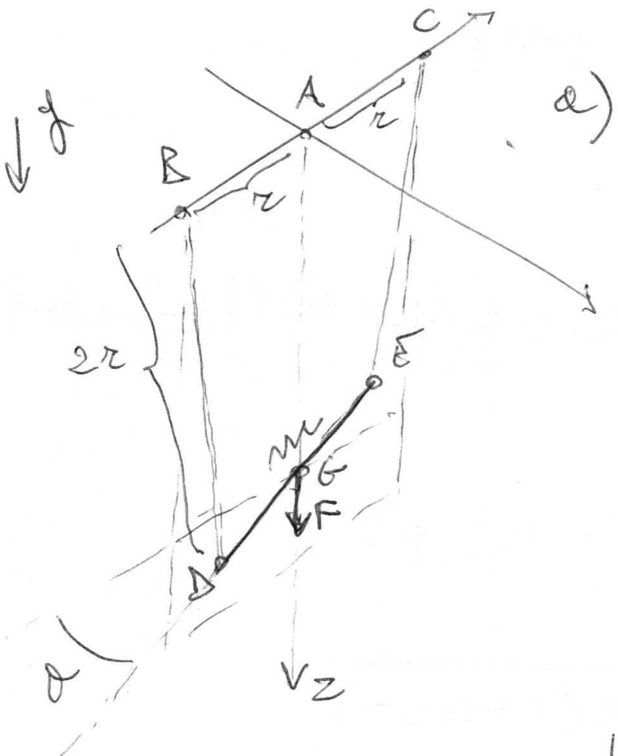
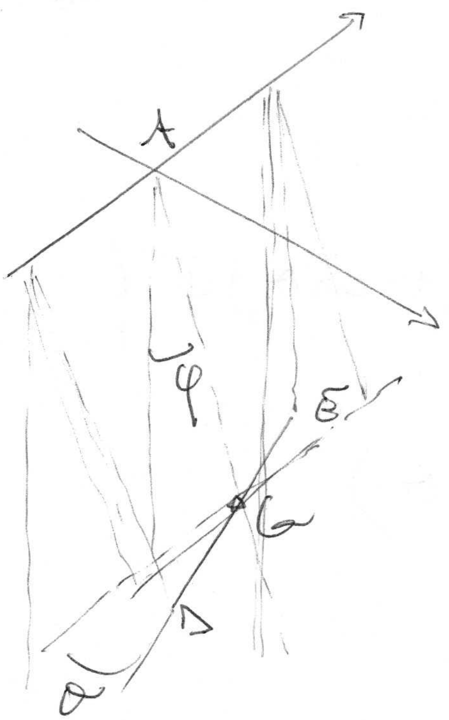
1) ALTALENA SOTTOPOSTA A TORSIONE

$$\bar{F} = mg \frac{z}{r} \hat{k}$$



a) IPOTESI: $x_G = 0$, G SU VERTICALE DI A, DE ⊥ AG.
SUPPOSTO NOTO z_G SCRIVERE COORDINATE DI B E D, IMPORRE BD = 2r TROVANDO LA RELAZIONE $z_G(\theta)$



b) SUPPONIAMO ORA CHE CI SIA UN ANGOLO φ TRA AG E LA VERTICALE. USARE COORDINATE GENERALIZZATE θ E φ. $x_G = 0$.

- SCRIVERE V, TROVARE PUNTI STAZIONARI
- SCRIVERE T
- PULSAZIONI PICCOLE OSCILLAZIONI.

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a)

$$D = (-r\cos\theta,\ r\sin\theta,\ z_G)$$

$$B = (-r,\ 0,\ 0)$$

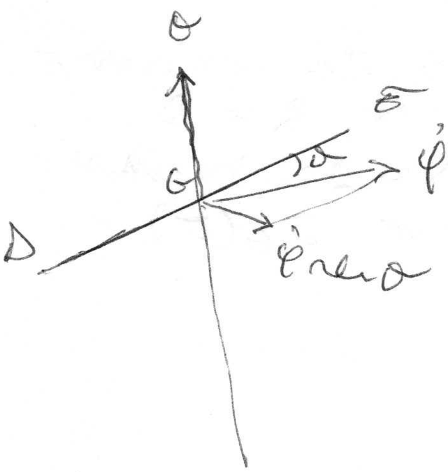
$$B - D = (-r + r\cos\theta,\ -r\sin\theta,\ -z_G)$$

$$4r^2 = BD^2 = (-r + r\cos\theta)^2 + r^2\sin^2\theta + z_G^2 = 2r^2(1-\cos\theta) + z_G^2$$

$$\Rightarrow z_G = r\sqrt{2(1+\cos\theta)}$$

$$V = -mgr\sqrt{2(1+\cos\theta)}\,\cos\varphi$$

$$-\frac{mg}{2r}\,r^2\,2(1+\cos\theta)\cos^2\varphi$$



ADDIZIONE VEL. ANGOLARI

$$T = \frac{1}{2}m v_G^2 + \frac{1}{2}\left(\frac{1}{12}m(2r)^2\right)\left(\dot\theta^2 + \sin^2\theta\,\dot\varphi^2\right)$$

DOVE

$$v_G^2 = \dot r_G^2 + r_G^2\,\dot\varphi^2$$

CON

$$r_G = r\sqrt{2(1+\cos\theta)}$$

$$\dot r_G = \frac{r}{\sqrt{\cdots}}(-\sin\theta)\,\dot\theta = -\frac{r^2}{r_G}\sin\theta\,\dot\theta$$

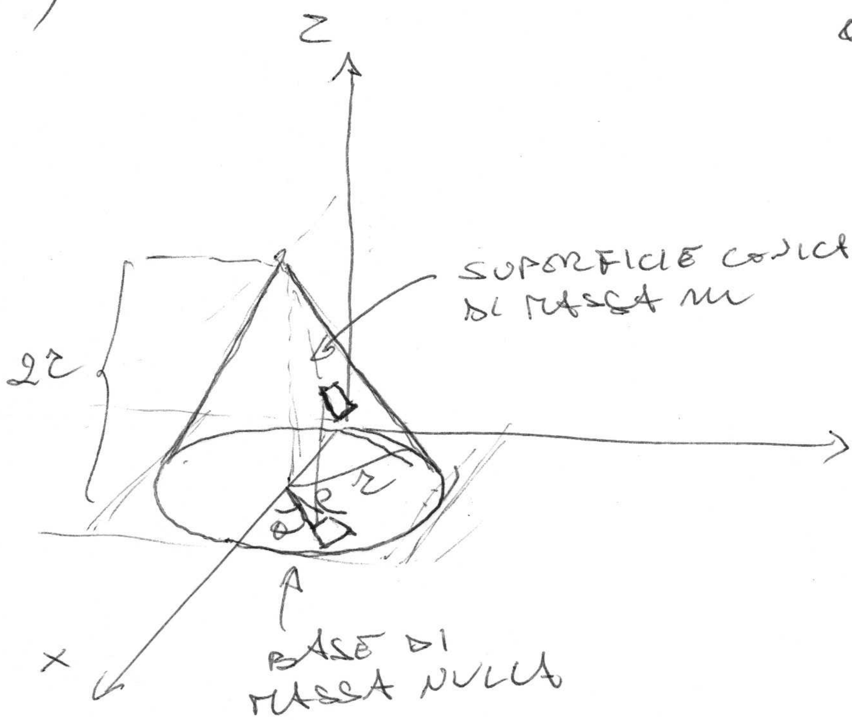
$$v_G^2 = \frac{r^4}{r_G^2}\sin^2\theta\,\dot\theta^2 + r_G^2\,\dot\varphi^2$$

$$T = \frac{1}{2}mr^2\left\{\frac{\sin^2\theta\,\dot\theta^2}{2(1+\cos\theta)} + 2(1+\cos\theta)\,\dot\varphi^2\right\}$$

$$+\frac{1}{6}mr^2\left(\dot\theta^2 + \dot\varphi^2\sin^2\theta\right)$$

2)

a) TROVARE I_x, I_z, I_{xz}



MASSA dm DEL RETTANGOLO CHE PROIETTA NEL RETTANGOLO DI LATI $\rho\, d\theta$ E $d\rho$

$$dm = m \frac{\rho\, d\rho\, d\theta}{\pi r^2}$$

INFATTI SCHIACCIANDO LUNGO z SI OTTIENE UN DISCO OMOGENEO DI MASSA m.

$$I_z = \frac{1}{2} m r^2 + m r^2 = \frac{3}{2} m r^2$$

$$x = r + \rho \cos\theta$$

$$z = 2(r-\rho)$$

$$y = \rho \sin\theta$$

$$I_x = \int (y^2+z^2)\, dm = \int \left\{ \rho^2 \sin^2\theta + 4(r-\rho)^2 \right\} dm$$

$$= \frac{m}{\pi r^2} 2\pi \int_0^r \left\{ \frac{\rho^2}{2} + 4(r-\rho)^2 \right\} \rho\, d\rho = \frac{11}{12} m r^2$$

$$I_{xz} = -\int x z\, dm = -\int (r + \rho\cos\theta)\, 2(r-\rho)\, dm$$

$$= -\frac{m}{\pi r^2} 2\pi \int_0^r 2r(r-\rho)\, \rho\, d\rho = -\frac{2m}{3} r^2$$