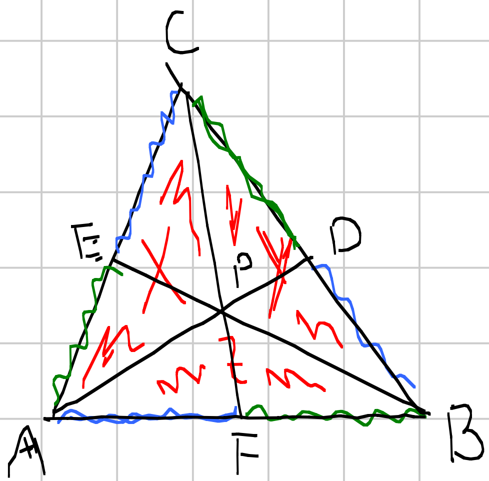
Titolo nota 11/10/2011

TEOR. DI CEVA

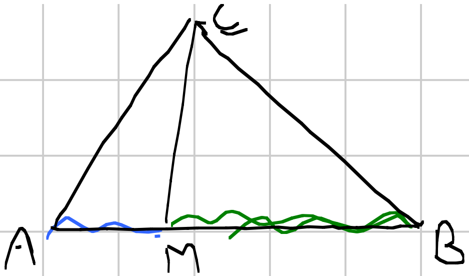


$$\frac{AF}{BF} \cdot \frac{BD}{CD} \cdot \frac{CE}{AE} = 1$$

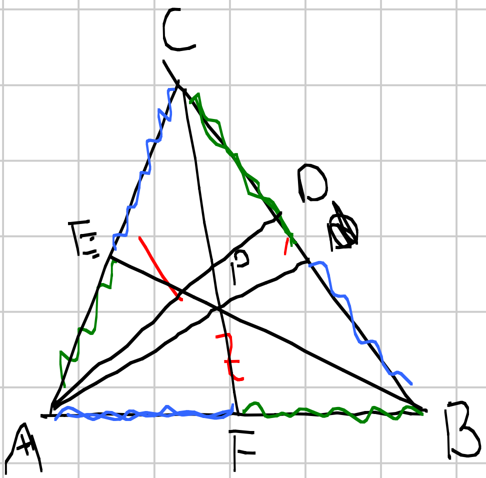
$$\frac{x}{y} \cdot \frac{z}{x} \cdot \frac{y}{z} = 1$$

1. concorrenza ⇒ relaz. metrica

2. viceversa



$$\frac{AM}{BM} = \frac{AM \cdot h}{BM \cdot h} = \frac{[AMC]}{[BMC]}$$

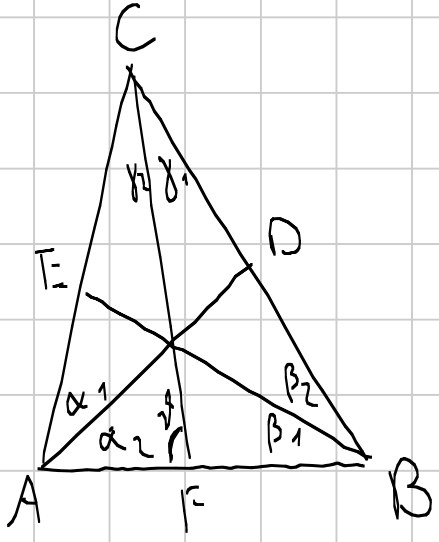


$$\Pi = \Pi$$

$$\wedge \qquad \vee$$

$$\Pi' = \Pi'$$

$\vartheta \mathrel{\hat{=}} AFC$



$$\sin\alpha_1 \cdot \sin\beta_1 \cdot \sin\gamma_1$$
$$= \sin\alpha_2 \cdot \sin\beta_2 \cdot \sin\gamma_2$$

$$\frac{AF}{BF}$$

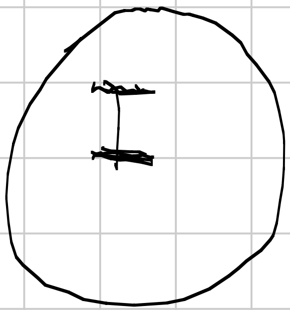
$$\underbrace{\frac{AF}{\sin\gamma_2} = \frac{AC}{\sin\vartheta} \qquad \frac{BF}{\sin\gamma_1} = \frac{BC}{\sin\vartheta}}$$

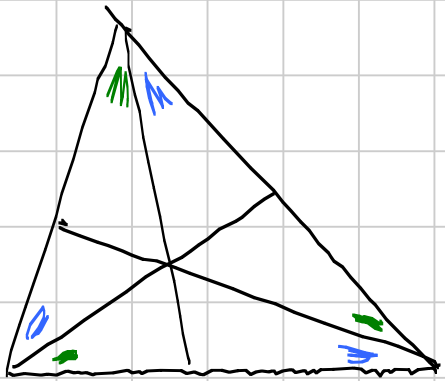
$$\frac{AF}{BF} = \frac{\sin\gamma_2}{\sin\gamma_1} \cdot \frac{AC}{BC}$$

$$\frac{AF}{BF} \cdot \sim \cdot \sim = 1$$

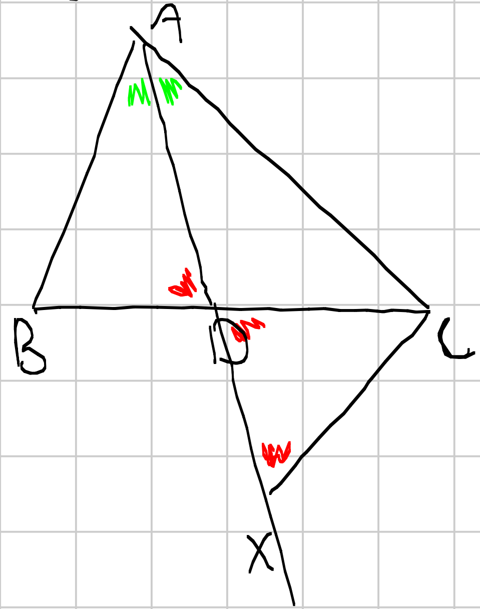
$$\|$$

$$1 = \frac{\sin \gamma_2}{\sin \gamma_1} \cdot \frac{\sin \alpha_2}{\sin \alpha_1} \cdot \frac{\sin \beta_2}{\sin \beta_1} \cdot \frac{A\cancel{C}}{B\cancel{C}} \cdot \frac{B\cancel{A}}{\cancel{C}A} \cdot \frac{\cancel{C}B}{\cancel{A}B}$$





Th. bisettrice



$$\frac{BD}{CD} = \frac{AB}{AC}$$

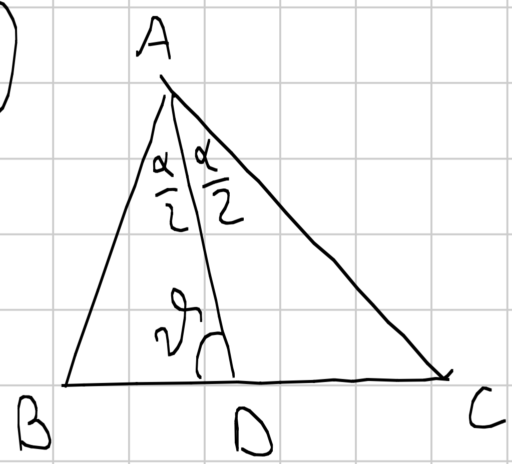
DIM 1

$$CD = CX$$

$$A\hat{B}D \approx A\hat{C}X$$

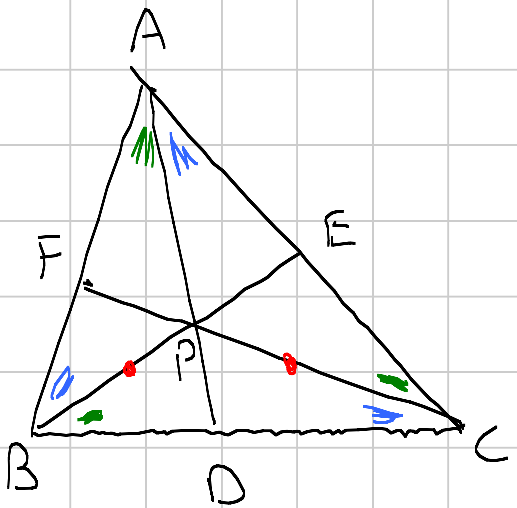
$$\frac{BD}{CX} = \frac{AB}{AC}$$

DIM 2



$$\frac{BD}{\sin\frac{\alpha}{2}} = \frac{AB}{\sin\vartheta}$$

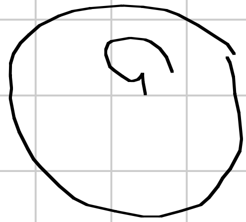
$$\frac{CD}{\sin\frac{\alpha}{2}} = \frac{\cancel{AB}\ AC}{\sin\vartheta}$$

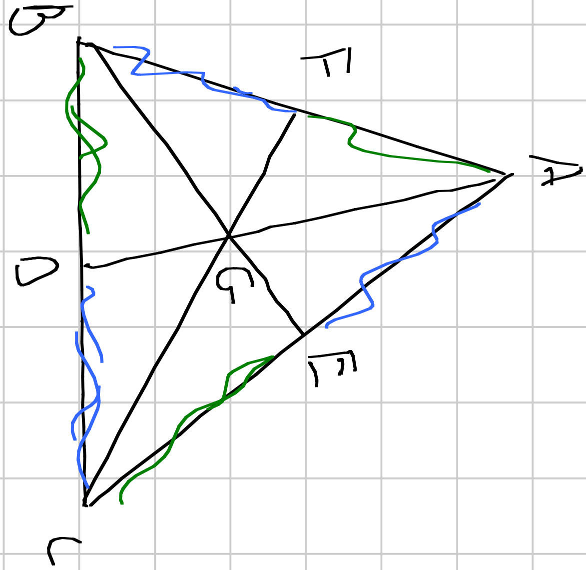


$$\frac{AF}{BF}\cdot\frac{BD}{CD}\cdot\frac{CE}{AE}\overset{\text{HOPTz}}{\overset{?}{=}}1$$

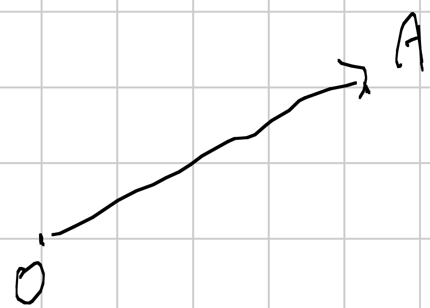
$$\frac{AC}{BC}\cdot\frac{AB}{AC}\cdot\frac{BC}{AB}\overset{!}{=}1$$

bisettrice come luogo di punti





VETTORI

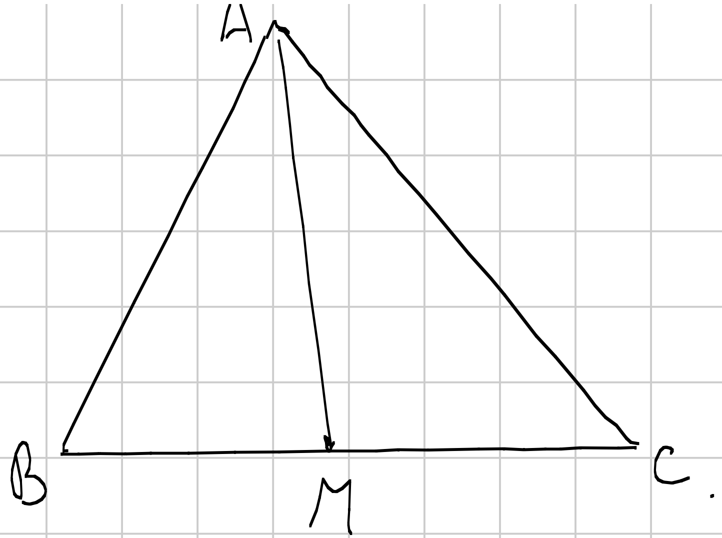


$\overrightarrow{OA} = \vec{A}$

$\vec{A}$, $\vec{B}$

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\vartheta$

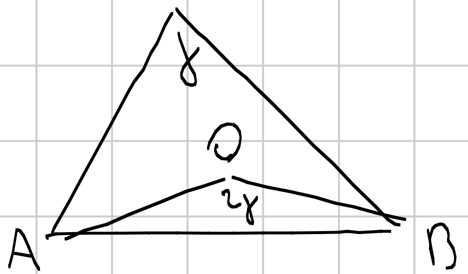
$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$



$$\vec{AM} \cdot \vec{AM} = |\vec{AM}|^2$$

$\vec{A}$, $\vec{B}$, $\vec{C}$ $\qquad |\vec{A}| = |\vec{B}| = |\vec{C}| = R$

$$\vec{A} \cdot \vec{B} = R^2 \cdot \cos 2\gamma = R^2(1 - 2\sin^2\gamma) =$$



$$= R^2 - \frac{1}{2} \cdot \underbrace{4R^2\sin^2\gamma}_{c^2} = R^2 - \frac{1}{2}c^2 .$$

$$\left|\overrightarrow{AM}\right|^2 \quad ?$$

$$\overrightarrow{AM} = \vec{M} - \vec{A} = \frac{\vec{B}+\vec{C}}{2} - \vec{A}$$

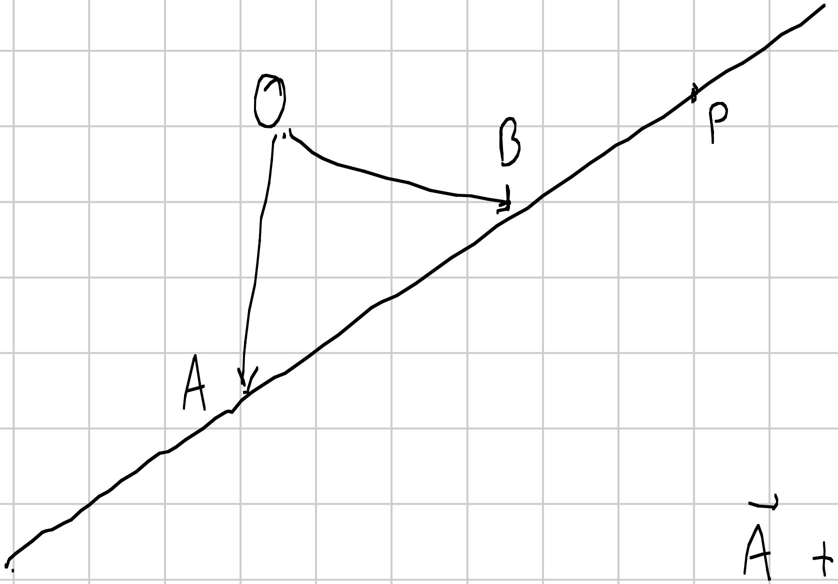
$$\left(\frac{\vec{B}+\vec{C}}{2} - \vec{A}\right)^2 = \frac{|\vec{B}|^2}{4} + \frac{|\vec{C}|^2}{4} + |\vec{A}|^2 + \frac{1}{2}\left(\vec{B}\cdot\vec{C}\right) -$$

$$- \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{C} =$$

$$\frac{R^2}{4} + \frac{R^2}{4} + R^2 + \frac{1}{2}R^2 - \frac{1}{4}a^2$$

$$- R^2 + \frac{1}{2}c^2 - R^2 + \frac{1}{2}b^2 =$$

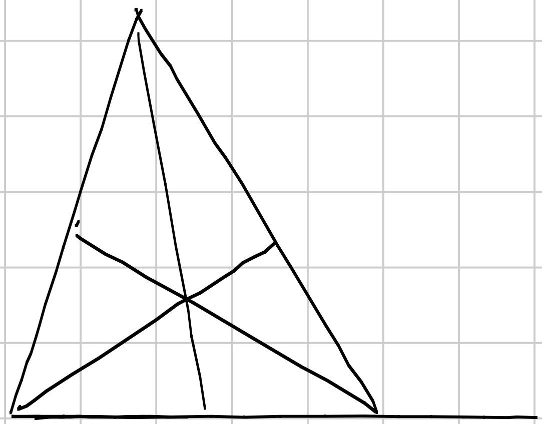
$$= \frac{1}{4}\left(2b^2 + 2c^2 - a^2\right)$$



$$t\vec{A} + (1-t)\vec{B}$$

$$\vec{A} + k\left(\overrightarrow{AB}\right) = \vec{A} + k\left(\vec{B} - \vec{A}\right) =$$

$$= k\vec{B} + (1-k)\vec{A}$$



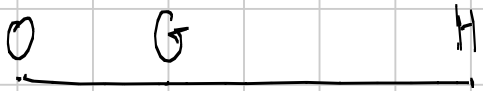
$\vec{A} \quad \vec{B} \quad \vec{C}$

$\frac{B+C}{2} \quad \frac{A+C}{2} \quad \frac{A+B}{2}$

$\left\{ tA + (1-t)\frac{B+C}{2} \right\}$ $\left\{ sB + (1-s)\frac{A+C}{2} \right\}$

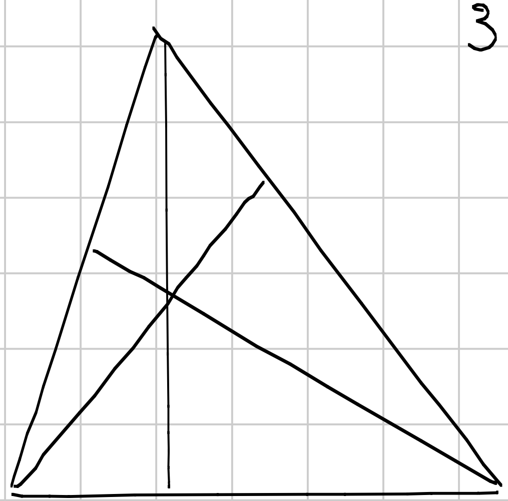
$\frac{A+B+C}{3}$ $t = \frac{1}{3}$ $s = \frac{1}{3}$

H



$$\frac{A + B + C}{3}$$

$$H = A + B + C$$



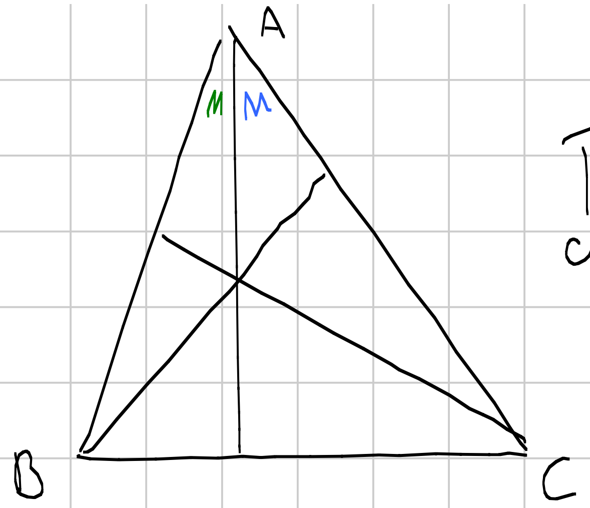
$$A + B + C = P$$

$$AP \perp BC$$

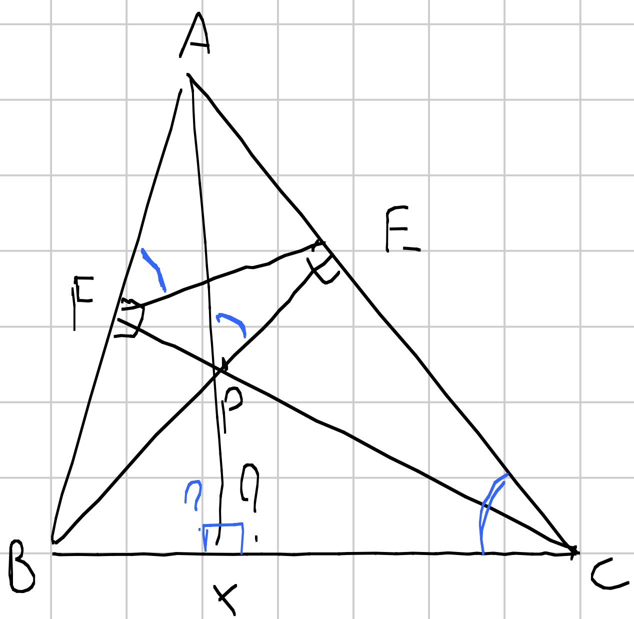
$BC \rightsquigarrow B - C$

$AP \rightsquigarrow A + B + C - A = B + C$

$(B - C)(B + C) = R^2 - R^2 = 0$

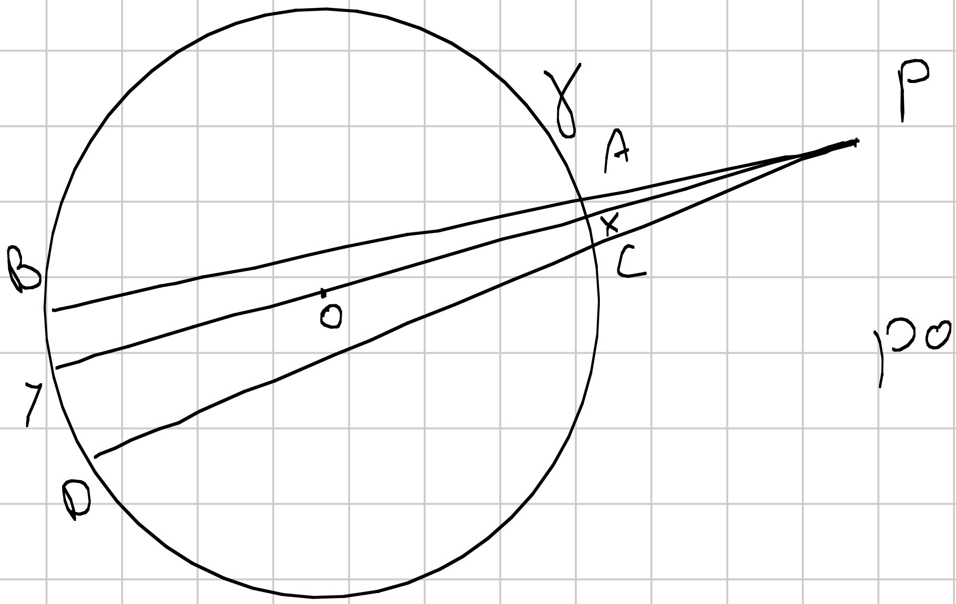


$$\prod_{cyc} \frac{\sin(90-\beta)}{\sin(90-\gamma)} = 1$$

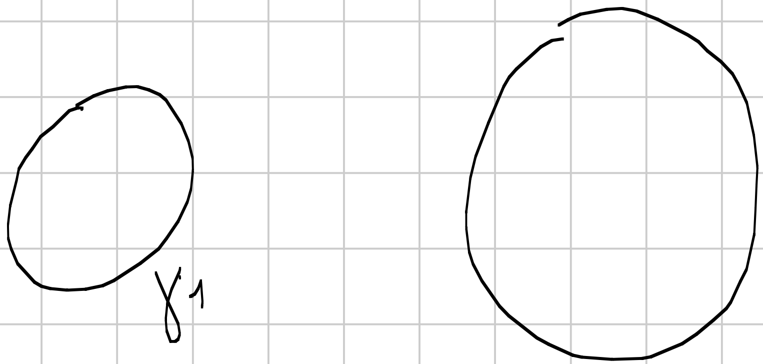


BCEF
AEPF ciclici

APE $\simeq$ ACX



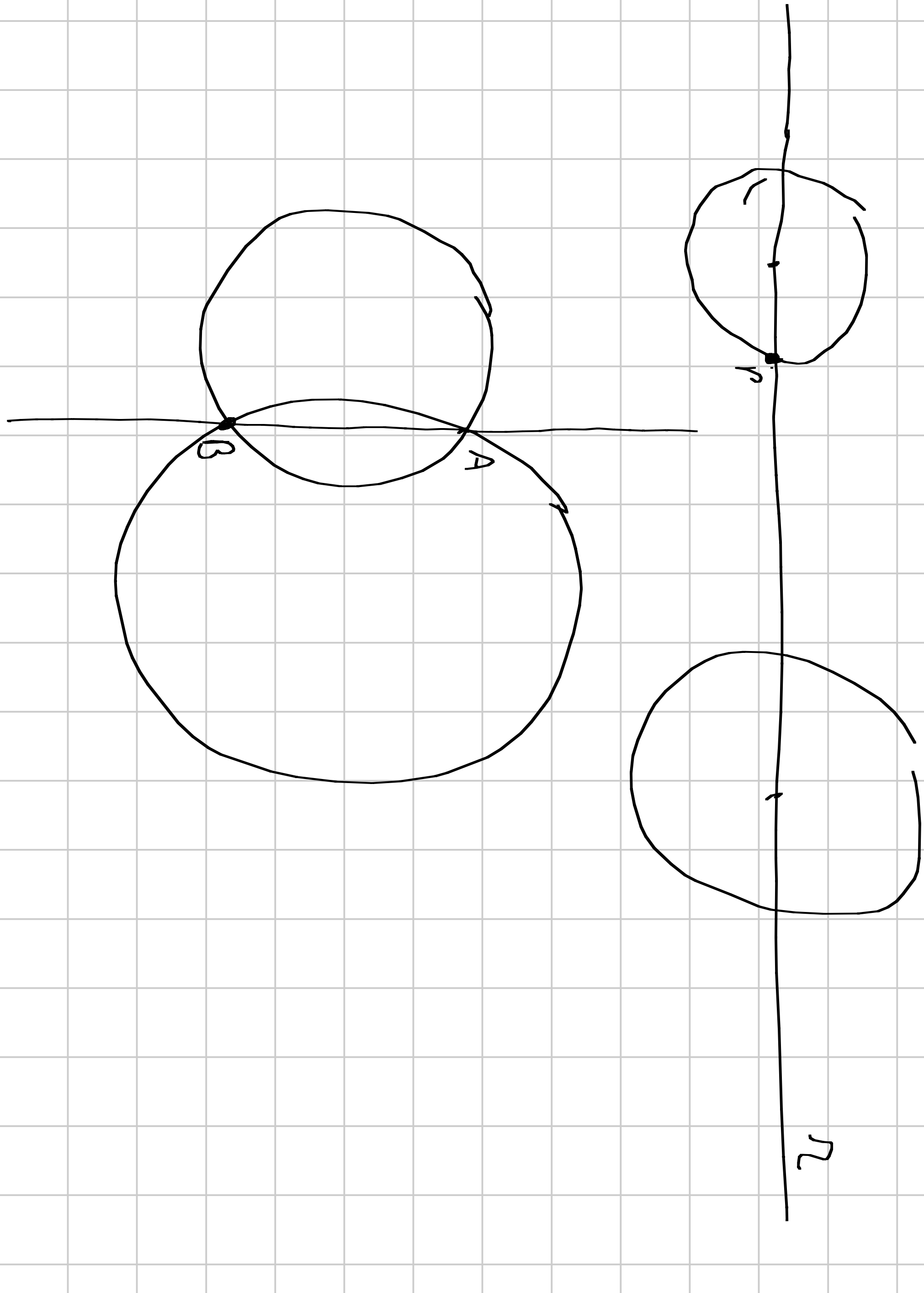
$$\mathrm{pow}_\gamma(P) = PA \cdot PB =$$
$$= d(P,O) - r^2$$
$$(d-r)(d+r)$$

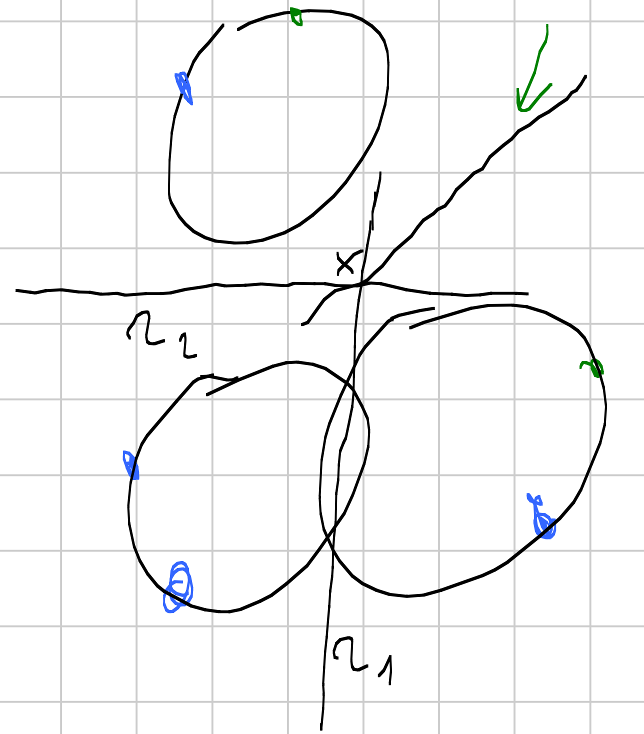


ℓ t.c. $pow_{\gamma_1}(P) = pow_{\gamma_2}(P)$

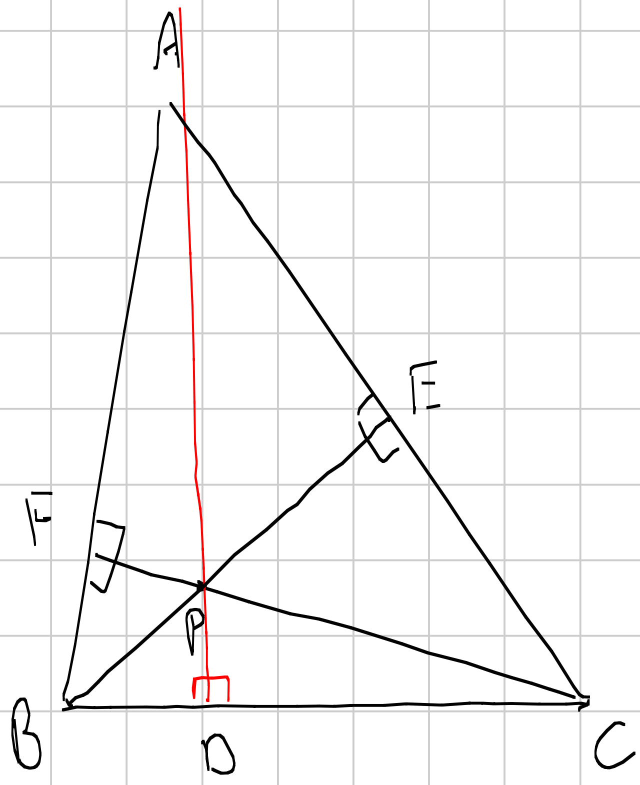
$d(P, O_1)^2 - r_1^2 = d(P, O_2)^2 - r_2^2$ $\qquad P(x, y)$

$(x - a_1)^2 + (y - b_1)^2 - r_1^2 = (x - a_2)^2 + (y - b_2)^2 - r_2^2$


B
A



$X = \Omega_1 \cap \Omega_2$



2 t.c. - passa per P
- ⊥ a BC

BDPF
CDPE

BCEF