

SUCCESSIONI PER RICORRENZA

$$a_0 \ a_1 \ a_2 \ \dots \ a_n \ \dots$$

1. ARITMETICA

$$a_n = a_{n-1} + h$$
$$a_0$$

→ formula chiusa : $a_n = a_0 + nh$

2. GEOMETRICA

$$a_n = r a_{n-1} ; a_0$$

3. PERMUTAZIONI

$$a_n = n a_{n-1}$$

$$a_0 = 1$$

4. FIBONACCI

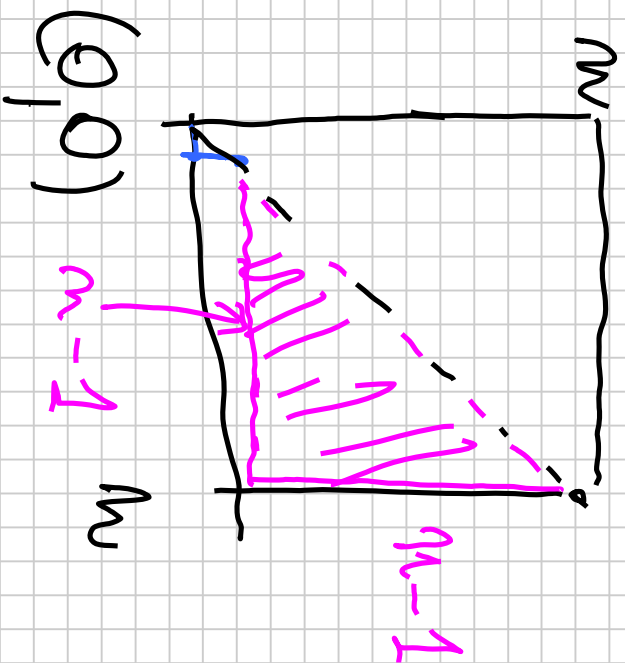
$$F_n = F_{n-1} + F_{n-2}$$

$$F_0 = F_1 = 1$$

5.

$$a_n = \frac{a_{n+1} + a_{n-1}}{2}$$

6. CATALAN



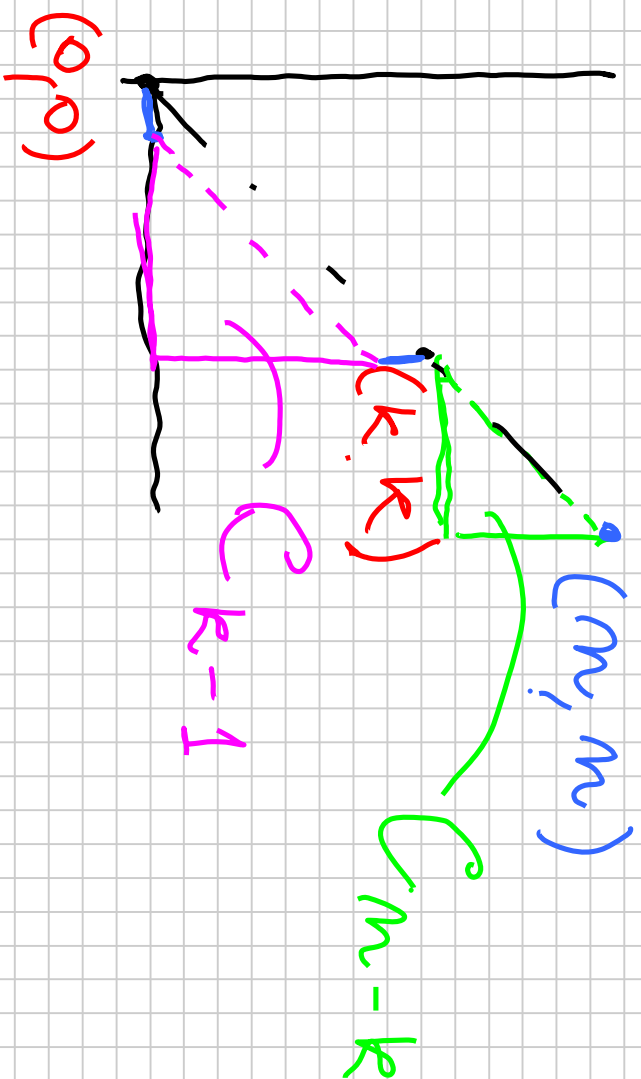
$$C_n = \binom{2n}{n} \frac{1}{n+1}$$

$$C_0 = 1 \quad C_1 = 1$$

$$C_n = C_{n-1} C_0 +$$

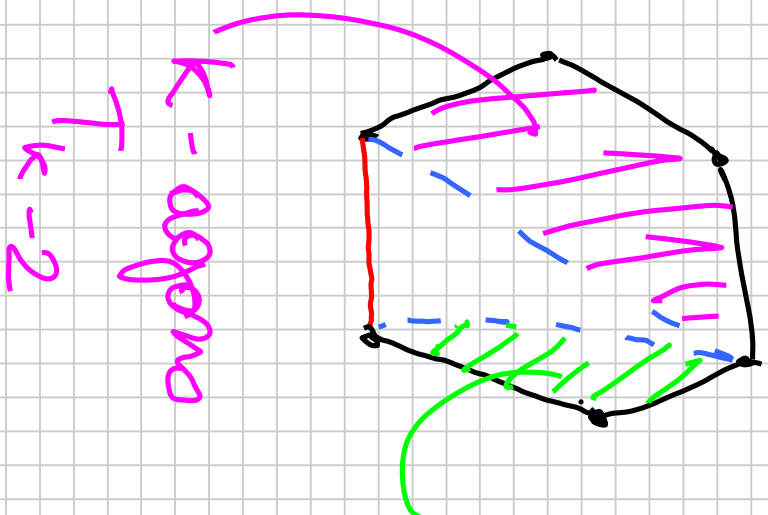
$$C_{n-2} C_1 +$$

$$C_{n-3} C_2 + \dots + C_0 C_{n-1}$$



$$\rightarrow C_{k-1} \cdot C_{n-k}$$

modi di vibrazione $n+2$ -agons ($= T_n$)



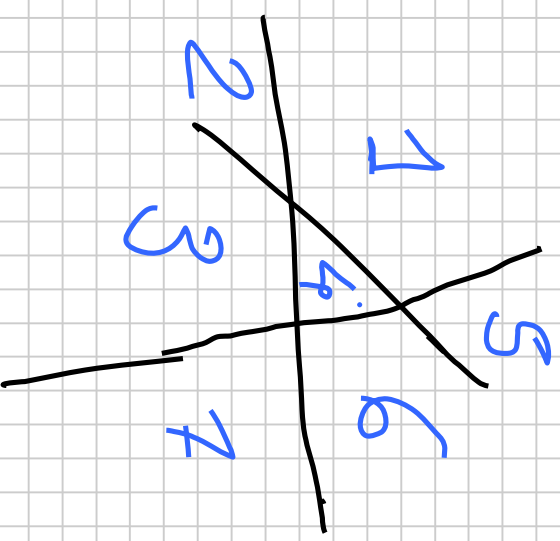
T_{n-k+1}
 $(n+2-k+1)$ -agons

$$T_n = T_0 T_{n-1} + T_1 T_{n-2} + \dots + T_{n-1} T_0$$

7 n rette nel piano
max no di regioni = ?

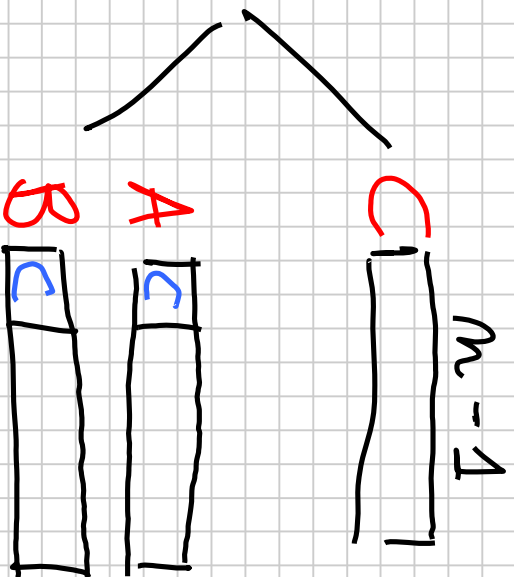
$$P_n = P_{n-1} + n$$

$$P_0 = 1$$



8. Stensione di n lettere in A B C che
non abbiamo AA o BB o AB o BA

$$S_n$$

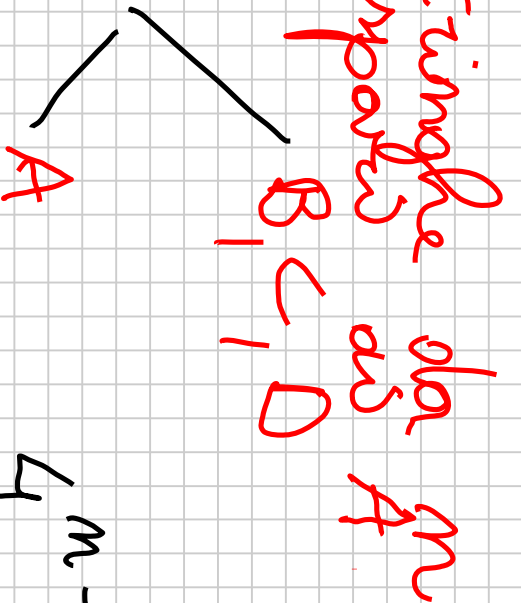


$$S_{n-1}$$

$$2S_{n-2}$$

$$S_n = S_{n-1} + 2S_{n-2}$$

$$S_0 = 1 \quad S_1 = 3$$



$$3Q_{n-1}$$

$$Q_{n-1} - Q_{n-1}$$

$$Q_n = 2Q_{n-1} + Q_{n-2}$$

$$Q_0 = 0$$

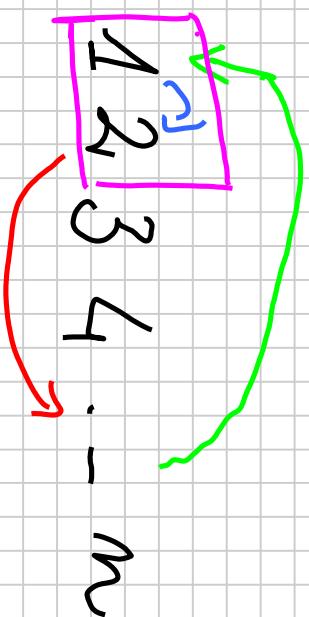
g. Strings of length n in A, B, C, D can be formed

10. Derangements (permutácie bez žiadnej fixnej pozície) D_n

1 2 ... n

2 → 1

D_{n-2}



D_{n-1}

$$D_n = (n-1)(D_{n-2} + D_{n-1})$$

$$D_2 = 1$$

$$D_1 = 0$$

iii. formula above ??

$$7 \quad p_n = p_{n-1} + n \quad p_0 = 1$$

$$\begin{aligned} p_n &= 1 + (1 + \dots + n) = \\ &= 1 + \frac{(n+1)n}{2} \end{aligned}$$

$$9. \quad Q_n = 2Q_{n-1} + 4^{n-1}$$

$$Q_n = 2^n q_n$$

$$\rightarrow \cancel{2^n} q_n = \cancel{2^n} q_{n-1} + \frac{2^n}{4}$$

$$q_n = q_0 + \sum_{i=1}^n \frac{4}{2^i} = \frac{1}{4} \sum_{i=1}^n 2^i$$

$$= 0$$

$$\leftarrow 2^{i-2}$$

$$=$$

$$\rightarrow Q_n = 2^{n-2} (2^{n+1} - 1)$$

$$\frac{1}{4} (2^{n+1} - 1)$$

8. $S_n = S_{n-1} + 2S_{n-2}$

$$\lambda^n = \lambda^{n-1} + 2\lambda^{n-2}$$

polynomialo
korotsheniye

$$\lambda^2 = \lambda + 2$$

$$2^n, (-1)^n$$

$$A 2^n + B (-1)^n$$

→ kroyng
konstonty
nizhnyos

$$A + B = S_0$$

$$2A - B = S_1$$

$$\frac{1 \pm \sqrt{9}}{2}$$

2

-1

5.

$$a_n = 2a_{n-1} - a_{n-2}$$

$$\hookrightarrow \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2$$

$$I_n \quad | \quad n I_n$$

$$\textcolor{red}{A} I_n + \textcolor{green}{B} n I_n$$

$$\begin{cases} A = a_0 \\ A + B = a_1 \end{cases}$$

$$\rightarrow \lambda^n + a\lambda^{n-1} + b\lambda^{n-2}$$

$$\lambda^n + a(n-1)\lambda^{n-1} + b(n-2)\lambda^{n-2}$$

$\underbrace{\hspace{1.5cm}}_{b_{n-1}}$
 $\underbrace{\hspace{1.5cm}}_{b_{n-2}}$

b_n
 b_{n-1}
 b_{n-2}

→ Ricorrenza lineare a n termini

→ caso non omogeneo

10.

$$D_n = (n-1)(D_{n-1} + D_{n-2})$$

$$\begin{aligned} & \hookrightarrow D_n - nD_{n-1} = -D_{n-1} + (n-1)D_{n-2} \\ & \parallel H_n \end{aligned}$$

$$= (-1)(D_{n-1} - (n-1)D_{n-2})$$

$$= (-1)H_{n-1}$$

$$D_n - nD_{n-1} = (-1)^n$$

