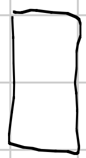
ESERCIZI COMBINATORIA / 2

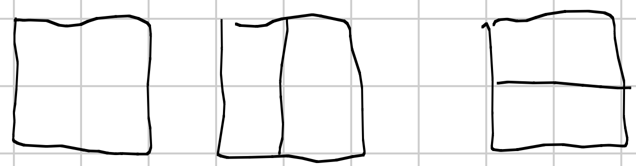
4 scacchiera $2 \times n$ 2×1 2×2

T_n = n° di possib. piastrellature

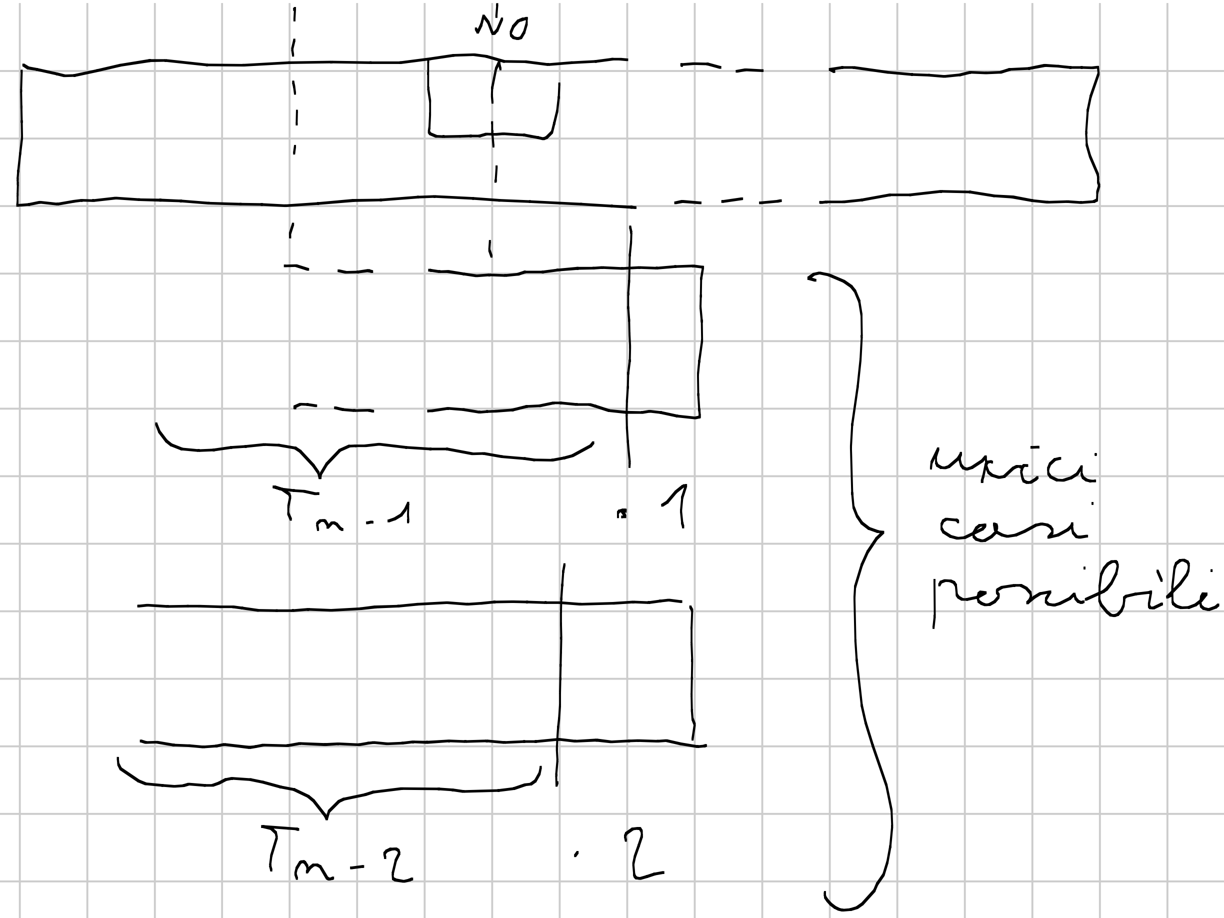
$T_1 = 1$

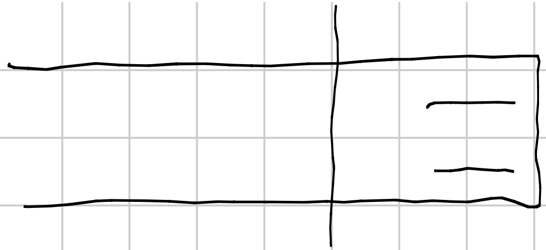


$T_2 = 3$



T_n





$$T_n = T_{n-1} + 2T_{n-2}$$

$$x^2 - x - 2 = 0 \qquad \lambda_1 = 2 \quad \lambda_2 = -1$$

$$T_n = a\lambda_1^n + b\lambda_2^n = a \cdot 2^n + b \cdot (-1)^n$$

$$T_1 = a \cdot 2 \cancel{+} - b = 1$$

$$T_2 = 4a + b = 3$$

$$T_2 = T_1 + 2T_0$$

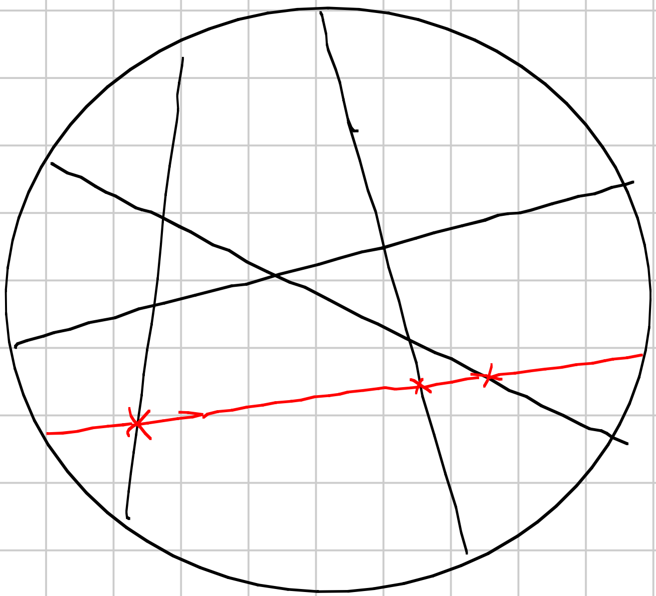
$$T_0 = (3-1)\frac{1}{2} = 1$$

$$a + b = 1 = T_0$$
$$2a - b = 1 = T_1$$

$$a = \frac{2}{3} \qquad b = \frac{1}{3}$$

$$T_n = \frac{2}{3} \cdot 2^n + \frac{1}{3}(-1)^n = \frac{2^{n+1} + (-1)^n}{3}$$

$$2^{n+1} + (-1)^n \equiv 0 \quad (3)$$
$$(-1)^n \left[(-1) + (+1)\right] \equiv 0 \quad (3)$$



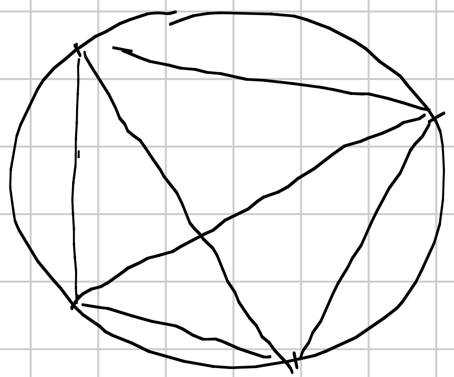
X aree

$\binom{n}{2}$ corde

aggiungo una corda
1 + n° intersezioni

$1 + \binom{n}{2} + A$

A = n° tot di intersezioni



A = n° di possibili quaterne =

$$= \binom{n}{4}$$

$$1 + \binom{n}{2} + \binom{n}{4}$$

1

$$X = \sum_{i=1}^{n} \binom{n}{i} i^2$$

?

n persone $\longrightarrow$ commissione con "i" membri
1 segretario
1 presidente

$S = P \quad n \cdot \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \right) = n \cdot 2^{n-1} = 2n \cdot 2^{n-2}$

$S \neq P \quad n(n-1) \cdot \left(\sum_{i=0}^{n-2} \binom{n-2}{i} \right) = n^2 \cdot 2^{n-2} - n \cdot 2^{n-2}$

$$X = 2^{n-2} \left(n^2 + n \right) = 2^{n-2} \cdot n(n+1)$$

$$\sum \binom{n}{i} i(i-1) + \sum \binom{n}{i} \cdot i$$

3

n

$$\underbrace{0, 1, \dots, n-1}_{n \text{ cassetti}}$$

se pieno $\Longrightarrow$ vuoto

vuoto $\Longleftarrow$ se pieno

⑥ $0, 1, 2 \; \ldots \; 99$

100 classi di resto
(100 cassetti)

SUPPONGO PER ASSURDO
che tutti i cass. siano pieni

50 c_i dispari $\qquad c_i = a_i \cdot b_i$

$$A_d \updownarrow B_d \qquad A_p \updownarrow B_p \rightarrow \text{ASSURDO!}$$

7

$S_i \qquad 1 \le i \le 77$

S_i = problemi risolti fino al giorno i

$\longrightarrow 1 \le S_1 < S_2 < S_3 < \ldots < S_{77} \le 132$

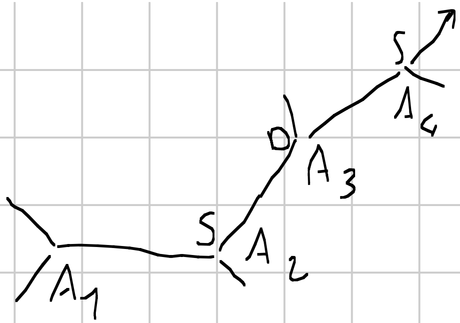
Th: $\exists\, i, j$ t.c. $S_i = S_j + 21$

$\longrightarrow 22 \le S_1 + 21 < S_2 + 21 < \quad < S_{77} + 21 \le 153$

ROSSI → 77
BLU → 77
} TOT = 154

$$\boxed{S_i = S_j + 21}$$

5

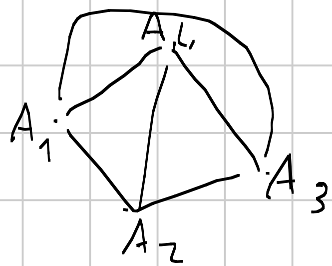


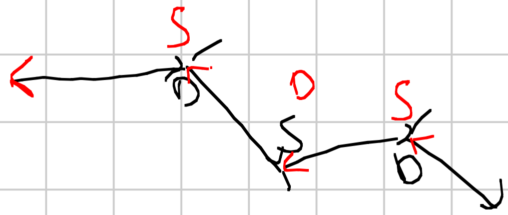
(A) cammino è periodico

(B) non c'è! antiperiodo

$$\underbrace{\left(\overrightarrow{A_i A_j}\ ,\ \overrightarrow{A_j A_k}\right)} \longrightarrow \text{NOTO QUESTO, NOTO TUTTO}$$

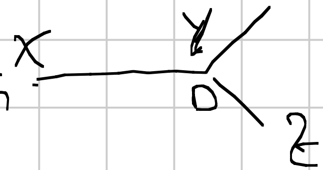
A_k





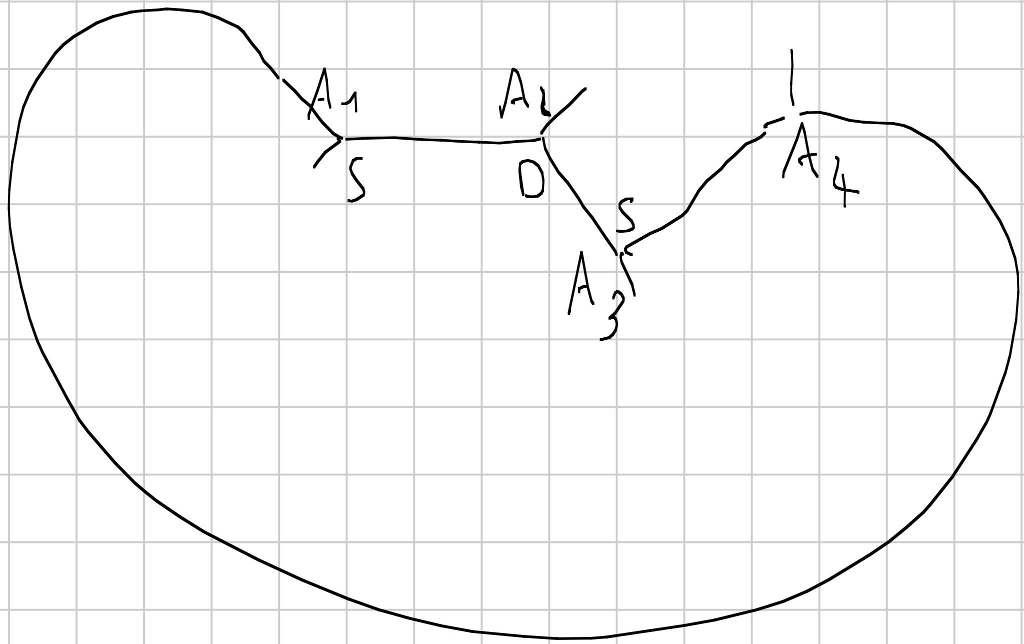
F_n periodici mod p

$\left(XY \, , \, YZ \right)$ arrivo



passi n $\quad$ $n+1$

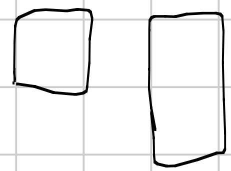
XY

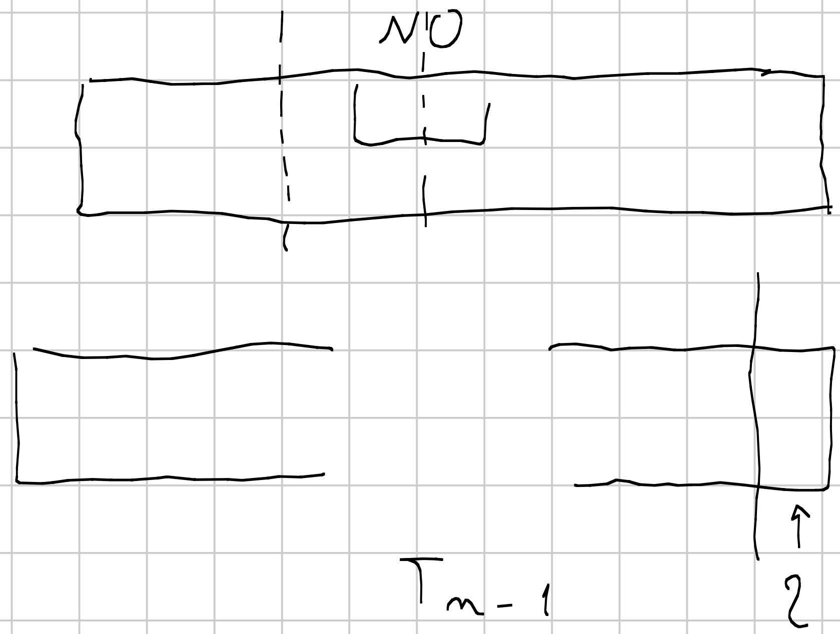

A1
S
A2
D
S
A3
A4

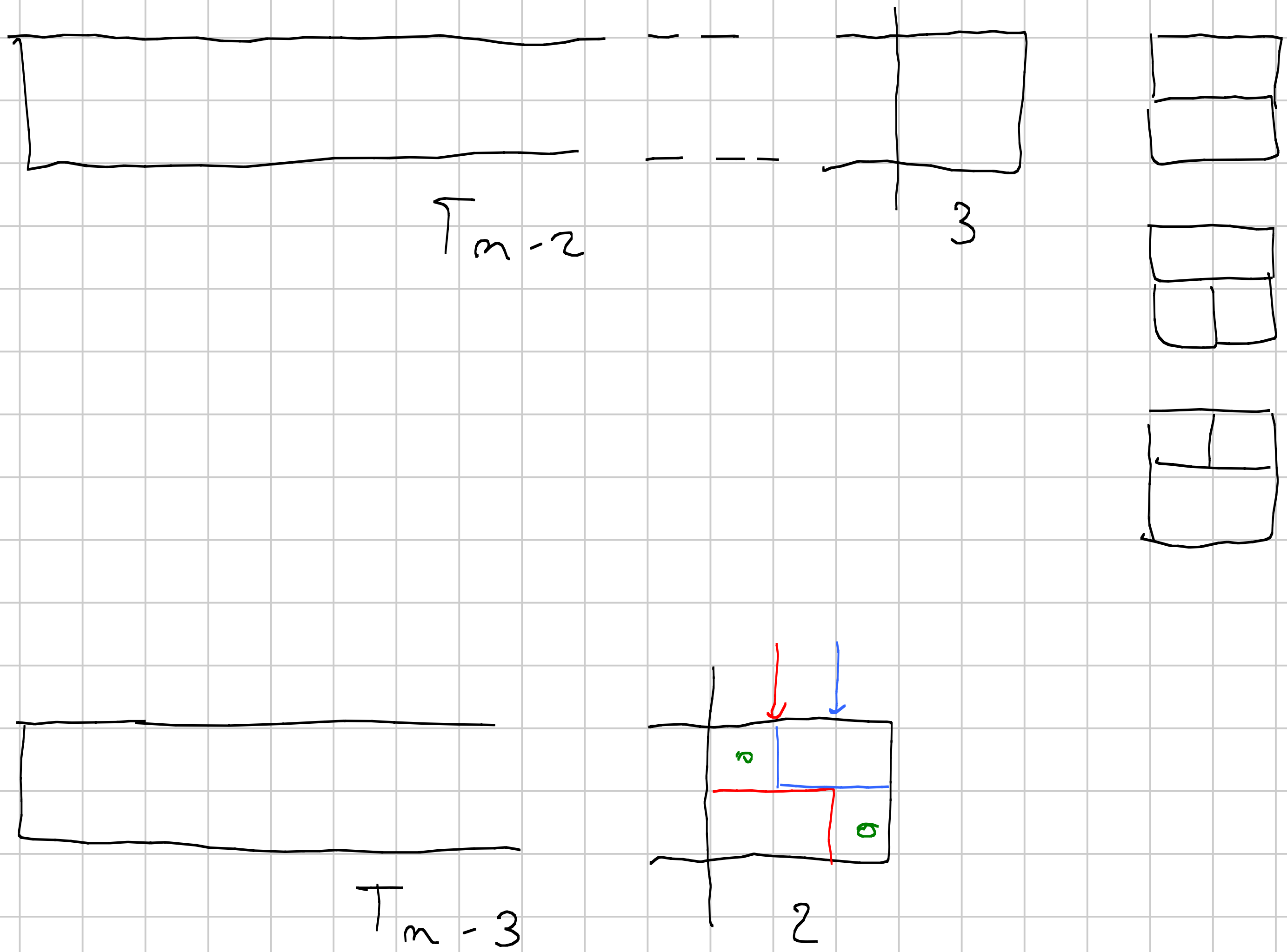
PROBLEMA 9 G.S.

T_n

$2 \times n$

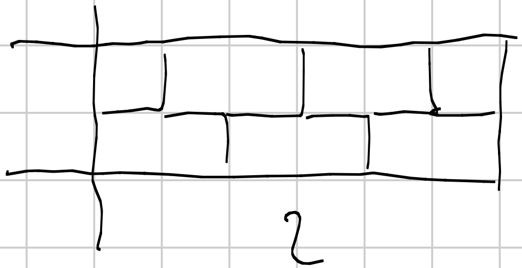






T_{n-2}
3
T_{n-3}
2

T_{n-6}



2

$T_0 = 1$

2

$$T_n = T_{n-2} + 2 \sum_{i=0}^{n-1} T_i \qquad S_n \stackrel{\text{DEF}}{=} \sum_{i=0}^{n} T_i$$

$$T_n = T_{n-2} + 2 S_{n-1}$$

$$S_{n-1} = T_{n-1} + S_{n-2} \longrightarrow T_n = 2T_{n-1} + T_{n-2} + 2S_{n-2}$$

$$T_{n-1} = T_{n-3} + 2S_{n-2}$$

$$T_n = 3T_{n-1} + T_{n-2} - T_{n-3}$$

$$x^3 - 3x^2 - x + 1$$

$$\lambda_1 \quad \lambda_2 \quad \lambda_3$$

$$T_n = a\lambda_1^n + b\lambda_2^n + c\lambda_3^n$$