Chronology violations and the origin of time

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A connected Hausdorff (paracompact) time-oriented Lorentzian four-dimensional (C^r, $r \geq 2$) manifold.

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They can be regarded as relations on M i.e. as subsets of $M \times M$

$$I^+ = \{(p, q) \in M \times M : p \ll q\}, \qquad \text{chronology relation}$$

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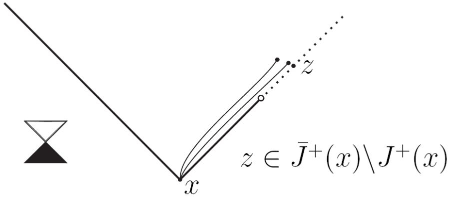
Theorem

Any causal curve which is not an achronal lightlike geodesic can be deformed to a timelike curve (keeping endpoints fixed).

Topological properties of causal relations

I^+ and J^+ are transitive. I^+ is open but J^+ and is not necessarily closed.
Remove a point from Minkowski spacetime



Main causality conditions

- **Chronology**: There are no closed timelike curves (i.e. I^+ is antisymmetric),
- **Causality**: There are no closed causal curves (i.e. J^+ is antisymmetric),
- **Stable causality**: Causality holds and is stable in a suitable interval topology on Lorentzian metrics (i.e. J_S^+ is antisymmetric),
- **Global hyperbolicity**: Causality holds and the causal diamonds $J^+(p) \cap J^-(q)$ are compact.

Chronology violating set and Carter's classes

Chronology violating set vI

$$vI = \{x : \text{ there is } y \text{ such that } x \ll y \ll x\}$$

that is, vI is made of all the points through which passes a closed timelike curve.

Carter's equivalence relation over vI

"$x \sim y$ iff $x \ll y \ll x$" is an equivalence relation over vI. The class of $x \in vI$ is denoted $[x]$ and is open.

The chronology violating set splits into classes. Two points belong to the same class iff there is a closed timelike curve passing through them.

Time functions

A function $t : M \to \mathbb{R}$ such that $x < y \Rightarrow t(x) < t(y)$ (that is, it increases over every causal curve).

Time functions, stable causality and closed and transitive relations

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J^+ is not both closed and transitive but ...

Seifert's relation $J_S^+ = \bigcap_{\{g' \text{ has wider light cones than } g\}} J_{g'}^+$

J_S^+ is closed, transitive and contains J^+. It is the smallest relation with this property.

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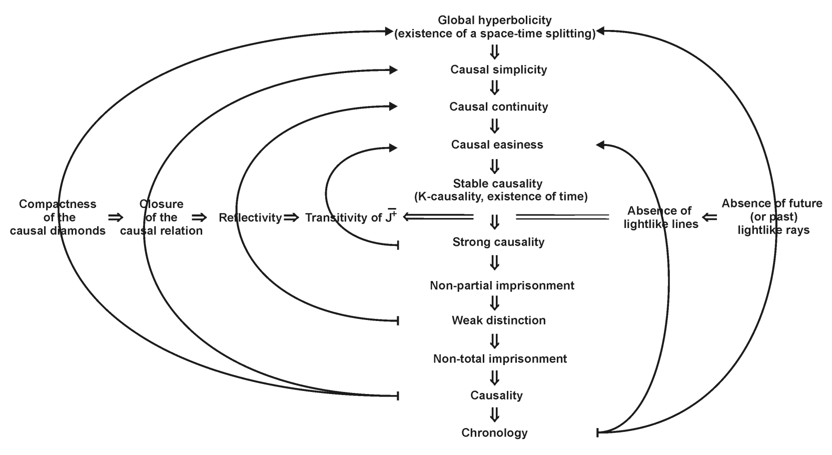
J^+_S is closed, transitive and contains J^+. It is the smallest relation with this property.

Theorem: the following are equivalent

- Stable causality,
- J^+_S is antisymmetric (partial order),
- There is a time function.



Below stable causality: levels protect against causality violation.
Above stable causality: levels protect against influence from infinity

To prove singularity theorems find lightlike lines

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Raychaudhuri equation for a beam of lightlike geodesics near $\gamma(t)$

Let $F(t) = \frac{1}{2} R(\gamma', \gamma') + \sigma^2$. The Raychaudhuri equation in Riccati form $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -2F - \frac{1}{2}\theta^2$ can be put in the form ($\theta = \frac{2}{y}\frac{\mathrm{d}y}{\mathrm{d}t}$, y is the size of the beam section)

$$\ddot{y} = -F(t) y \qquad \text{(Eq. of the elastic spring)}$$

thus if $F \geq 0$ and $F(t_0) > 0$ for some t_0, there are solution with 2 zeros (i.e. conjugate points).

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Hawking and Penrose's argument

If a *null geodesically complete* spacetime satisfies

- $R(n, n) \geq 0$ for every lightlike vector n (null energy condition),
- $n^c n^d n_{[a} R_{b]cd[e} n_{f]} \neq 0$ somewhere over every null geodesics (null genericity condition),

then every geodesic has two conjugate points, so it cannot be achronal i.e. NO LIGHTLIKE LINES.

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Role of shear

The shear satisfies

$$\frac{\mathrm{d}\sigma_{mn}}{\mathrm{d}t} = -C_{m4n4} - \theta\sigma_{mn} - \sigma_{mp}\sigma_{pn} + \delta_{mn}\sigma^2$$

thus if the condition "$F(t) > 0$ for some t" does not hold then $\sigma_{mn}(t) = 0$ for every t and hence $C_{m4n4} = 0$ over γ. That is, if the genericity condition fails, spacetime must be special indeed (rigidity theorems).

Kriele's theorem (1989)

Suppose that (M, g) satisfies the null energy condition and the null genericity condition. If the chronology violating set vI, or just a connected component of its boundary is *compact*, then M is null geodesically incomplete.

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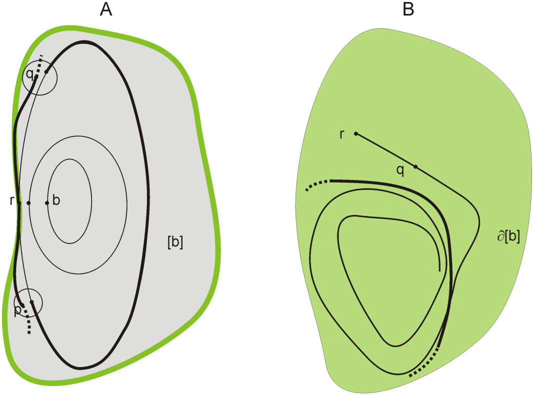
Warning: the genericity condition is applied over a compact set

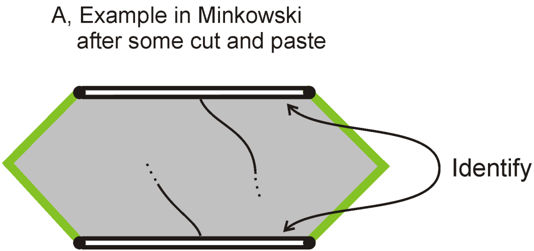
Contrary to conventional wisdom in my opinion we should not expect the null genericity condition to hold if the null incomplete geodesic is imprisoned in a compact, for in this case the geodesic does not probe arbitrarily far away regions of spacetime. Spacetime can be genuinely special there...



A: Through every point of the boundary passes a future or past lightlike ray which stays in the boundary.

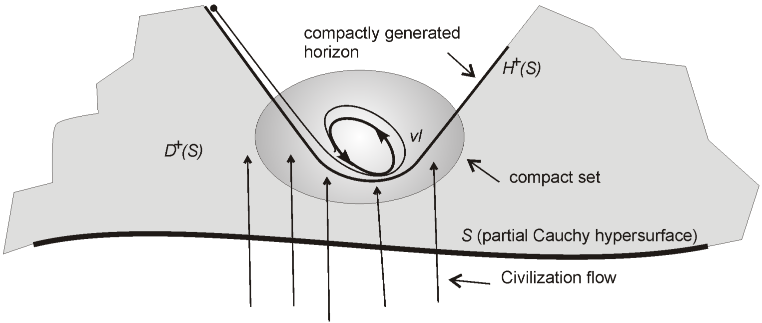
B: If the boundary is compact, it contains a lightlike line.

Example



Hawking's theorem (1992)

Let (M, g) be a spacetime admitting a non-compact partial Cauchy surface S such that $H^+(S)$ is smooth and compactly generated. Then the null energy condition is violated.



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Chronology protection conjecture

The laws of physics do not allow the appearance of closed timelike curves.

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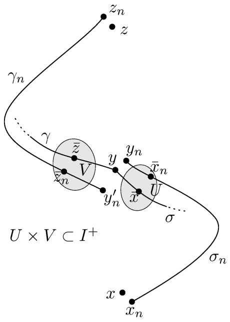
Chronology protection conjecture

The laws of physics do not allow the appearance of closed timelike curves.

However, it excludes their *formation*, not their *presence since the beginning of the universe*.

Key argument: No lightlike lines $\Rightarrow \overline{J^+}$ is transitive

Suppose $(x,y) \in \overline{J^+}$ and $(y,z) \in \overline{J^+}$. Let $(x_n, y_n) \in J^+$, $(x_n, y_n) \to (x,y)$, $(y'_n, z_n) \in J^+$, $(y'_n, z_n) \to (y,z)$, then $(x_n, z_n) \in J^+$ thus $(x,z) \in \overline{J^+}$



But then $\overline{J^+}$ is clearly the smallest closed and transitive relation containing J^+, hence $J_S^+ = \overline{J^+}$.

Is $\overline{J^+}$ antisymmetric (a partial order)?

Yes, if (M,g) is chronological (or yes outside $[r]$ if $M = I^+([r])$).

Lemma

Suppose that the universe is generated *by a chronology violating class, i.e.* $M = I^+([r])$, *then* $[r]$ *is unique.*

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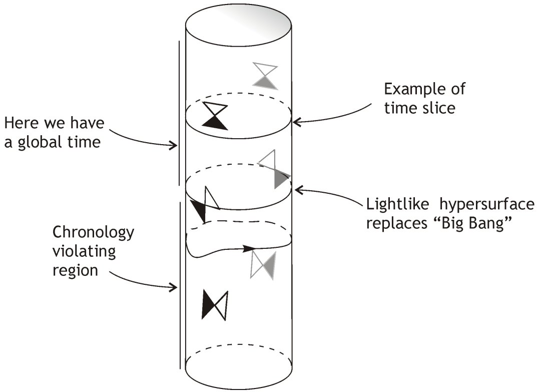
Time begins just outside the chronology violating set at the beginning of the universe

Theorem. Let (M, g) be a spacetime which is either chronological or admits a chronology violating class which generates the whole universe, i.e. $M = I^+([r])$. Assume that on every inextendible geodesic entirely contained in $M \backslash \overline{[r]}$, the null energy condition and the null genericity condition hold true.
Then either there is a null incomplete geodesic contained in $M \backslash \overline{[r]}$, or the spacetime $M \backslash \overline{[r]}$ is stably causal and hence admits a time function.

General picture



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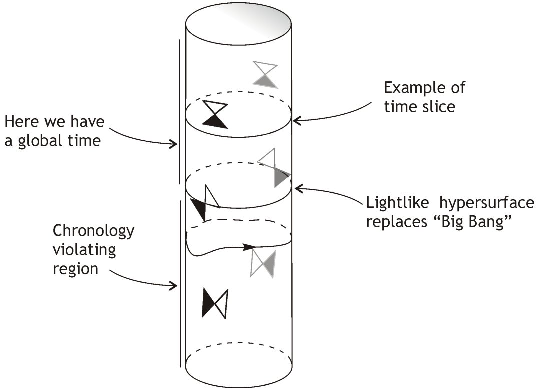
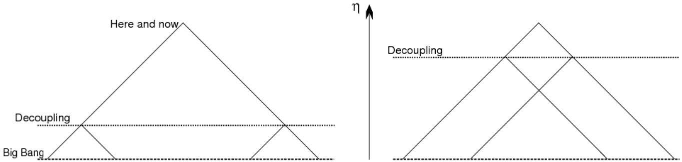


Solves nicely the homogeneity and entropy problems of cosmology

Cosmic microwave background

Approximate black body radiation at 2.7K, variations of order 10^{-5} for angles as large as 80°.

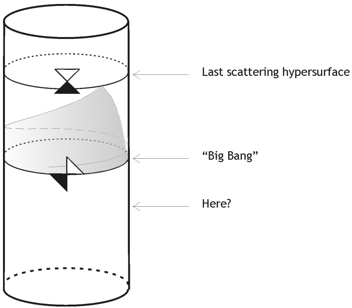
Light moves at 45° in spacetime diagrams with conformal time $\eta = \int_0^t \frac{\mathrm{d}t}{a(t)}$.



Inflation by acting only on the dependence $a(t)$ pushes up the decoupling time (up to rescaling). Inhomogeneities have time to interact and smooth out (thermalization). Note: according to this solution entropy from Big Bang to decoupling increases so the Big Bang has to be more "special" then the universe at decoupling.

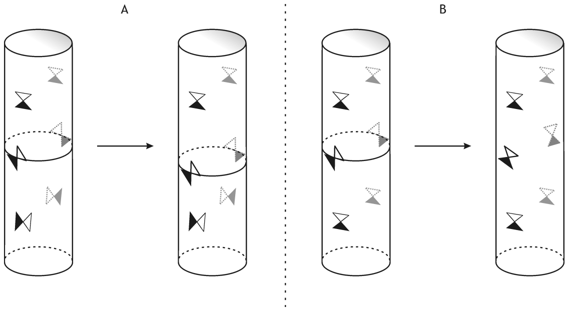
A simple causality solution: The Big Bang hypersurface is null

Inflation solves the problem of homogeneity/isotropy inducing a modification of the conformal factor $a(t)$, here we just tilt the cones so that the whole null hypersurface is in the past of every point of the last scattering hypersurface.



Exact solutions of Einstein equations with similar causal behavior exist, e.g. Taub-NUT and λ-Taub-NUT.
Between “Big Bang” and decoupling there is no thermalization so there is no such increase of entropy as in the inflationary case, a fact which made the problem of the small entropy at Big Bang even worse in the inflationary scenario.

This mechanism for homogeneity in order to be stable under small perturbations of the metric requires that behind the Big Bang null hypersurface there be a chronology violating region.



The matching could be conformally smooth but metrically singular.

How special is the Big bang?

Interesting evolution is possible only if the initial entropy is small. A system already at maximum entropy is dynamically trivial. The second law of thermodynamics holds because in the coarse graining we start from a small cell of phase space.

Entropy problem (Penrose's argument) II

Estimate of the maximum entropy of the universe

Use Bekenstein-Hawking formula $S_{BH} = \frac{kc^3}{4G\hbar}A$, and assume that every baryon will fall into a black hole. Comparison with entropy at decoupling gives that the cell at decoupling is about one part of $10^{10^{123}}$ than that at end of universe.

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In which way could the Big Bang be special? The role of gravity

Gravitational degrees of freedom had small entropy. Homogeneity evolves into *clumping* as gravitation is attractive. Thus small entropy is related to gravitational degrees of freedom rather that to the smallness of the universe.

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Penrose's Weyl tensor hypothesis

The Weyl tensor

$$C_{ab}{}^{dc} = R_{ab}{}^{cd} - 2R_{[a}{}^{[c} g_{b]}{}^{d]} + \frac{1}{3} R g_{[a}{}^{c} g_{b]}{}^{d}$$

is small for homogeneous matter distributions thus it is expected to be small at the beginning of the universe.

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Yet the problem remains: If the Big Bang is made by causally unrelated events on the boundary how it could be special?

The role of rigidity

The solution stand on the achronality of the boundary given by the Big Bang hypersurface. This hypersurface can be proved to be generated by achronal inextendible lightlike geodesic. This fact implies *rigidity*: the metric is special near the boundary e.g. its transverse section gives the Euclidean flat metric. As the metric is special, gravitational degrees of freedom are frozen, exactly the way out to the entropy problem that Penrose suggested (but we don't need a cyclic cosmology).

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Role of shear

In an achronal curve γ one has $\sigma^2 = 0$ to avoid conjugate points. But the shear satisfies

$$\frac{\mathrm{d}\sigma_{mn}}{\mathrm{d}t} = -C_{m4n4} - \theta\sigma_{mn} - \sigma_{mp}\sigma_{pn} + \delta_{mn}\sigma^2$$

thus $C_{m4n4} = 0$ over γ.

Connection with Augustine of Hippo's philosophy I

Augustine of Hippo (Thagaste 354 - Hippo 430), romanized berber philosopher.

Augustine's time studies. Confessions book XI

"What, then, is time? If no one ask me, I know; if I wish to explain to him who asks, I know not."

Connection with Augustine of Hippo's philosophy II

A few assumptions on God's nature

1. There is an entity which we call God that satisfies the following points.
2. God created the world.
3. God cannot be wrong.

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Manichæans' question: What was God doing before creating the world?

His will must be eternal since God cannot change his mind. Thus if God creates something, that something has to have been already created, so the world had no beginning (contradiction).

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Augustine's solution

Yes, the will of God is eternal, but God does not perceive time as we do since he creates the time itself, thus God precedes his creation causally but not temporally. For God time is still.

In other words Augustine separates temporality from causality! Something usual in GR but philosophically it was a very non-trivial argument.

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The connection

Assume God is represented by the chronology violating class at the beginning of the universe. A closed timelike curve cannot represent an observer (conflicts with free will) but can represent a God since its will is eternal thus confirms previous path (choices). The null Big Bang represents the creation of world and time.

Conclusions

I have advocated that a picture for the beginning of the universe whose causal aspects are as in the figure can solve the homogeneity/isotropy problem and the entropy problem, while fitting nicely Augustine's considerations on the beginning of time.