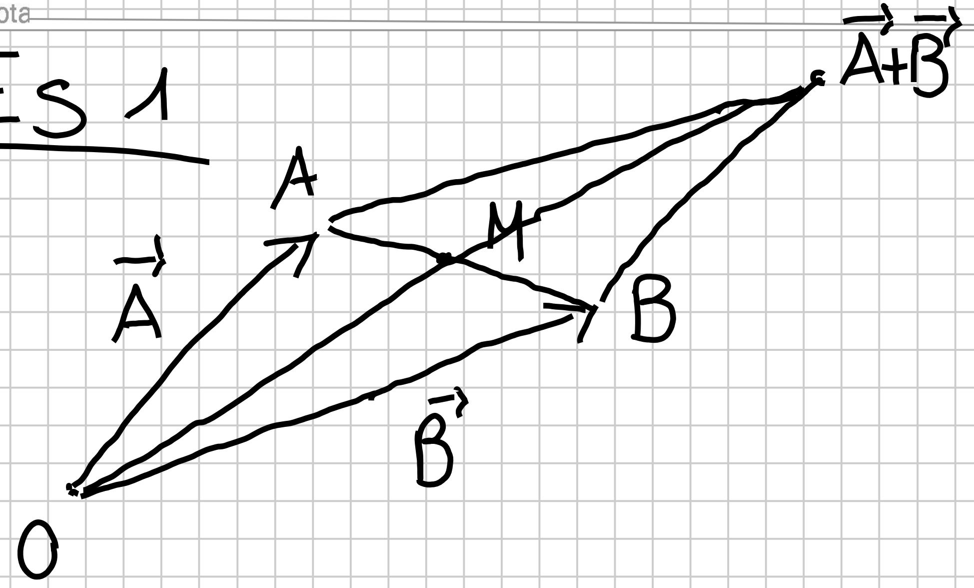
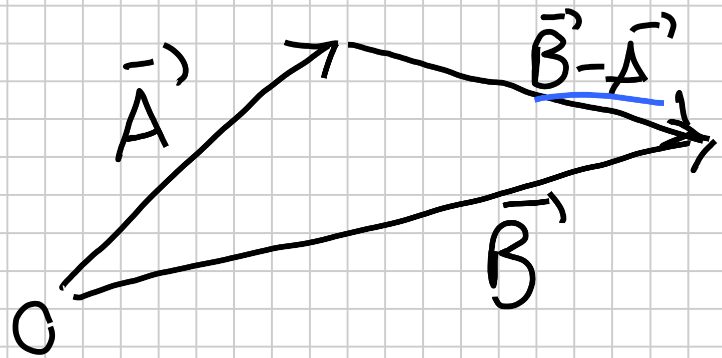
Titolo nota 19/10/2010

Esercitazione di geometria

Es 1



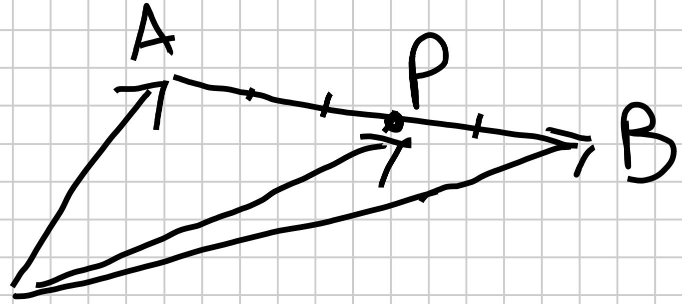
$$\vec{M} = \frac{\vec{A}+\vec{B}}{2}$$



$$\vec{M} = \underbrace{\vec{A}}_{= \vec{OA}} + \underbrace{\frac{\vec{B}-\vec{A}}{2}}_{= \vec{AB}} = \frac{\vec{A}+\vec{B}}{2}$$

$$\vec{M} = \vec{B} + \frac{\vec{A}-\vec{B}}{2}$$

Es 2



$$PA : AB = 3 : \underset{2+3}{\underset{\shortparallel}{5}}$$

$$\vec{P} = \vec{A} + \left(\vec{B} - \vec{A}\right)\frac{3}{5} =$$

$$= \frac{2}{5}\vec{A} + \frac{3}{5}\vec{B} = \frac{2\vec{A} + 3\vec{B}}{2+3}$$

Es 3

$$PA : PB = t : (1-t) \implies PA : AB = t : 1$$

$$PA : PB = k_1 : k_2$$

$$= \frac{k_1}{k_1+k_2} : \frac{k_2}{k_1+k_2}$$

$$\vec{P} = \vec{A} + t\left(\vec{B} - \vec{A}\right)$$

$$= (1-t)\vec{A} + t\vec{B}$$

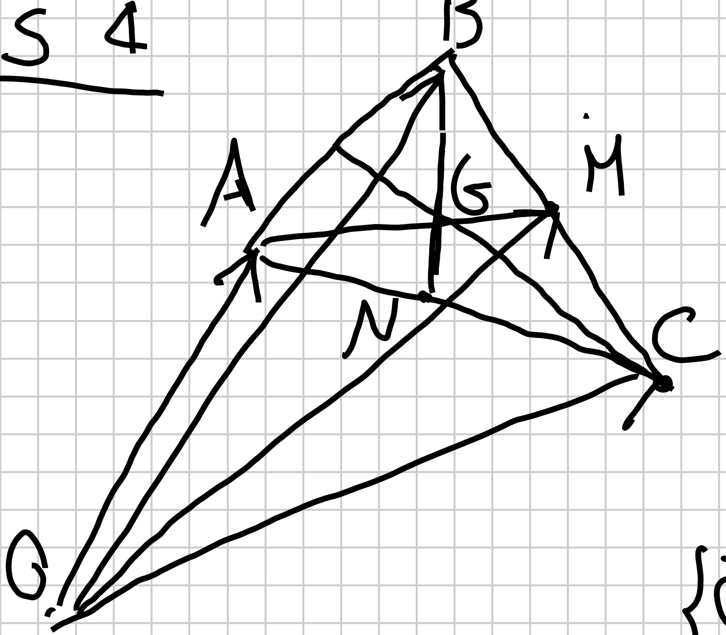
$$\vec{P} = \frac{k_2\vec{A} + k_1\vec{B}}{k_1+k_2}$$

Segmento AB: $t\vec{A} + (1-t)\vec{B} \quad t \in [0,1]$

Retta: $\vec{P} = \vec{A} + t(\vec{B} - \vec{A}) \quad t \in \mathbb{R}$

$$= (1-t)\vec{A} + t\vec{B}$$

Es 4



$$\vec{M} = \frac{\vec{B}+\vec{C}}{2}$$

$$\cancel{\{AM\}} \quad \{\vec{P} \in AM\} = \{t\vec{A} + (1-t)\vec{M}\} =$$

$$= \left\{ t\vec{A} + (1-t)\frac{\vec{B}+\vec{C}}{2} \right\}$$

$$\{\vec{Q} \in BN\} = \left\{ s\vec{B} + (1-s)\frac{\vec{A}+\vec{C}}{2} \right\}$$

$$\left[\vec{G} \stackrel{?}{=} \frac{\vec{A}+\vec{B}+\vec{C}}{3}\right.$$

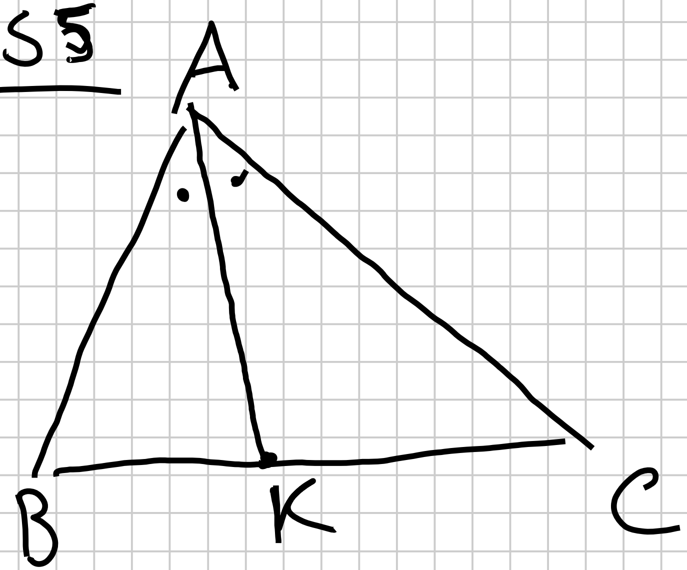
$$t = \frac{1}{3} \qquad s = \frac{1}{3}$$

$$\vec{P} = \frac{1}{3}\vec{A} + \frac{\cancel{2}}{3} \cdot \frac{\vec{B}+\vec{C}}{\cancel{2}} = \vec{G}$$

$$\vec{Q} = \frac{\vec{B}}{3} + \frac{\cancel{2}}{3}\,\frac{\vec{A}+\vec{C}}{\cancel{2}} = \vec{G}$$

$$t = \frac{1}{3} = \frac{GM}{AM}$$

Es5



$$\vec{K} = ?$$

$$BK : KC = c : b$$

$$\vec{K} = \frac{c\vec{C} + b\vec{B}}{b+c}$$

1° metodo

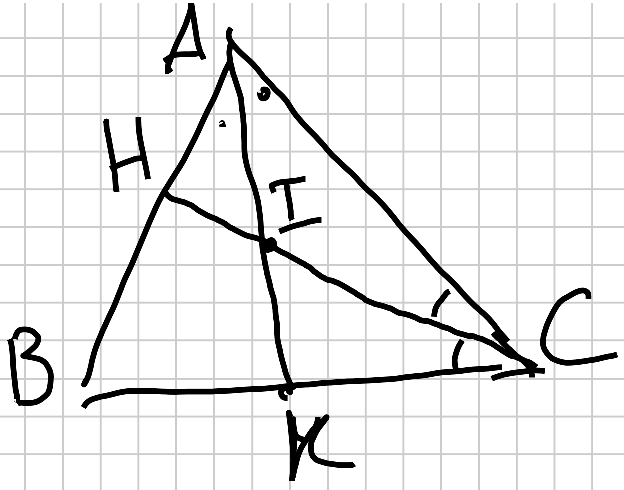
$$t\vec{A} + (1-t)\,\frac{c\vec{C} + b\vec{B}}{c+b}$$

$$t = \frac{a}{a+b+c} \qquad 1-t = \frac{b+c}{a+b+c}$$

Quindi $\cancel{\vec{P}} = \vec{I} = \dfrac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$ sta sulla bisettrice per A. Ripetendo il ragionamento sta su ~~b~~ tutte $\Rightarrow$ è l'incontro

2° metodo.

Guardo $C\hat{K}A$. I é il piede della bisettrice in C.

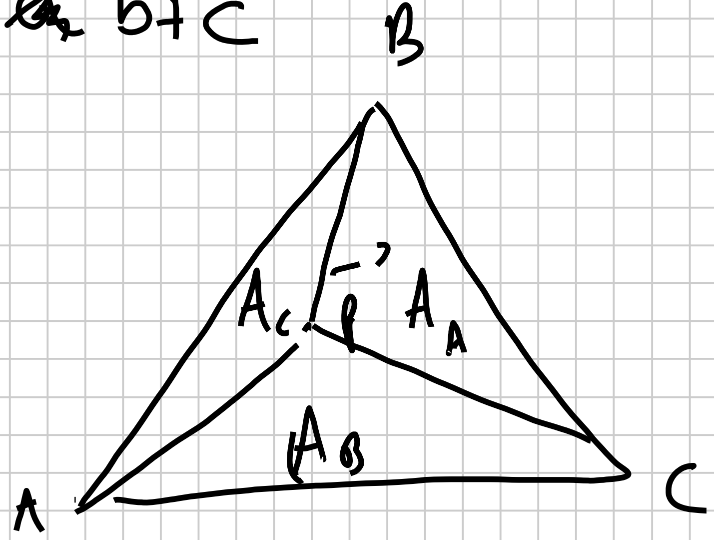


$$\vec{I} = \frac{\overline{CK}\,\vec{A} + b\,\vec{K}}{\overline{CK} + b}$$

$$CK : BK = b : c$$

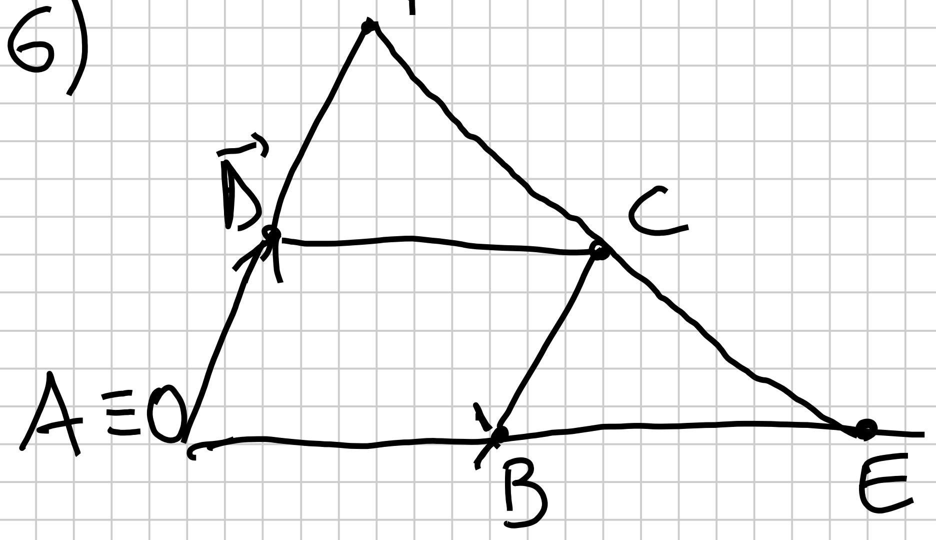
$$CK : BC = b : b + c$$

$$CK = \frac{ab}{\cancel{a}\; b + c}$$



$$\vec{P} = \frac{A_A\,\vec{A} + A_B\,\vec{B} + A_C\,\vec{C}}{A_{TOT}}$$

6)



$\vec{C} = \vec{B} + \vec{D}$

C è punto medio di EF

$\vec{E} = 2\vec{B}$

$\vec{F} = 2\vec{D}$

$\vec{M} = \dfrac{\vec{E}+\vec{F}}{2} = \vec{B} + \vec{D}$

Es 7: $\vec{E} = \dfrac{\vec{A}+\vec{B}}{2}$ $\quad F = \dfrac{B+C}{2}$ $\quad G = \dfrac{C+D}{2}$ $\quad H = \dfrac{D+A}{2}$

$\vec{M}_{AC} = \dfrac{\vec{A}+\vec{C}}{2}$ $\quad \vec{M}_{BD} = \dfrac{\vec{B}+\vec{D}}{2}$

$$\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C} + \vec{D}}{4}$$

$\vec{G}$ è punto medio di EG, FH, $M_{AC}M_{BD}$

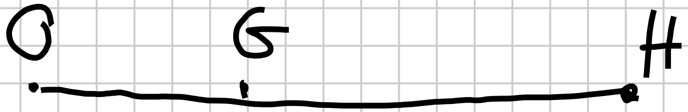
Es 8

G, H, O allineati $\overline{OH} = 3\,\overline{OG}$

Origine in O

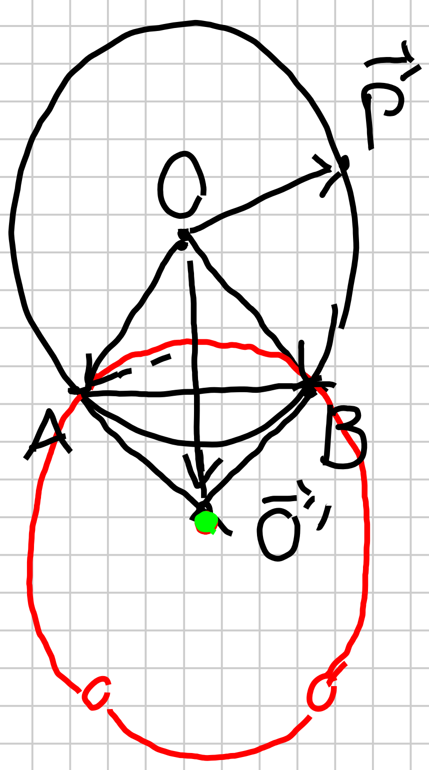
$$\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$$

$\vec{H} = ?$



$\vec{H} = 3\vec{G} = \vec{A} + \vec{B} + \vec{C}$

Solo se l'origine é circocentro



$\vec{H} = \vec{A} + \vec{B} + \vec{P}$

$\vec{O}' := \vec{A} + \vec{B}$ é il simmetrico di $\vec{O}$ risp ad AB.

$\vec{H}' = \vec{O}' + \vec{P}$

Omotetie

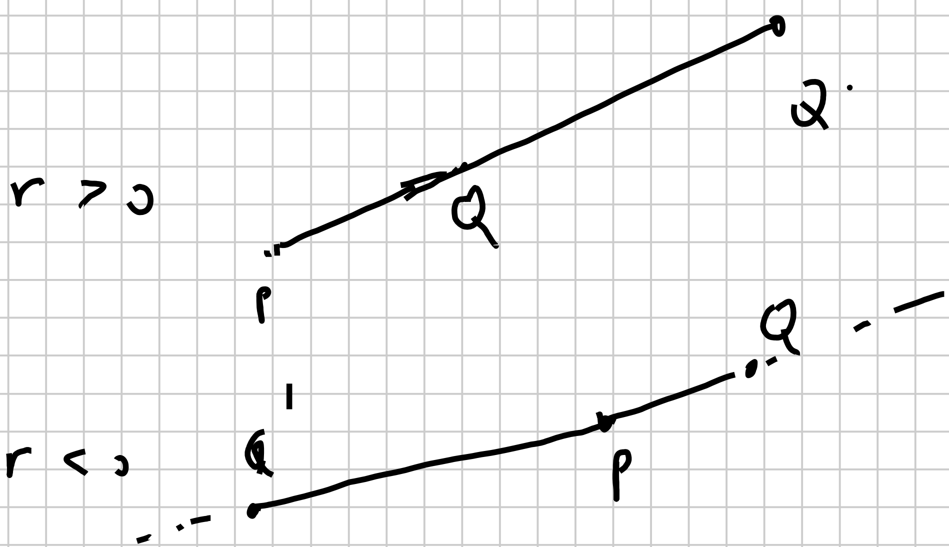
$O(P, r)$ $\quad Q \longmapsto Q'$ t.c.

P, Q, Q' sono allineati e

$\overline{PQ'} = \overline{PQ} \cdot r$

se $r > 0$ Q, Q' sono dalla stessa parte risp. a P

se $r < 0$ Q, Q' sono in part. opposte

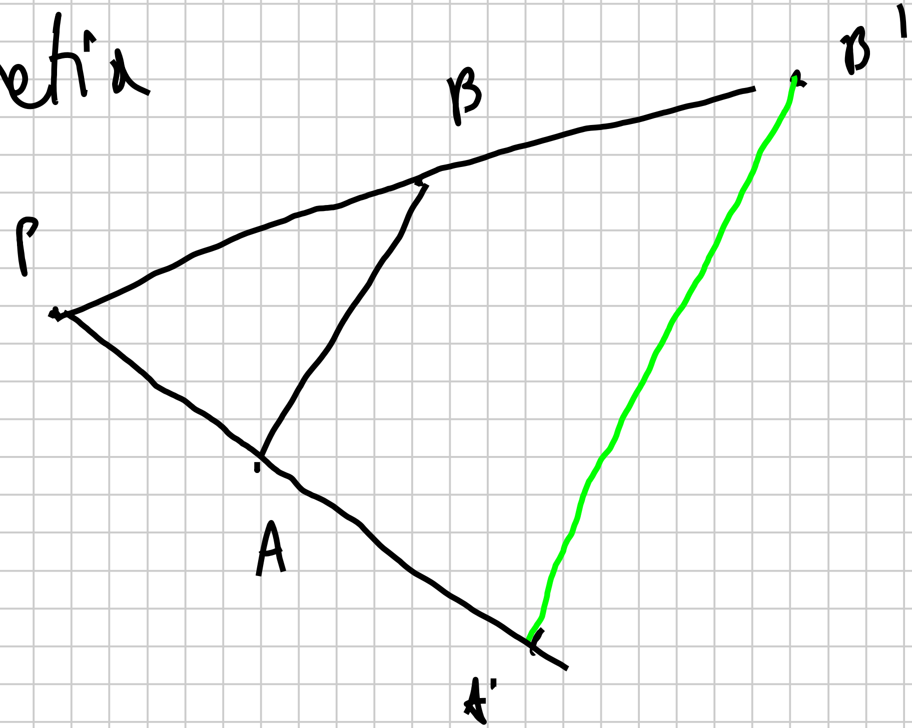


in particolare $r=1$ e' l'identità

$r=-1$ e' la simmetria centrale risp. a P

proprietà dell'omotetia

A, B $\quad$ A', B'



$$\frac{PB'}{PB} = r = \frac{PA'}{PA} \qquad AB \parallel A'B'$$

① segmenti vanno in segmenti paralleli a se stessi.

② $\overline{A'B'} = |r| \, \overline{AB}$

$\Rightarrow \Gamma$: circonferenza di centro O e raggio s

va in una circonf Γ' di centro O' e raggio $|r|s$

$P \in \Gamma \implies PO = s$

$\implies P'O' = |r|\,PO = |r|s$

$\implies P' \in$ circ. centro O' raggio $|r|s$

(3) le omotetie preservano gli angoli

A, B, C $\qquad A\hat{B}C = A'\hat{B'}C'$

Considerando $\overset{\triangle}{ABC}$ e $\overset{\triangle}{A'B'C'}$

$$\frac{AB}{A'B'} = |r| = \frac{BC}{B'C'} = |r| = \frac{AC}{A'C'}$$

3° cri. di sim. $\Rightarrow$ $\overset{\triangle}{ABC} \sim \overset{\triangle}{A'B'C'}$

4 $\mathcal{A}$ $\mathcal{B}$

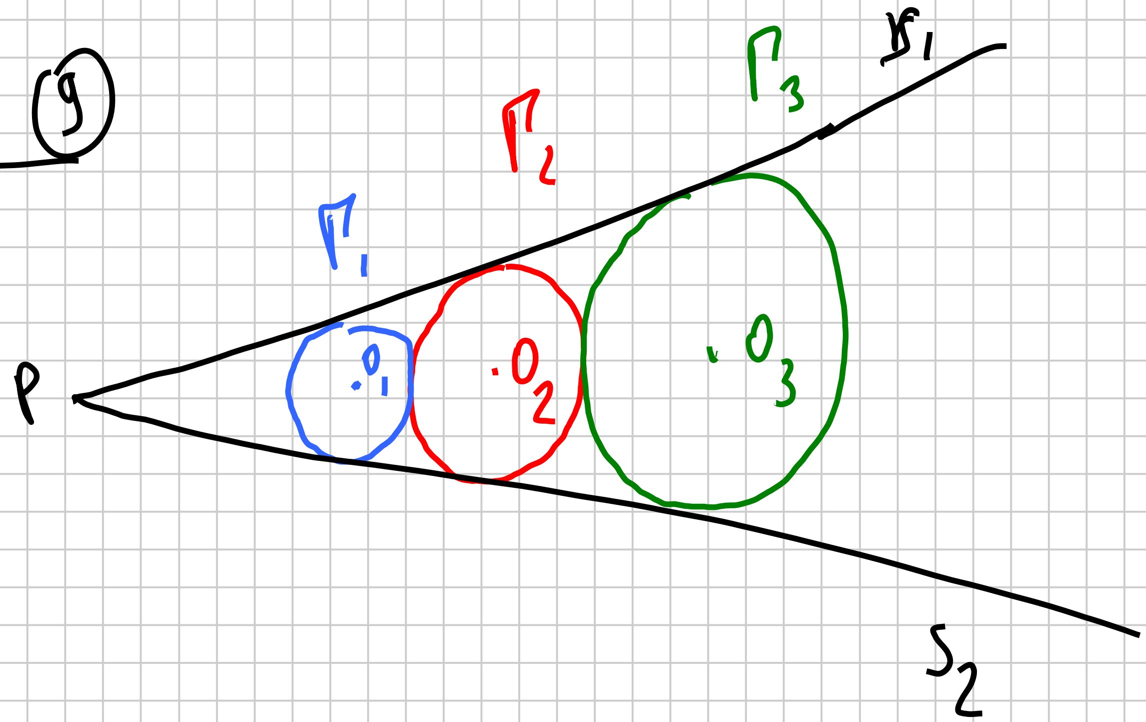
$$(\mathcal{A} \cap \mathcal{B})' = \mathcal{A}' \cap \mathcal{B}'$$

$$(\Pi_1 \cap \Pi_2)' = \Pi_1' \cap \Pi_2'$$

$$\{A\}' = \{A'\} = \Pi_1' \cap \Pi_2'$$

La tangenza si preserva.

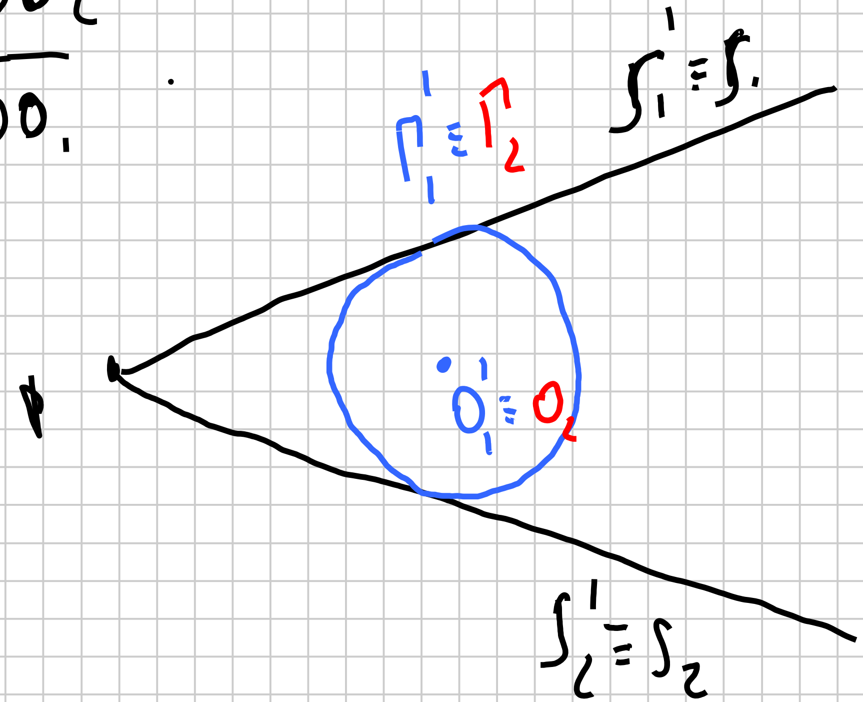
Es. 9



Prendo l'omotetia di centro P di ragione r di modo che: $O_1' = O_2$

$$OO_1' = r\, OO_1$$
$$\| $$
$$OO_2$$

$$r = \frac{OO_2}{OO_1}.$$



Γ_1 tange s_1 $\longrightarrow$ Γ_1' tange s_1'

Γ_1 tange s_2 $\longrightarrow$ Γ_1' tange s_2'

Γ_1' è la cfr. che ha centro in O_1' e tange s_1' e s_2'

Γ_1' è la cfr. che ha centro in O_2 e tange s_1 e s_2

Γ_2 tange Γ_1 $\longrightarrow$ Γ_2' tange $\Gamma_1' \equiv \Gamma_2$

Γ_2 tange s_1 $\longrightarrow$ Γ_2' tange S_1

Γ_2 tange s_2 $\longrightarrow$ Γ_2' tange S_2

$\Gamma_2' = \Gamma_3$

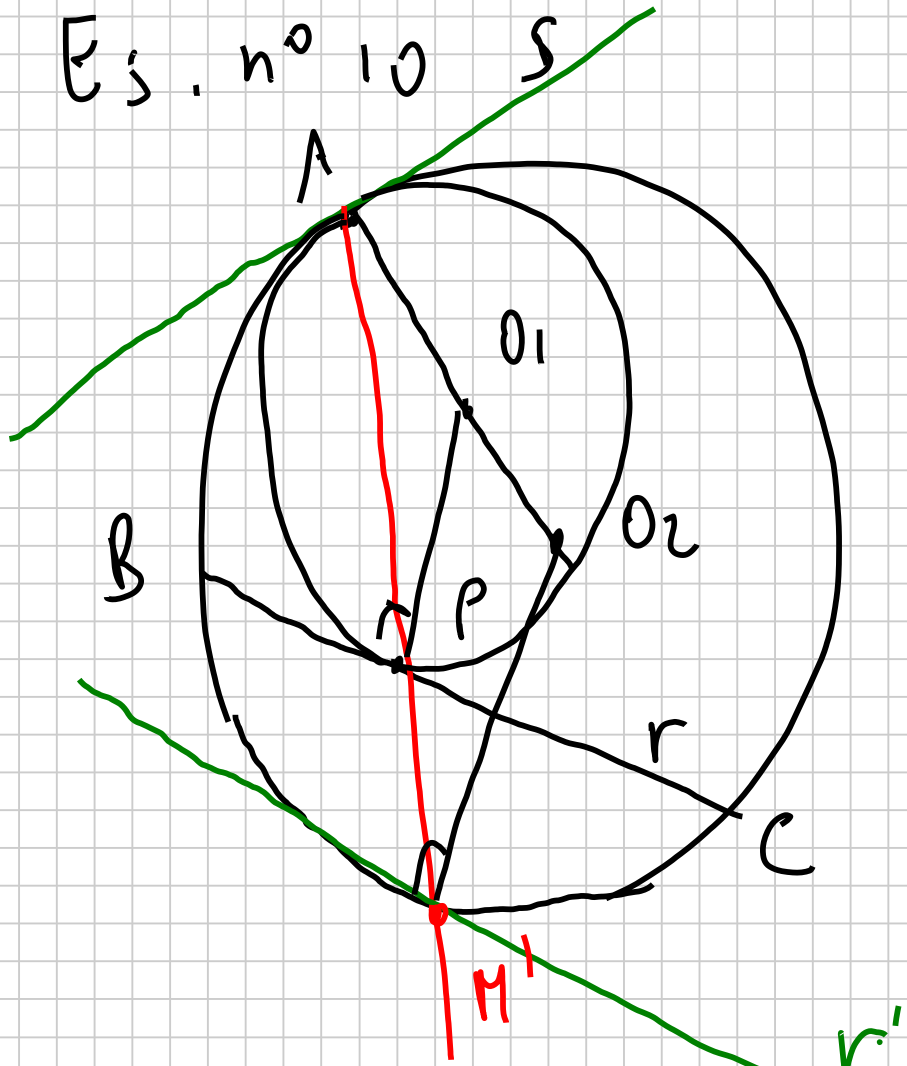
O_2 va in O_2', come da $\Gamma_2' = \Gamma_3$

$$O_2' \parallel O_3$$

$$pO_3 = pO_2' = |r|\, pO_2 = \frac{pO_2}{pO_1} \cdot pO_2$$

$$pO_3 \cdot pO_1 = pO_2^2$$

Es. n° 10



Nuova ip: M', P, A allineati
Nuova Tesi: $M'B = M'C$

Omotetia in A che manda O_1 in O_2

$s \to s$ $\quad \Gamma_1'$

$\rightsquigarrow \quad \Gamma_1 \to \Gamma_2$

Il punto P andrà in un punto sulla retta AP

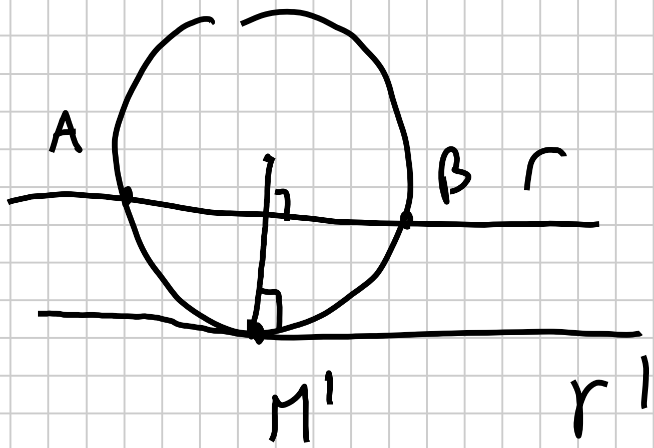
$P \in \Gamma_1 \rightsquigarrow P' \in \Gamma_1' = \Gamma_2$

$\rightsquigarrow$ ovviamente $P' = M'$

r tangente a Γ_1 in P

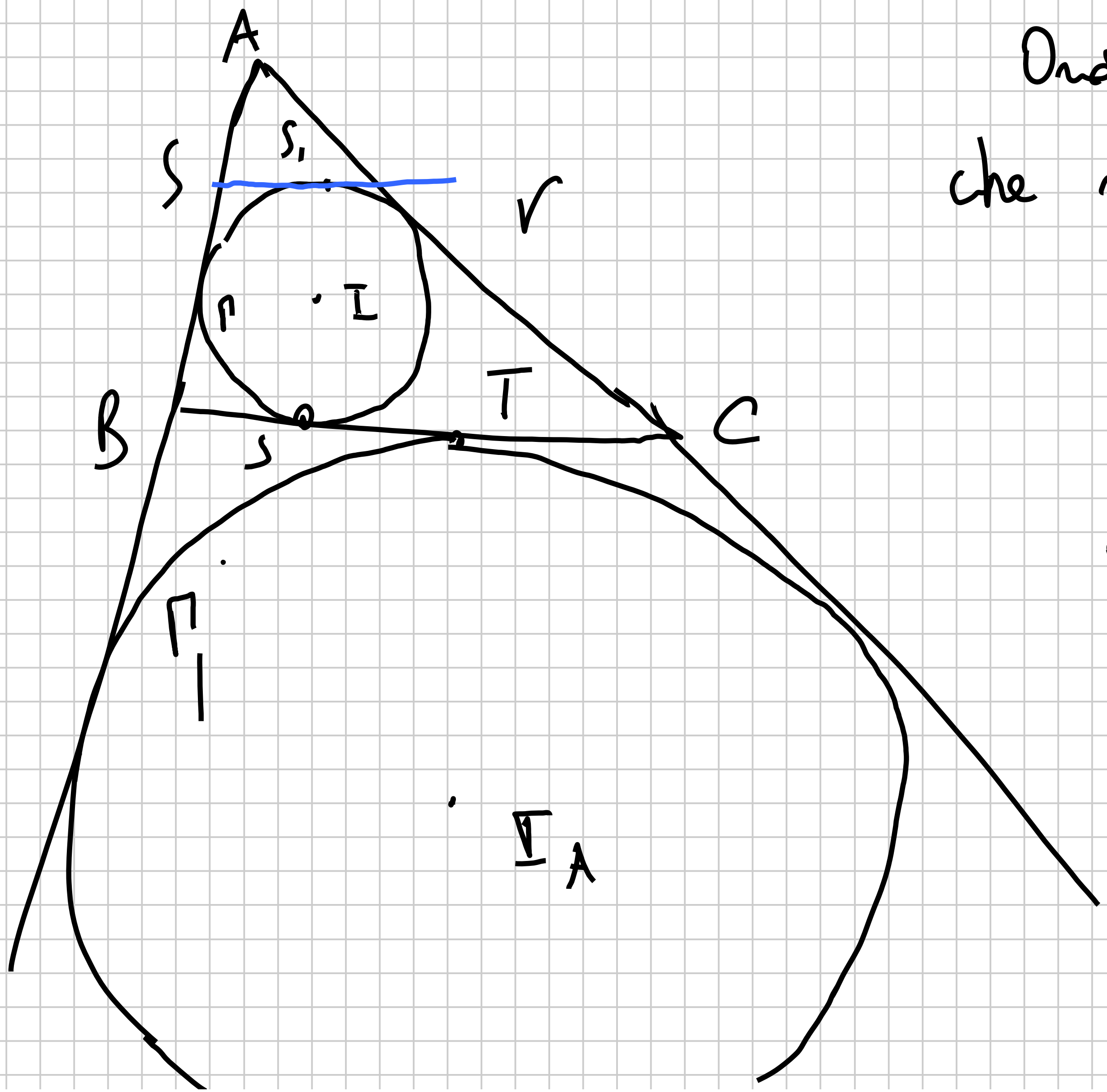
r' tangente a $\Gamma_1' = \Gamma_2$ in $P' = M'$

$r \parallel r'$



$\Rightarrow$ M' e' punto medio dell'arco $\widehat{AB}$

$M' \equiv M$

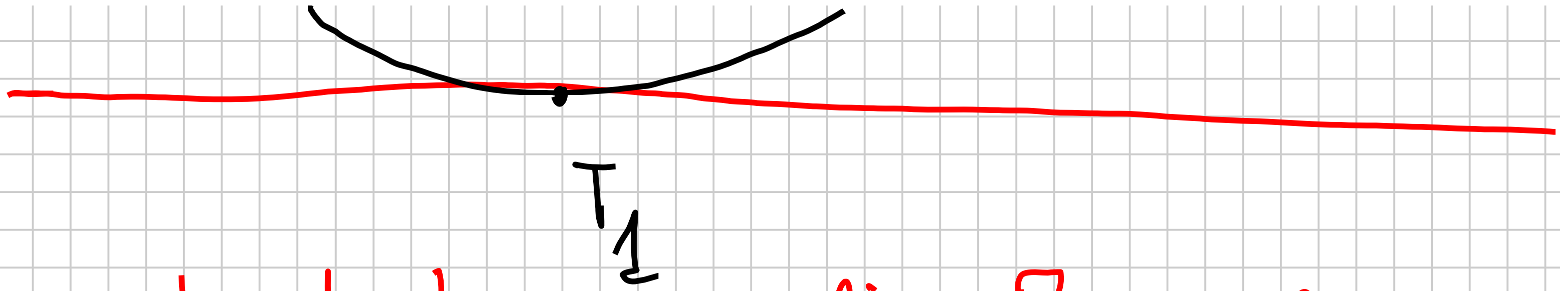


Omot. di centro A che manda I in I_A

$s \longmapsto s$

$r \longmapsto r$

$\Gamma \longmapsto \Gamma_1$

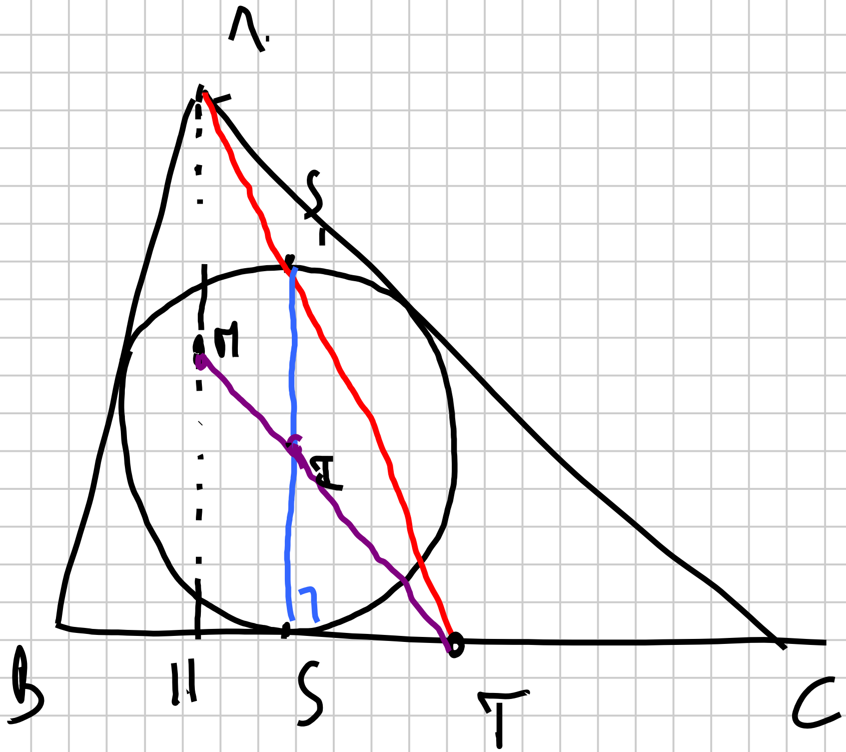


p.to di tangenza di Γ a BC = S

p.to di tangenza di Γ_1 a $B'C'$ = T_1

$T_1 = S'$ $\Rightarrow$ A, T_1, J sono allineati

A, T, S_1 sono allineati



Omotetia di centro T che manda S_1 in A.

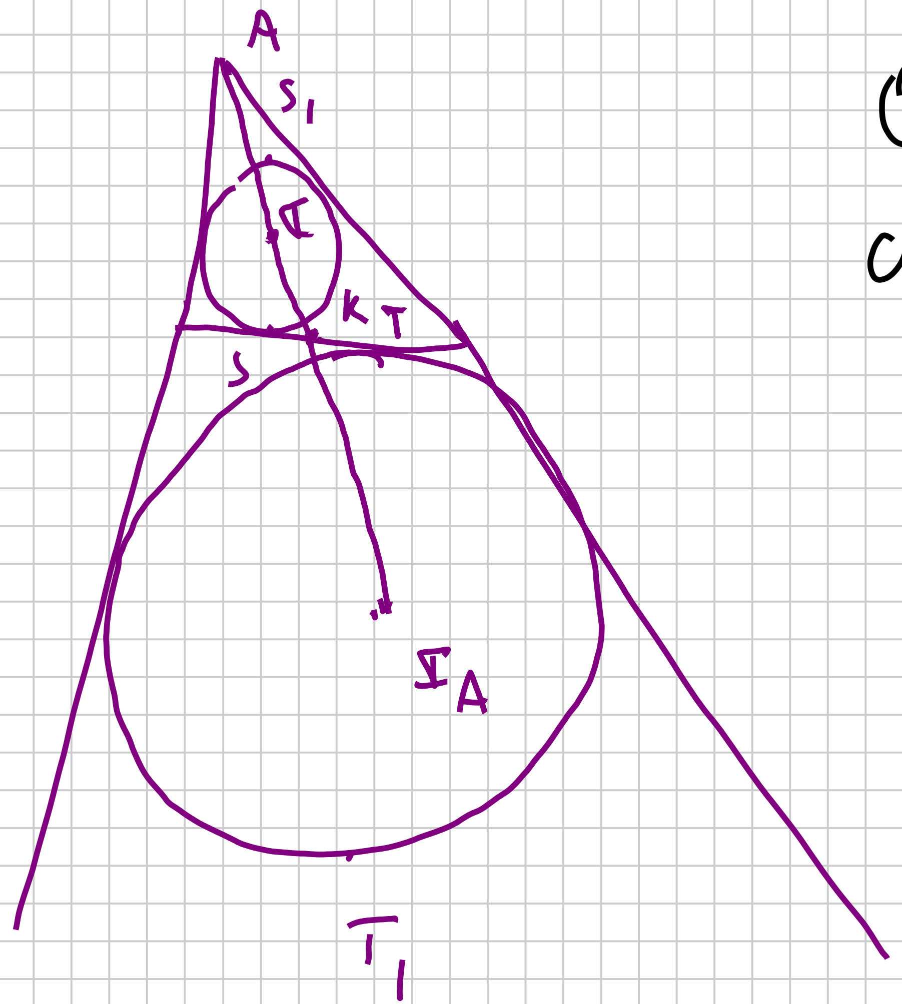
Dove va a finire S_1S?

$BC \longrightarrow BC$

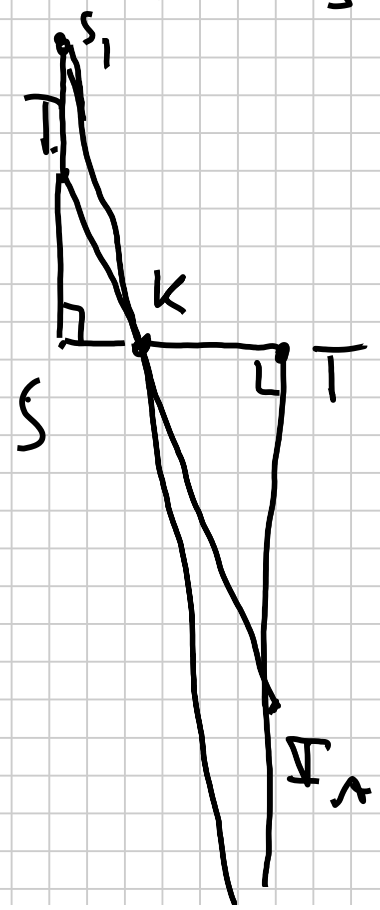
$S_1S \perp BC \Longrightarrow S_1'S' \perp BC$

$AS' \perp BC$

$S' \in AH$
$S' \in BC$ $\Rightarrow$ $S' \equiv H$



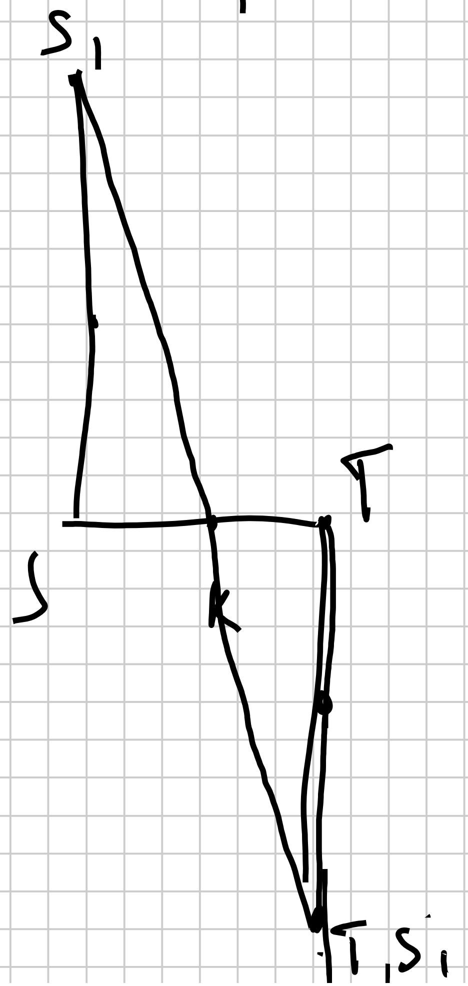
Omotetia di centro K
che manda S in T



$$\frac{IK}{I_A K} = \frac{TK}{KS} = r$$

T_1

$\Gamma \longrightarrow \Gamma_1$



$$\frac{TT_1}{SS_1} = \frac{I_1 T}{I S} = r$$

$$TT_1 = 2 I_1 T$$

$$SS_1 = 2 I S$$

S, K, F collinear.

$\square$