www.mat.uniroma2.it/~olimpiadi

Titolo nota 10/10/2011

Problema 1

Prendo $\{1, 2, 3, \ldots, 100\}$

Faccio 50 coppie, moltiplico gli elementi di ogni coppia e sommo.

Trovare minimo e massimo.

$1, 2, 3, 4$

$(1, 2) \quad (3, 4) \longrightarrow 14$

$(1, 3) \quad (2, 4) \longrightarrow 11$

$(1, 4) \quad (2, 3) \longrightarrow 10$

$\cancel{a < b < c < d}$

$0 < a_1 < b_2$

$0 < b_1 < b_2$

a_1, a_2

b_1, b_2

$$a_1 b_1 + a_2 b_2 \geq a_1 b_2 + a_2 b_1$$

$$a_1 b_1 + a_2 b_2 - a_1 b_2 - a_2 b_1 \geq 0$$

$$-a_1(-b_1 + b_2) + a_2(b_2 - b_1) \geq 0$$

P. A. prendo copp che realizzano il max.
e suppongo che 19 e 193 [illegible] [illegible]

$(a_1, a_2), \dots, (a_i, a_j), \dots, (a_p, 99), (a_q, 100)$ ecc. ogni vol.

$a_1 \cdot a_2 + \dots + a_i \cdot a_j + \dots + a_p \cdot 99 + a_q \cdot 100$

$a_p a_q + 99 \cdot 100$

(*) diminuire

Problema

$a, b, c > 0$ $a < b < c$

$$\boxed{a^b \cdot b^c \cdot c^a \ ? \ a^a \cdot b^b \cdot c^c}$$

$$\ln\left(a^b b^c c^a\right) \ ? \ \ln\left(a^a b^b c^c\right)$$

$$b \ln a + c \ln b + a \ln c \ ? \ a \ln a + b \ln b + c \ln c$$

Diseg. Riarrangiamento

Dati $a_1, a_2, \dots, a_n$ e $b_1, b_2, \dots, b_n$ t.c.

$$a_1 < a_2 < \dots < a_n \qquad b_1 < b_2 < \dots < b_n$$

Allora $\forall\, \sigma$ permutazioni non mediane,

$$\sum_{i=1}^{n} a_i b_{n+1-i} < \sum_{i=1}^{n} a_i b_{\sigma(i)} < \sum_{i=1}^{n} a_i b_i$$

Dim

[morale]

$[\qquad\qquad] + a_n b_i + a_j b_n$

$a_j b_i + a_n b_n$

$a_1 \le a_2 \le a_3 \le \cdots \le a_n = A$ $\quad A$

$b_1 \le b_2 \le \cdots \le b_n = B$

$$a_1 \le a_2 \le a_3 \le \dots \le a_k < \overbrace{A = A = \dots = A}^{n-k}$$

$$b_1 \le b_2 \le b_3 \le \dots\dots \le b_h < \underbrace{B = B = \dots = B}_{n-h}$$

$h \le k$

P.A.

$$[\qquad\qquad] + B \cdot a_i + b_j \cdot A$$

$i \le k \qquad j \le h$

$0 < a < b$

$\alpha < \beta$

$a^{\beta} b^{\alpha} \overset{?}{<} a^{\alpha} b^{\beta}$

$a^{(\beta - \alpha)} \overset{?}{<} b^{(\beta - \alpha)}$

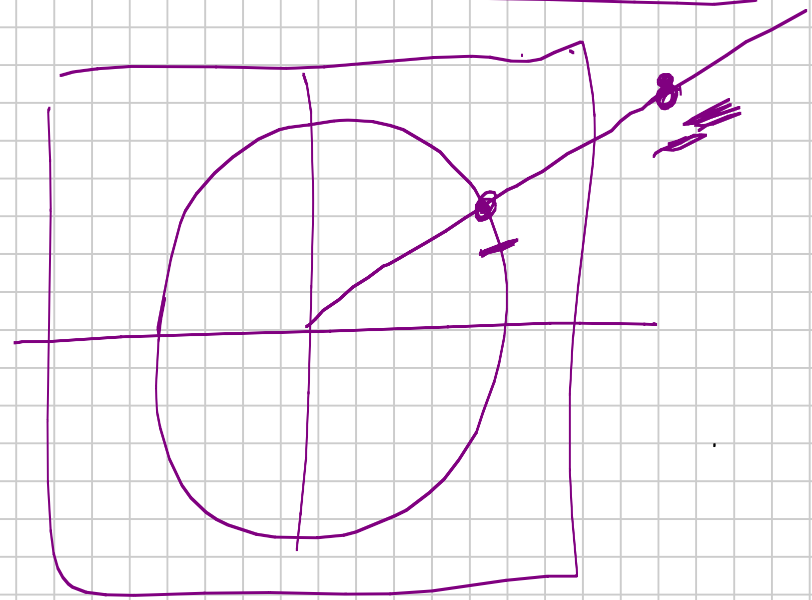
Vu

$a_1, a_2, \dots, a_n$

$b_1, b_2, \dots, b_n$

$$\left|a_1 b_1 + a_2 b_2 + \dots + a_n b_n\right|^2 \le \overbrace{\left(a_1^2 + a_2^2 + \dots + a_n^2\right)}^{=1} \overbrace{\left(b_1^2 + b_2^2 + \dots + b_n^2\right)}^{=1}$$

$$\left|\frac{xy}{x^2+y^2}\right| \le \frac{1}{2}$$



$$-2 \leq 2a_1b_1 + 2a_2b_2 + \cdots + 2a_nb_n \leq 2$$

sapendo che

$$a_1^2 + a_2^2 + \cdots + a_n^2 = 1$$

$$b_1^2 + b_2^2 + \cdots + b_n^2 = 1$$

$$-a^2 - b^2 \leq 2ab \leq a^2 + b^2$$

$$\underbrace{-a_1^2-b_1^2-\dots-a_n^2-b_n^2}_{-2} \le 2a_1b_1+\dots+2a_nb_n \le \underbrace{a_1^2+b_1^2+\dots+a_n^2+b_n^2}_{2}$$

$$0$$

$$(a_1+tb_1)^2+(a_2+tb_2)^2+\dots+(a_n+tb_n)^2 \ge 0$$

$$(b_1^2+\dots+b_n^2)t^2+2(a_1b_1+\dots+a_nb_n)t+(a_1^2+\dots+a_n^2) \ge 0$$

$$(a_1b_1+\dots+a_nb_n)^2-(a_1^2+\dots+a_n^2)(b_1^2+\dots+b_n^2) \le 0$$

$a_1, a_2, \dots, a_n$

$b_1, b_2, \dots, b_n$ linth positiv

$c_1, c_2, \dots, c_n$

$$a_1 b_1 c_1 + \dots + a_n b_n c_n \le \underbrace{\sqrt[3]{a_1^3 + \dots + a_n^3}}_{=1} \cdot \underbrace{\sqrt[3]{b_1^3 + \dots + b_n^3}}_{=1} \cdot \underbrace{\sqrt[3]{c_1^3 + \dots + c_n^3}}_{=1}$$

$$\boxed{abc \le \frac{1}{3}(a^3 + b^3 + c^3)}$$

$$abc + abc + abc \leq a^3 + b^3 + c^3$$