Titolo nota — 10/10/2011

- CETRARO 2011 · CORREZIONE ALGEBRA

$$\sum_{i=1}^{n} F_i^2 = F_n \cdot F_{n+1}$$

INDUZIONE

VERIFICATA $n=0,\ n=1$

(1) $F_1^2 + \dots + F_n^2 = F_n \cdot F_{n+1}$ $\longleftarrow$ IPOTESI INDUTTIVA

(2) $F_1^2 + \dots + F_n^2 + F_{n+1}^2 = F_{n+1} \cdot F_{n+2}$ $\longleftarrow$ TESI

(2) $-$ (1) $\longrightarrow$ $F_{n+1}^2 = F_{n+1} \cdot F_{n+2} - F_n \cdot F_{n+1}$

$F_{n+1} = F_{n+2} - F_n \quad \vdash \quad F_{n+2} = F_n + F_{n+1}$

$\square$

ES. 2

$x^2 - 2011x + 1102$

α, β RADICI

$\alpha^{2011} + \beta^{2011} \in \mathbb{Z}$

$\alpha^n + \beta^n \in \mathbb{Z} \quad \forall n \in \mathbb{N}$

$\alpha + \beta = 2011 \qquad \alpha \cdot \beta = 1102$

$n = 0 \qquad \alpha^0 + \beta^0 = 2$

$n = 1 \qquad \alpha + \beta = 2011$

$\alpha^n + \beta^n \in \mathbb{Z}$ $\qquad$ $\alpha^{n+1} + \beta^{n+1} \in \mathbb{Z}$

$$\alpha^{n+2} + \beta^{n+2} = (\alpha^{n+1} + \beta^{n+1})(\alpha + \beta) - \alpha\beta(\alpha^n + \beta^n)$$

$\in \mathbb{Z}$ $\qquad\qquad$ $\in \mathbb{Z}$

3

$$(\sqrt{3}+2)^{101011} = \alpha \qquad \beta = (2-\sqrt{3})$$

$$\alpha+\beta = 4 \qquad \alpha \cdot \beta = 4 - 3 = 1$$

1° $\alpha^n + \beta^n \in \mathbb{Z}$

2° $0 < \beta < 0{,}3 < 10^{-\frac{1}{4}}$

Proviamo a dimostrare

$$\beta^{100.000} < 10^{-30}$$

$$\beta^{100.000} < \left(10^{-\frac{1}{4}}\right)^{100.000} = 10^{-25.000} < 10^{-30}$$

4 $\quad x+\frac{1}{x}\in\mathbb{Z} \implies \forall n \quad x^n+x^{-n}\in\mathbb{Z}$

COME PRECEDENTE

5

$(x-a)^2(x-b)^2+1$ é irriducibile in $\mathbb{Z}[x]$

$= P(x) \cdot Q(x)$

1° caso $\deg(P) = 1$ o 3

No! perché non ha radici

2° $\deg P = 2$ $\deg Q = 2$

possiamo supporre P, Q monici

CASO $a = b$

$$(x-a)^4 + 1 = P(x)\,Q(x)$$

$$P'(x) = P(x+a)$$

$$y = x - a$$

$$x^4 + 1 = P(x) \cdot Q(x) \qquad P(0) \cdot Q(0) = 1$$

$$P(x) = x^2 + ax + b$$

$$Q(x) = x^2 + cx + d$$

$b = 1$, $d = 1$ | $b = -1$ $d = -1$

$$(x^2 + ax + 1)(x^2 + \cancel{b}^{c}x + 1) = x^4 + 1$$

$$x^4 + (a+c)x^3 + (ac+2)x^2 + \dots$$

$a \neq b$

$$\begin{cases} P(a) \cdot Q(a) = 1 \\ P(b) \cdot Q(b) = 1 \end{cases}$$

$P(a), P(b), Q(a), Q(b) = \pm 1$

1° $P(a) = P(b) = 1$ $Q(a) = Q(b) = 1$

$P(x) - 1 = (x-a)(x-b)$

$P(x) = (x-a)(x-b) + 1$

SOSTITUIRE $\longrightarrow$ NON VERIFICA!

$P(a) = 1 \qquad P(b) = -1$

$\alpha, \beta \in \mathbb{Z}$

$P(x) = (x-a)(x-\alpha_1) + (x-b)(x-\beta_1)$

$1 = (a-b)(a-\beta)$

$-1 = (b-a)(b-\alpha)$

3

$P(x) = x^3 + ax^2 + bx + c$

$= (x-\alpha)(x-\beta)(x-\gamma)$

$\alpha + \beta = 0$

$\alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = b$

$\alpha\beta\gamma = -c$

$\gamma = -a \quad -ab = \alpha\beta\gamma = -c$

$\alpha\beta = b$

4

$$3(ab+bc+ca) \leq (a+b+c)^2$$

$$ab+bc+ca \leq a^2+b^2+c^2$$

1°

$$0 \leq \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

2°

$$\frac{a^2+b^2}{2} \geq \sqrt{a^2 b^2} = ab$$

3°

(a, b, c) (a, b, c)

$$x^3 - 3x + n = (x-a)(x-b)(x-c) \qquad a, b, c \in \mathbb{Z}$$

$$\begin{cases} a+b+c = 0 \\ ab+bc+ca = -3 \\ abc = -n \end{cases}$$

$$6 = (a+b+c)^2 - 2(ab+bc+ca) = a^2+b^2+c^2$$

$a = \pm 1 \quad b = \pm 1 \quad c = \pm 2$ (CASI FINITI

(e permutazioni)

$n = \pm 2$