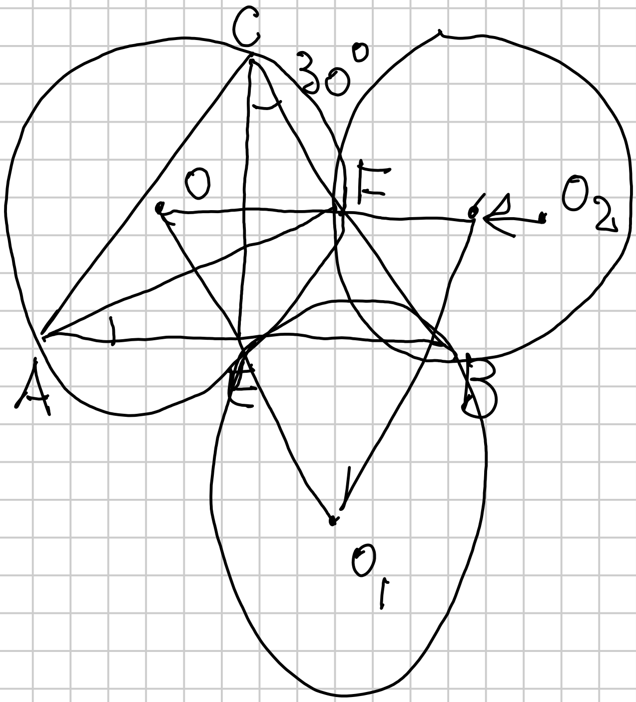
CORREZ. ESERCIZI GEOMETRIA

Titolo nota 06/12/2009



$\triangle OO_1O_2$ equil.

$C\hat{B}A = 60°$

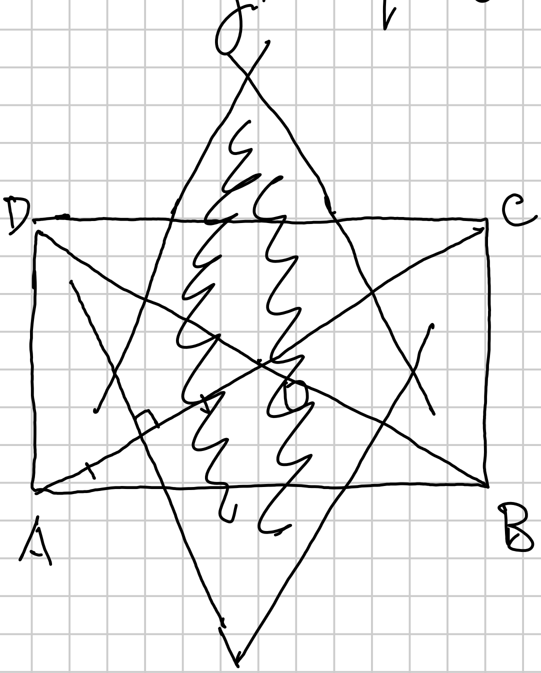
$OF = FO_2$

$OE = OF = O_1E$

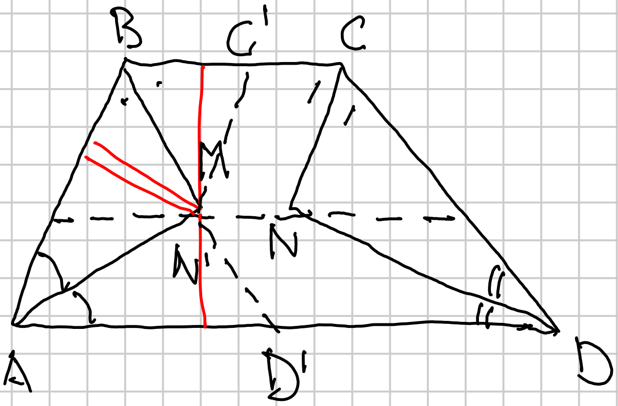
$O_1B = O_2B$

$O_1\hat{O}O_2 = 60°$

triang. equil.: OEF, O_1EB, O_2FB EFB



$AM + BM + CM + DM \geq OM$



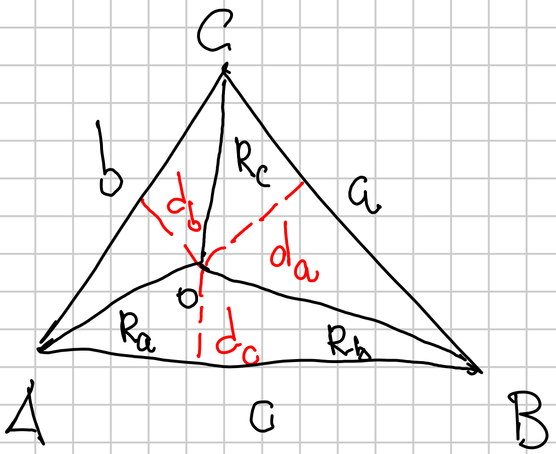
$2MN = |AB + CD - (AD + BC)|$

N traslato su M

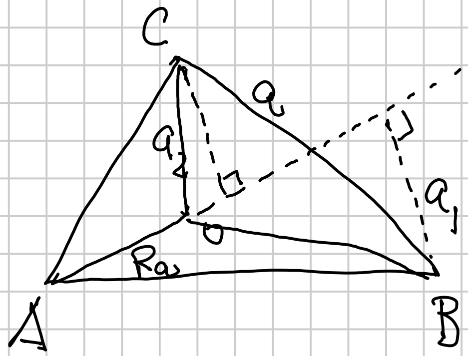
$N' = M$

$$2MN' = 0 = |AB + C'D' - (AD' + BC')|$$

$$2MN = |AB + CD - (AD + BC)|$$



$$aR_a \geq b d_b + c d_c$$



$$R_a(a_1 + a_2) \leq a R_a$$

$$R_a a_2 = 2S_{ACO} = b d_b$$

$$R_a a_1 = 2S_{ABO} = c d_c$$

$$a \geq b \geq c, \quad d_a \geq d_b \geq d_c$$

$$aR_a \geq b d_b + c d_c \geq$$

$$\geq c d_b + b d_c$$

$$\rightarrow bR_b \geq a d_c + c d_a \qquad cR_c \geq a d_b + b d_a$$

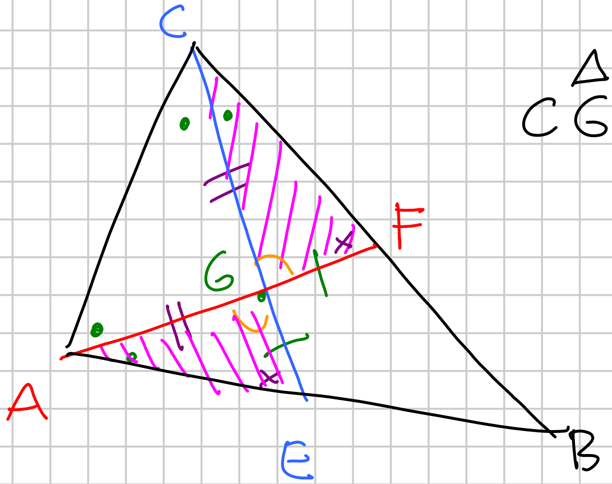
$$\begin{cases} R_a \geq \frac{b}{a} d_c + \frac{c}{a} d_b \\ R_b \geq \frac{a}{b} d_c + \frac{c}{b} d_a \\ R_c \geq \frac{a}{c} d_b + \frac{b}{c} d_a \end{cases}$$

$$R_a + R_b + R_c \geq$$

$$\geq \left(\frac{b}{a} + \frac{a}{b}\right) d_c + \left(\frac{b}{c} + \frac{c}{b}\right) d_a + \left(\frac{c}{a} + \frac{a}{c}\right) d_b \geq$$

$$\geq 2(d_c + d_a + d_b)$$

$$\cdot\ x, y > 0 \Rightarrow \frac{x}{y} + \frac{y}{x} \geq 2$$



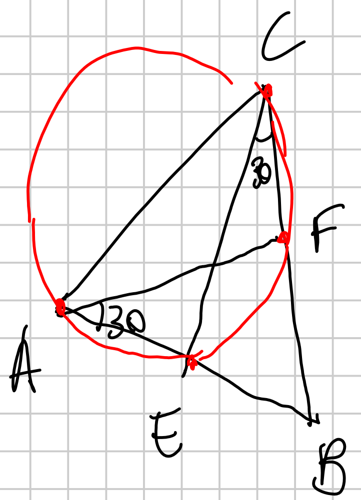
$\overset{\triangle}{CGF}$ è simile ad $\overset{\triangle}{AGE}$

$$\frac{AG}{GE} = \frac{CG}{GF} \qquad \frac{\cancel{2}GF}{GE} = \frac{\cancel{2}GE}{GF}$$

$$CG = 2GE \qquad GE = GF$$
$$AG = 2GF$$

$$CG = 2GF \qquad C\hat{F}G = 90^\circ \qquad C\hat{G}F = 60^\circ$$

$\overset{\triangle}{CGA}$ è isoscele e $C\hat{G}A = 120^\circ$



$$pow_\Gamma(B) =$$
$$= BE \cdot AB = BE \cdot 2BE =$$
$$= 2FB^2$$

$\rightarrow BE = BF$
$AB = BC$

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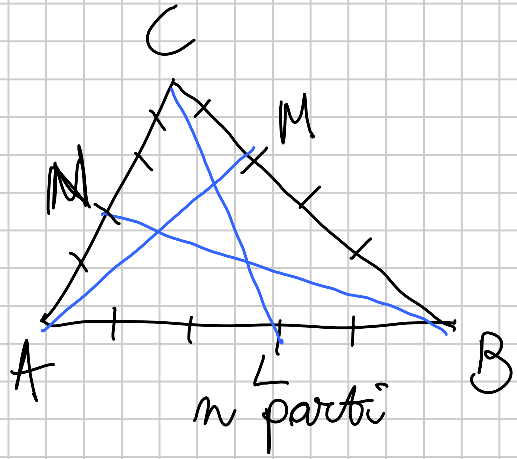
5 ...



n PRIMO > 2

$$\frac{AL}{LB}\,\frac{BM}{MC}\,\frac{CN}{NA} =$$
$$= \frac{x}{n-x}\,\frac{y}{n-y}\,\frac{z}{n-z}$$

$$xyz \overset{?}{=} (n-x)(n-y)(n-z) = n(\text{cose}) - xyz$$

$2xyz \overset{?}{=} n$ (cose)

$\begin{pmatrix} x < n \\ y < n \\ z < n \end{pmatrix}$

$n \mid 2xyz$

PRIMO

non può essere!

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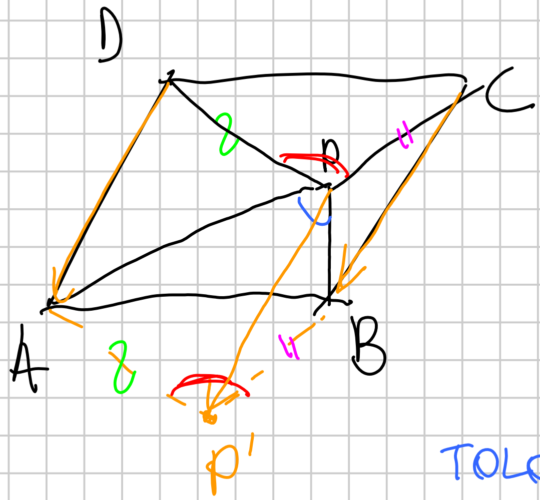
(6)



$\angle + \angle = 180°$

$AP'BP$ CICLICO

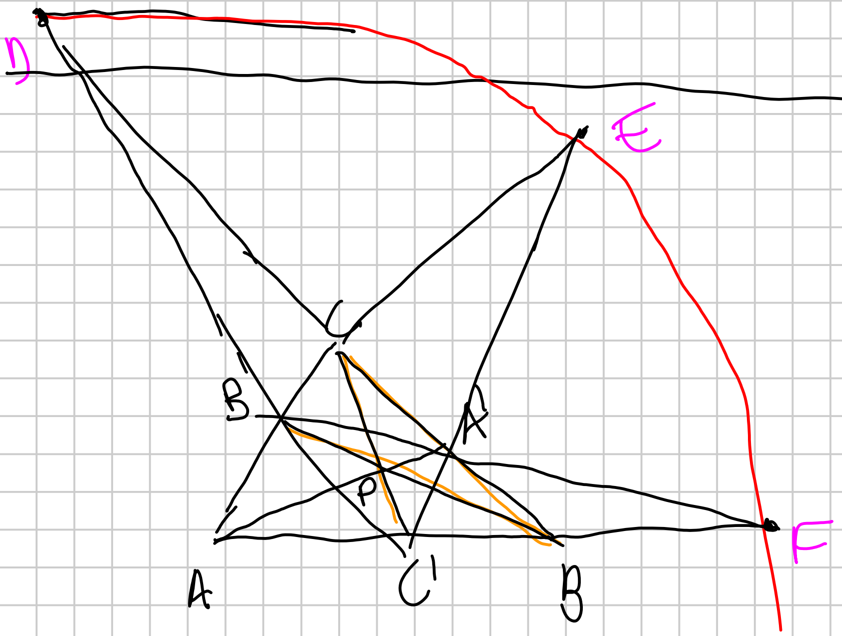
TOLOMEO

$\Downarrow$

$AP' \, BP + AP \, P'B = AB \, PP'$

$DP \, BP + AP \, PC = AB \, AD$

$\square$



- CEVA su ABC (A', B', C')

- MENELAO

ABC — F, A', B'

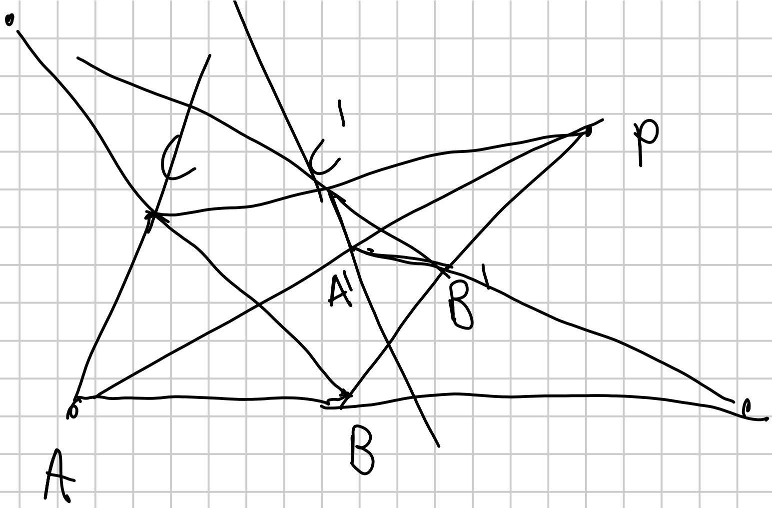
— E, A', C'

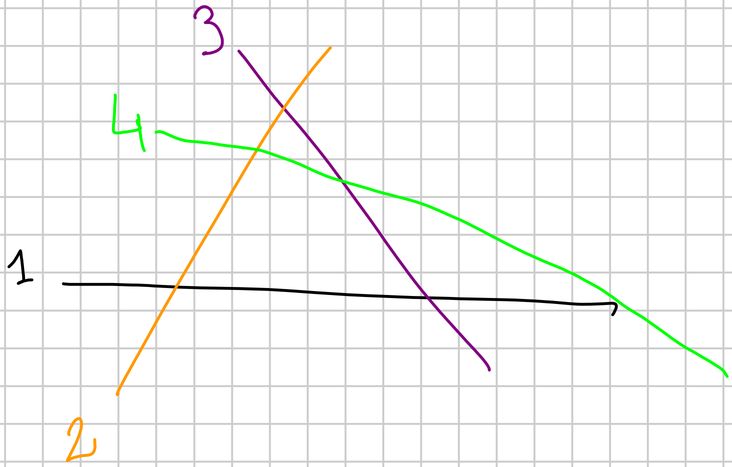
— D, B', C'

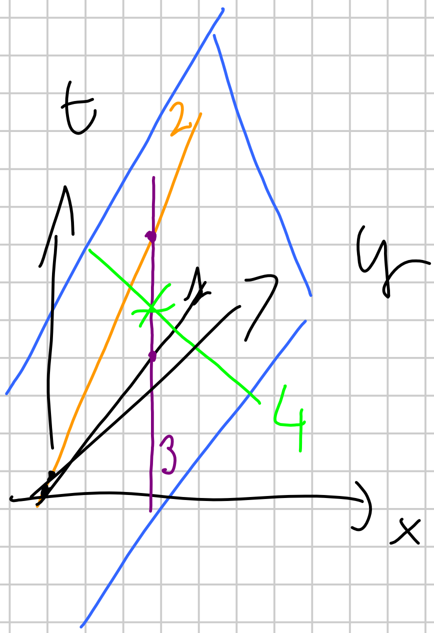
OPPURE MENELAO

triangolo	retta
B, P, C	C', B', D
C, P, A	C', A', E
A, P, B	F, A', B'

TEOREMA DI DESARGUES







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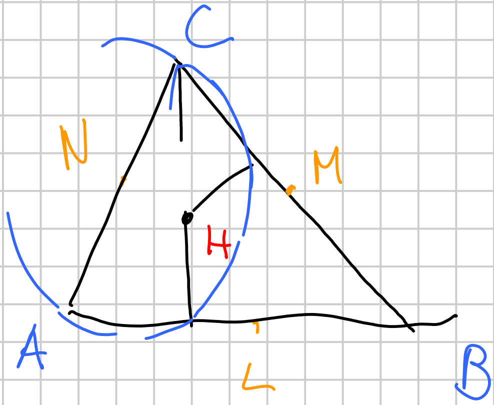


$$HL^2 + HM^2 + HN^2 < AL^2 + BM^2 + CN^2$$

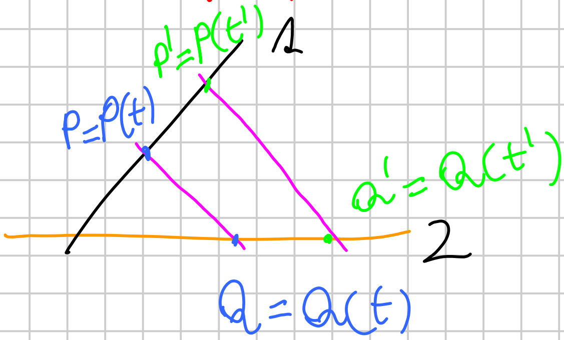
$$HN^2 - CN^2 = $$

$$= p$$

$3p < 0 \iff$ H interno! (a circonfer.)

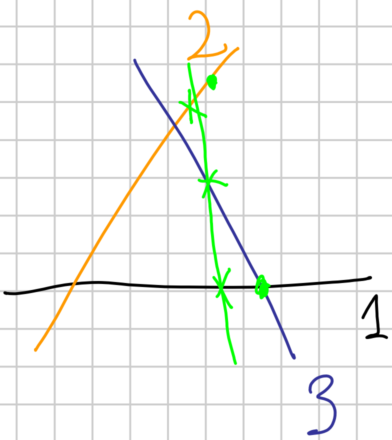
ABC acutangolo $\Longleftrightarrow$ è interno al triangolo

LEMMA 1



(TALETE)

LEMMA 2



viaggiatori 1, 2, 3 si incontrano 2 a 2

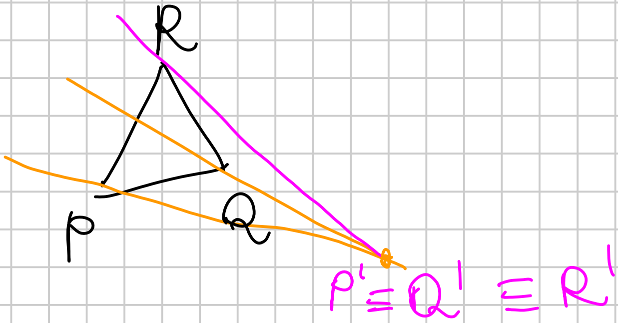
$\Longrightarrow$ 1, 2, 3 sono sempre allineati

oppure

le strade concorrono

dim

$PQ \parallel P'Q'$ $\quad PR \parallel P'R'$ $\quad RQ \parallel R'Q'$



CONCLUSIONE