

$$\underline{2} = \frac{1}{A_C} + \frac{1}{A_D}$$

$$A_B = \frac{2}{\frac{1}{A_C} + \frac{1}{A_D}}$$

A_C, A_B, A_D

prop. armonica

$$\frac{1}{A_C}, \frac{1}{A_B}, \frac{1}{A_D}$$



$$(A, B, C, D) = -1$$

$$\frac{AC}{CB} = -\frac{AD}{DB} \iff$$

$$AC \cdot DB + AD \cdot CB = 0$$

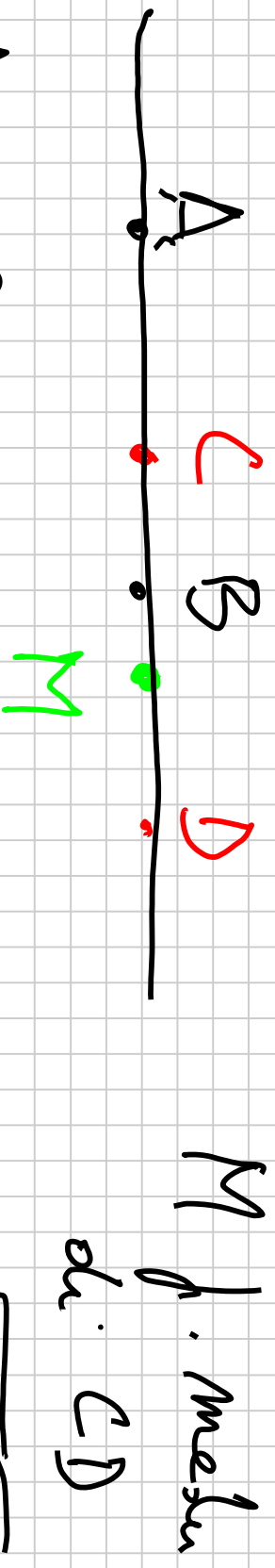
$$(c-a)(b-a) + (d-a)(b-c) = 0 \iff$$

$$c(b-a) + d(b-c) = 0 \iff$$

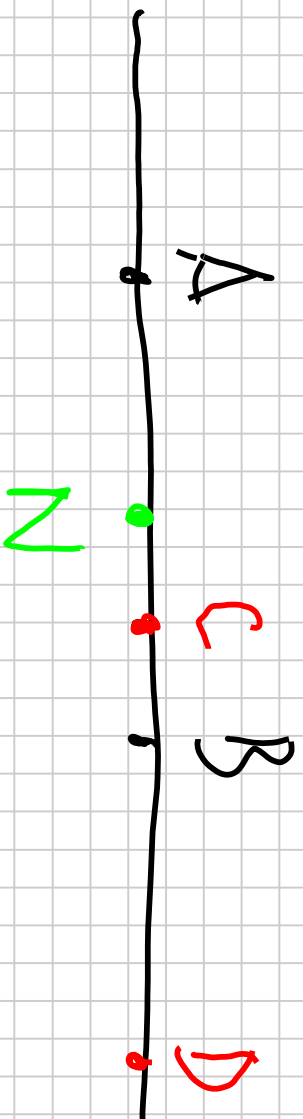
$$cb - cd + bd - cd = 0 \Leftrightarrow$$

$$2cd = bc + bd \Leftrightarrow$$

$$\frac{2}{b-a} = \frac{1}{d-a} + \frac{1}{c-a} \Leftrightarrow \frac{2}{AB} = \frac{1}{AC} + \frac{1}{AD}$$



$$MA \cdot MB = MC^2 \Leftrightarrow MC = \sqrt{MA \cdot MB}$$



Relazione di Mac Lennan

$$(A, B, C, D) = -1 \Leftrightarrow$$

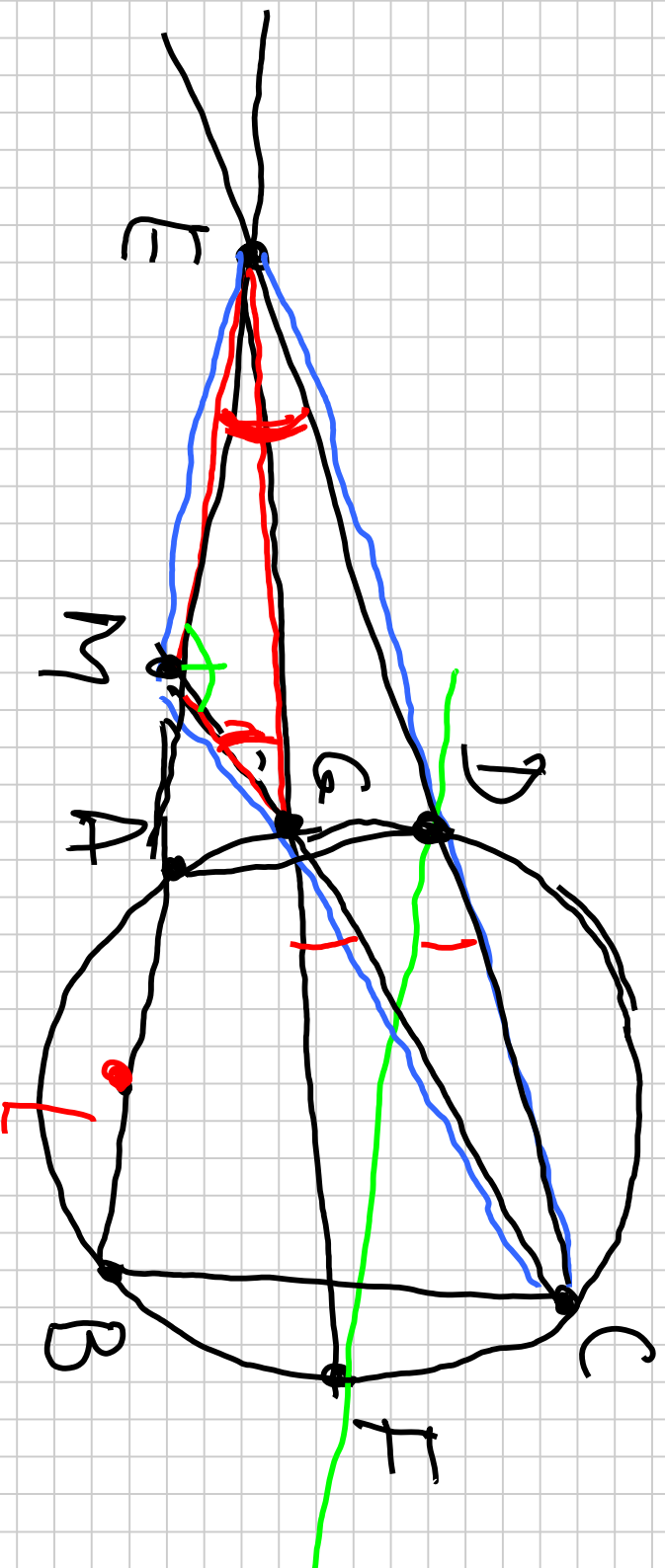
$$CA \cdot CB \neq CD \cdot CN$$

N h.t.
medio h.

$A B$

Sia $ABCD$ un quadrilatero convesso inscritto in un cerchio γ ,
 sia $E = AB \cap CD$, sia $F \in \gamma$ con $DF \parallel AB$, sia $G = \gamma \cap EF$,
 siam $M = AB \cap CG$. Dimostrare che

$$\frac{1}{EM} = \frac{1}{EA} + \frac{1}{EB}$$



$$\frac{E_M}{M_G} = \frac{M_C}{E_M}$$

$$\Leftrightarrow E_M^2 = M_C \cdot M_G$$

$$ME^2 = M_A \cdot M_B \Rightarrow \text{Newton.}$$

$$(E_L, AB) = -1$$

Per la relaz. di Contorno

$$\frac{1}{2} = \frac{1}{E_A} + \frac{1}{E_B}$$

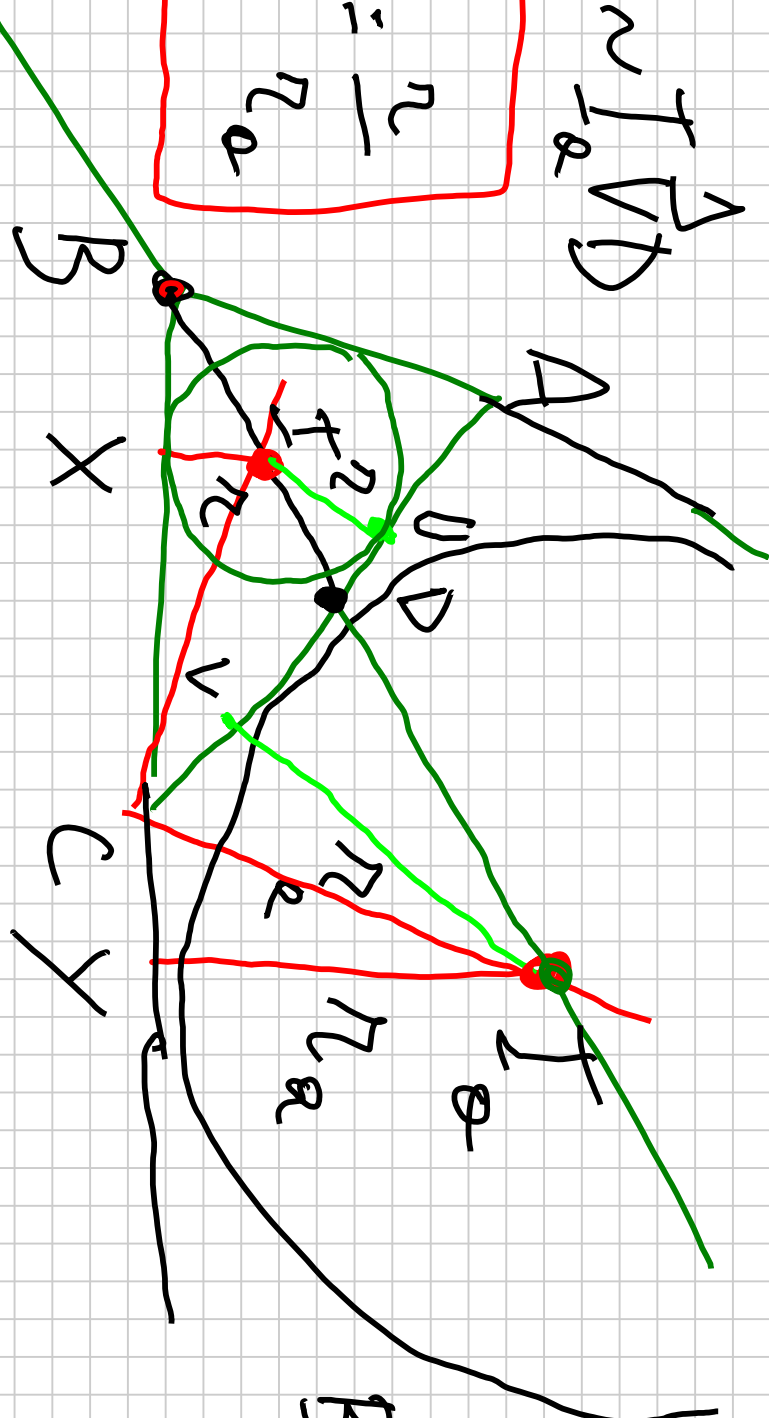
$$2 \cdot E_M \rightarrow \textcircled{E_L}$$

$$E_A \cdot E_B$$



$$\vec{I} \vec{U} \vec{D} \sim \vec{I}_a \vec{V} \vec{D}$$

$$\frac{\vec{I} \vec{D}}{\vec{D} \vec{I}_a} = \frac{\pi}{\pi_a}$$



$$\vec{B} \vec{I} \vec{X} \sim \vec{B} \vec{I}_a \vec{Y}$$

$$(B_D, \vec{I}_a) = -1 \Leftrightarrow$$

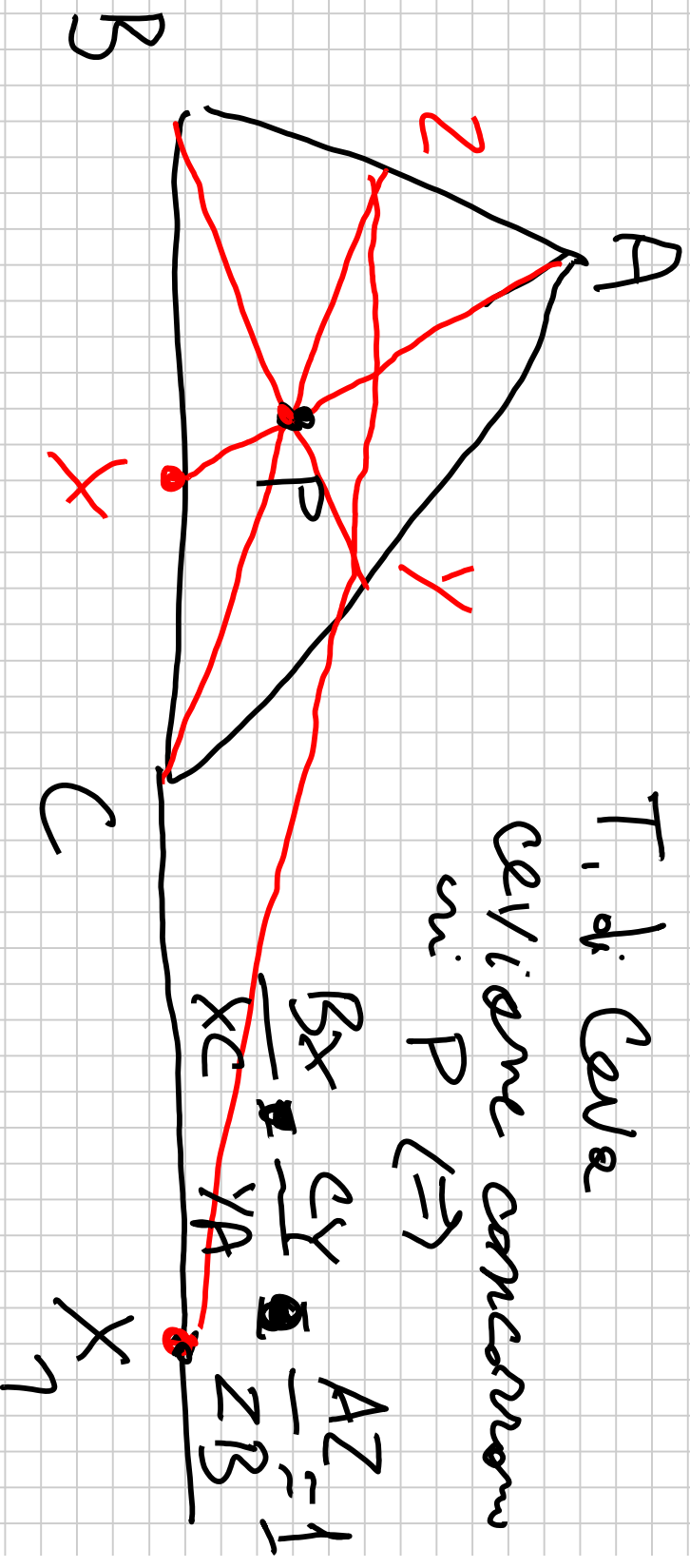
$$\frac{\vec{B} \vec{I}}{\vec{B} \vec{I}_a} = \frac{\pi}{\pi_a}$$

$$\frac{\vec{B} \vec{I}}{\vec{I} \vec{D}} = \frac{\vec{B} \vec{I}_a}{\vec{I}_a \vec{D}}$$

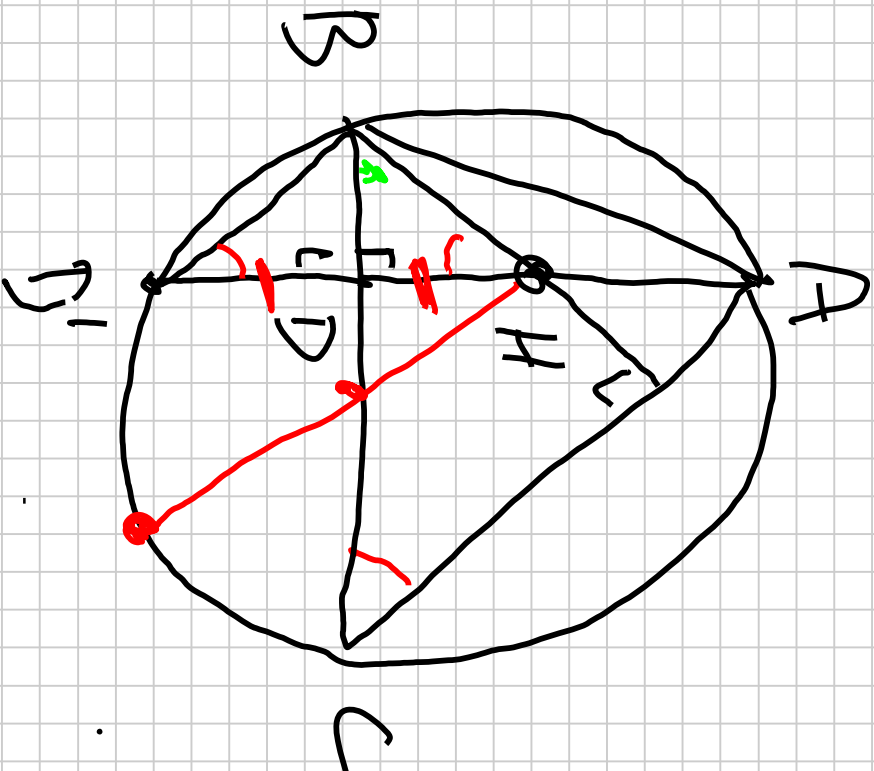
$$\frac{I_D}{DT_a} = \frac{BI}{BT_a}$$

$(\Rightarrow)$

$$\frac{BT}{I_D} = \frac{BT_a}{DT_a} = \frac{BT}{I_a D}$$



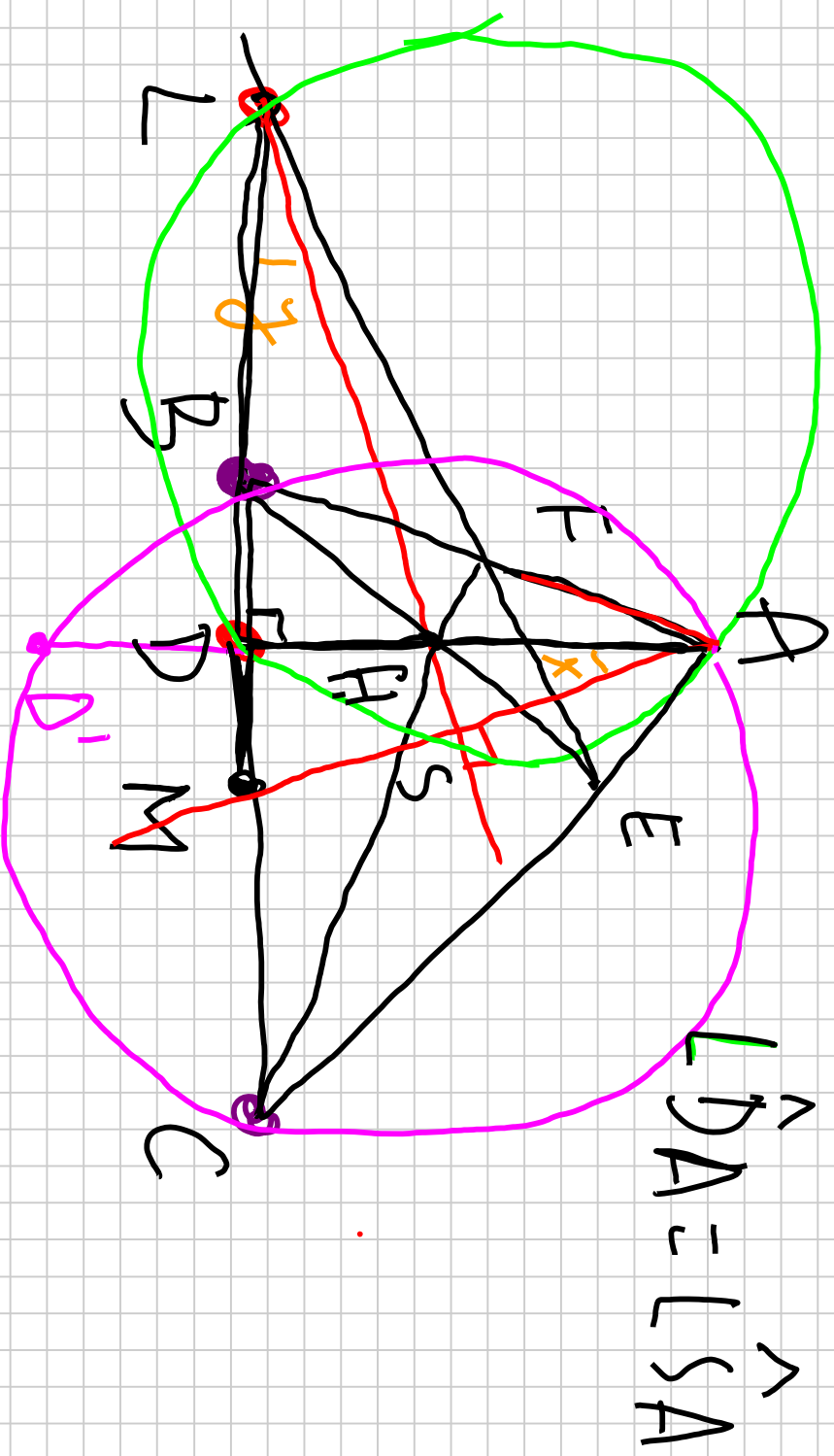
Lemma dell'ortocentro



$$TO: HD = OD'$$

$$\triangle BHD \cong \triangle CD'D'$$

Sia ABC un triangolo, sia M il punto medio di BC e siano D , E , F i piedi delle altezze relative ai lati BC , CA , AB . Se H è l'ortocentro di ABC ed $L = EF \cap BC$ dimostrare che $AM \perp LH$.



$$(L_D, B_C) = -1$$

Relazione di Mac Laurin

$$DB \cdot DC = DL \cdot DM$$

(1)

Teorema delle corde.

$$BD \cdot DC = AD \cdot DD'$$

$$BD \cdot DC = AD \cdot HD$$

(2)

$$DB \cdot DC = AD \cdot DH$$

$$DL \cdot DM = AD \cdot DH \Rightarrow$$

$$\frac{AD}{DM} = \frac{DL}{DH} \Rightarrow \cot x = \cot y$$

$$\Rightarrow x = y$$