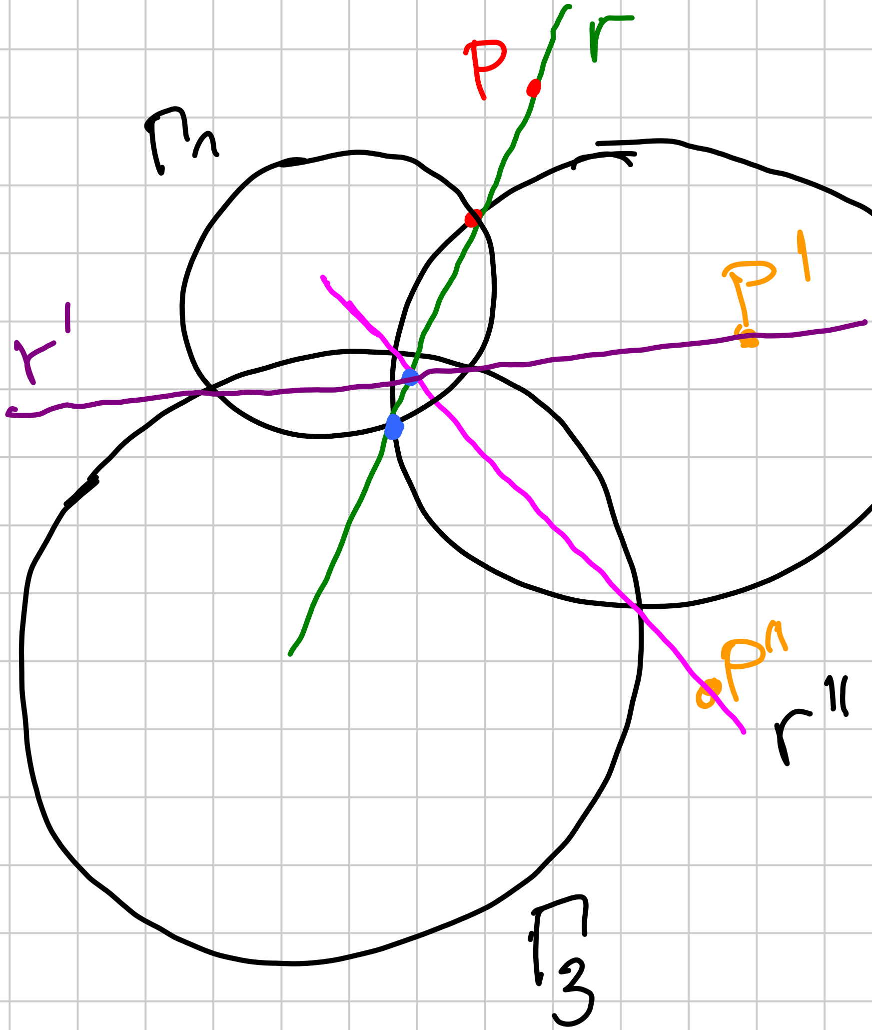
potenza di P

$$pow_\Gamma(P) = d^2 - R^2$$

punti che hanno la stessa potenza rispetto a una stessa Γ?



tutti i punti che $\in$ sulla circonferenza di centro O e raggio OP hanno la stessa potenza di P rispetto a Γ



$$\mathrm{pow}_{\Gamma_1}(P) = \mathrm{pow}_{\Gamma_2}(P)$$

$$\mathrm{pow}_{\Gamma_1}(P') = \mathrm{pow}_{\Gamma_3}(P')$$

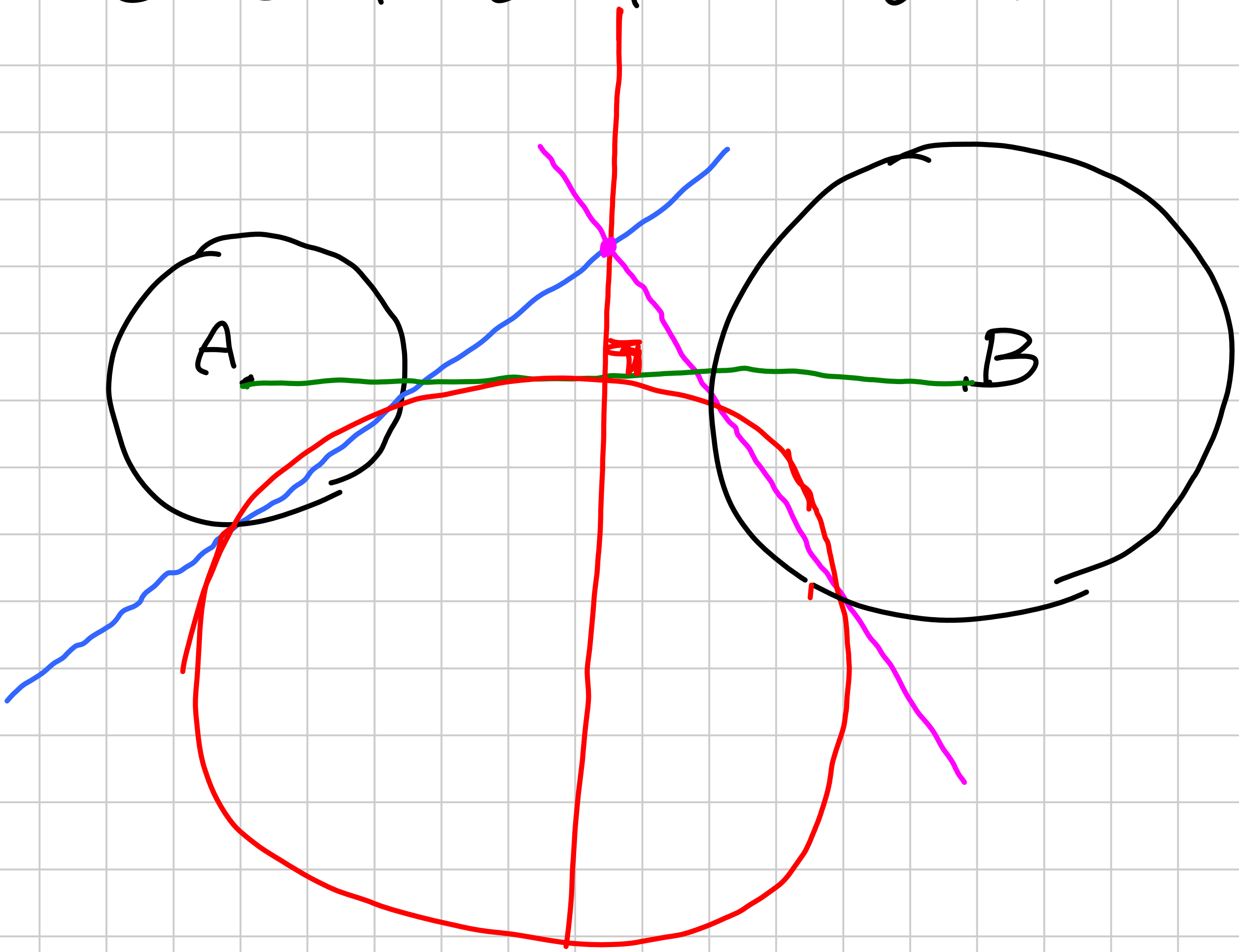
$$\mathrm{pow}_{\Gamma_2}(P'') = \mathrm{pow}_{\Gamma_3}(P'')$$

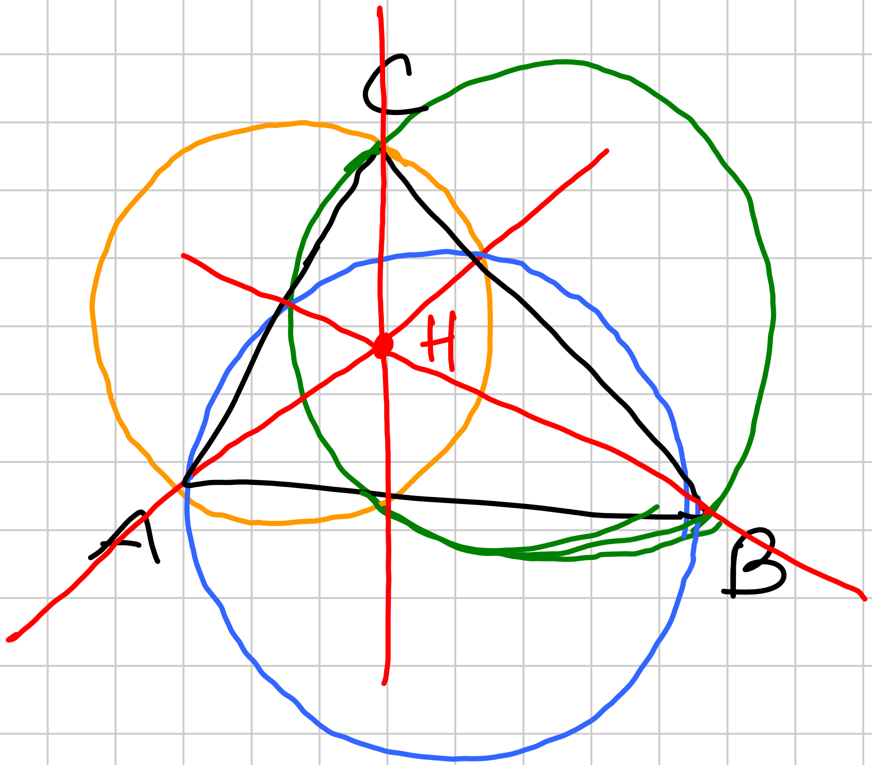
$$\bar{P} = r \cap r'$$

$\mathrm{pow}(\bar{P})$ è la stessa rispetto a tutte e 3

r'' deve passare per $\bar{P}$

$\overline{P}$ è detto centro radicale




C
H
A
B