Titolo nota 20/10/2010

1. ab $\quad 10a+b=3(a+b)$ $\quad 1\leq a\leq 9$

$0\leq b\leq 9$

$7a=2b$

$b=7$

$a=2$

2. $m^2=\ldots 6$ $\quad (10a+4)^2$ $\quad 10a+6$

$\underbrace{a_m a_m \ldots a_1}_{k} 4$

$100a^2+80a+16$

$10(10a^2+8a+1)+6$

$N = a_m b^m + a_{m-1} b^{m-1} + \dots \; a_1 b + a_0 \qquad 0 \leq a_i < b$

$a_m a_{m-1} \sim a \, a_0 \qquad \forall i = \{0 \dots 9\}$

3. $A : a\underbrace{bcde}_{k}$ $\qquad B = \underbrace{bcde}_{k}a \qquad 41 \mid B$?

$41 \mid A$

$A = a \cdot 10^4 + k \qquad B = k \cdot 10 + a$

$41 \mid A \qquad \underline{10A} - B = 10^5 a + \cancel{10k} - \cancel{10k} - a =$

$41 \mid 10A \qquad \underline{99.999\,a}$

$41 \mid B$

4. $\overline{AB} \times \overline{CD} = \overline{BA} \times \overline{DC}$ $\qquad 1 \leq A, B, C, D \leq 9$

5. $\left(\underbrace{99999999999999999}5\right)^2$ $\qquad 35^2$

$(10a+5)^2 = 100a^2 + \ldots \qquad a(a+1)100 + 25$ $\qquad 1225$

$\underbrace{9 \ldots 99}_{17}0\underbrace{\ldots 00}_{17}25$

$17 \cdot 9 + 2 + 5 = 160$

6. $\underline{a}\,\underline{b}\,\underline{a}\,\underline{b} = n$ $\qquad (ab)^2 \mid n^2$ $\qquad 1 \le a \le 9$

$n = 1000a + 100b + 10a + b = 101(10a+b)$

$ab \mid n$

$ab \mid 101(10a+b)$ $\qquad$ $MCD(a, 101) = 1$

$(b, 101) = 1$

$a \mid 10a+b$ $\qquad a \mid b$

$b \mid 10a+b$ $\qquad b \mid 10a$

$a \mid b \mid 10a$

$a = b$

1111

$ab \mid 10a+b$ $\qquad a^2 \mid 11a$

$a \mid 11$ $\qquad a = 1$

$b = 2a$

$b = 5a$

$\cancel{b = 10a}$

$2a^2 \mid 12a$ $\quad a \mid 6$

$\cancel{1}$

$2 \quad 4$

$3 \quad 6$

$\cancel{6}$

2424 $\quad$ 3636

7.

a, b $\qquad 0 \leq a, b \leq 9$

$10a + b = 19b + a$

$9a = 18b$ ....

$a = 2b$

8.

$f: \mathbb{Z}^+ \to \mathbb{Z}^+$

$f(1) = 1 \qquad f(3) = 3$

$f(2n) = f(n)$

$f(4n+1) = 2f(2n+1) - f(n)$

$f(4n+3) = 3f(2n+1) - 2f(n)$

1	1
2	1
3	3
4	1
5	5

$f(n) = n$

$n \le 1988$

6	3
7	7
8	1
9	9
10	5
11	13

DIVISIBILITÀ $\rightarrow \mathbb{Z}$

$b : a = q$

r

$a, b \quad a \neq 0 \quad \exists! \, r, q :$

$b = aq + r$

$0 \leq r < |a|$

$r = 0 \qquad a \mid b \qquad \exists h : b = ha$

- $a \mid b \wedge b \mid c \longrightarrow a \mid c$

  $b = \underline{ha} \qquad c = kb \qquad c = k \cdot ha \qquad c = (hk)a \;,$

  $a \mid c$

- $a \mid b \wedge b \mid a \qquad a = b \vee a = -b$

- $a \mid b \qquad |a| \leq |b|$

- $a \mid b \wedge a \mid c \longrightarrow a \mid b + c$

  $a \mid b - c$

  $a \mid 2010b + 300c$

$a \mid kb + hc \quad \forall (h,k) \in \mathbb{Z}$

$a \mid b \qquad a \mid b + ka \quad \forall k \in \mathbb{Z}$.

- $a - b \mid a^n - b^n$

$a + b \mid a^{2n+1} + b^{2n+1}$

9. $2^m - 1$ primo $\longrightarrow$ m è primo

$\boxed{m = ab}$ $\quad 2^{ab} - 1 = (2^a)^b - 1^b \longrightarrow 2^a - 1 \mid 2^m - 1$.

PRIMO :

$p \neq \pm 1$ $\quad p \mid ab \rightarrow p \mid a \vee p \mid b$

10. $2^n + 1$ primo $\longrightarrow$ n è potenza di 2

$n = a\,d$ $\quad d$ dispari $\quad 2^{ad} + 1 = \left(2^a\right)^d + 1$

$2^a + 1$

n è potenza di 2

11. $\dfrac{21n+4}{14n+3}$ è irriducibile

$MCD(a,b) = d$

$d \mid a \quad d \mid b \quad \wedge \quad \forall k : k \mid a \wedge k \mid b \quad k \mid d$

$k \leq d$

- $MCD(a,b) = d$

  $d \mid a-b$

se $d \mid a \wedge d \mid a-b \rightarrow d \mid b$

$MCD(13,8) = (5;8) = (5;3) = (2;3) = 1$

$$\frac{21n+4}{14n+3}$$

$$(21n+4\,;\,14n+3) = (7n+1\,;\,14n+3) =$$

$$(7n+1\,;\,1) = 1$$

- $d = (a,b) \quad \exists h,k : \quad ha + kb = d$ (BEZOUT)

$$(a,b) = d$$

$$(79,\,19) =$$

$$(\underbrace{79-3\cdot 19}_{22}, 19) = (22, 19) = (\underbrace{22-19}_{3}; 19) = (3; 19) =$$

$$= (3; \underbrace{19-3\cdot 6}_{1}) = 1$$

$$1 = 19 - 3\cdot 6 = 19 - 6\cdot(22-19) = 7\cdot 19 - 6\cdot 22 =$$

$$7\cdot 19 - 6\cdot(79-3\cdot 19) = 25\cdot 19 - 6\cdot 79$$

$-\quad p \mid ab \rightarrow p \mid a \vee p \mid b$

$p \nmid a \qquad \text{mcd}(a,p) = 1 \qquad 1 = ha + kp$

$\underline{ha} = 1 - kp$

$p \mid ab$

$p \mid hab$

$p \mid (1-kp)b \qquad p \mid b - \underbrace{kpb}_{p} \qquad p \mid b$

12\. $x^2 - y^2 = 2010$

$(x-y)(x+y) = 2 \cdot 5 \cdot 67 \cdot 3$

$\begin{cases} x - y = \\ x + y = \end{cases}$

1	2	3	5
2010	1005	670	402

$N = p_1^{a_1} \cdot p_2^{a_2} \cdots p_k^{a_k}$ $\qquad 0 \le b_1 \le a_1$ $\qquad b_1 = a_1 + 1$

$d = p_1^{b_1} \cdot p_2^{b_2} \cdots p_k^{b_k}$ $\qquad - \quad b_2 \; \cdots$ $\qquad b_2 = a_2 + 1$

$\cdots$

$d \to (a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$

d d: 36

1	2	3	4	6
36	18	12	9	$\cancel{6}$

$x^2 - 12x + 32$

- 1	2	4
- 32	16	8

$$\begin{cases} x+y=a \\ x-y=b \end{cases}$$

$$x=\frac{a+b}{2}$$

$$y=\frac{a-b}{2}$$

$a \cdot b$

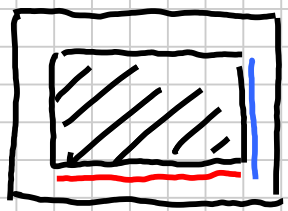
2010

a pari

b disp

14: $\dfrac{n^2+3n-2}{n+11} \in \mathbb{Z}$

13.



b

a

area nera = area bianca

$$ab = 2(a-2)\cdot(b-2)$$

$$ab = 2ab - 4b - 4a + 8$$

$$ab - 4b - 4a + 8 \overset{+8}{=} 0 + 8$$

$$b(a-4) - 4(a-4) = 8 \qquad (a-4)(b-4) = 8$$

$$ab - 4b - 4a + 8 = 0$$

$$a(b-4) = 4b - 8$$

$$a = \frac{4b-8}{b-4} = 4 + \frac{8}{b-4}$$

$$\frac{4b-8}{b-4} \qquad 4b-8 \,|\, b-4$$

14 $$\frac{m^2+3m-2}{m+11} \qquad R(-11) = 86$$

$$MCD(\text{num} \,;\, \text{den}) = d$$

$$d \mid m^2+3m-2 \qquad d \mid m+11 ,$$

$$d \mid \cancel{m^2}+3m-2 - \cancel{m^2} - 11m = -8m-2 .$$

$d \mid -8M - 2 + 8(M+1) = 86$ $\qquad 2 \cdot 43$

15 $\quad 3m^5 + 5m^3 - 8m \Rightarrow m(m-1)(m+1)(3m^2+8)$

$m = 0$	0
$m = 1$	0
$m = 2$	120
$m = 3$	840
$m = 4$	

$120 = 2^3 \cdot 3 \cdot 5$

m = pari

m dispari $\quad$ so

$\cancel{0}, \cancel{1}, 2, 3, 4, \cancel{5}, \cancel{6}, 7, 8, 9$