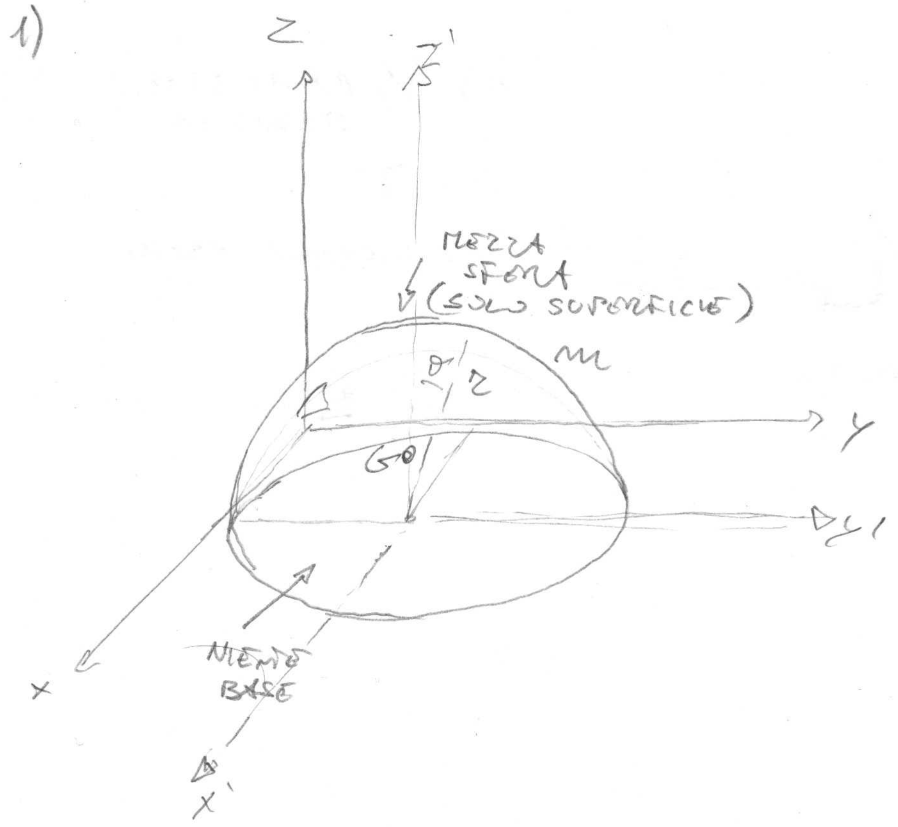
1)

a) MATRICE D'INERZIA

b) DISCUTERE SIMMETRIE



IL MOMENTO D'INERZIA DI UNA SFERA CAVA È $\frac{2}{3} m r^2$

QUINDI È FACILE DIMOSTRARE CHE

$$I' = \frac{2}{3} m r^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_z = I'_z + m\, 2r^2 = \frac{8}{3} m r^2$$

$$I_{xy} = I_{G\,xy} - m r^2 = -m r^2$$

COMPLETA LA SFERA

$$I_x = \frac{2}{3} m r^2 + m r^2 = \frac{5}{3} m r^2 = I_y$$

$$I_{xz} = -\frac{m}{2\pi r^2} \int x z \; r^2 \sin\theta \, d\theta \, d\varphi$$

$$= -\frac{m}{2\pi r^2} \int (r \sin\theta \cos\varphi + r)(r\cos\theta)\, r^2 \sin\theta \, d\theta \, d\varphi$$

$$= -\frac{m}{2\pi} r^2 \, 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = -m r^2 \left. \frac{\sin^2\theta}{2} \right|_{\theta=0}^{\theta=\pi/2}$$

$$= -\frac{m r^2}{2}$$

$$I = \frac{m r^2}{6} \begin{pmatrix} 10 & -6 & -3 \\ -6 & 10 & -3 \\ -3 & -3 & 16 \end{pmatrix}$$

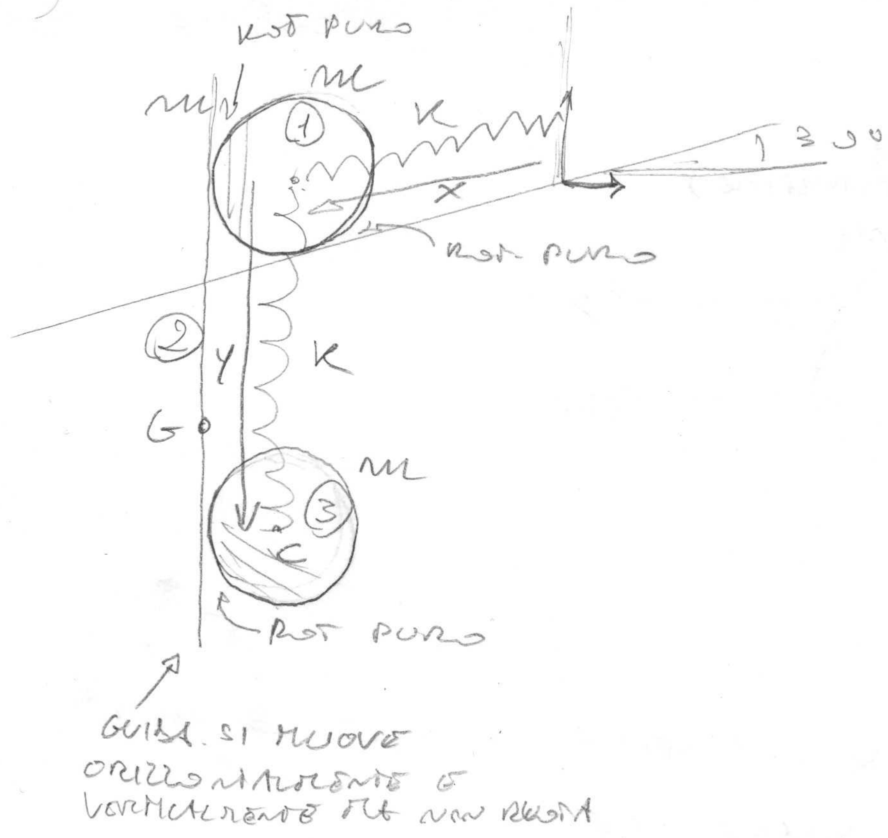
PER SIMMETRIA $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ È AUTOVETTORE DI MOMENTO PRINCIPALE $\frac{8}{3} m r^2$

$kr = mg$

a) V, punti staz. stabilità

b) T

c) piccole oscill.



$$V^{(1)} = -mg\frac{x}{2} + \frac{1}{2}kx^2 = -kr\frac{x}{2} + \frac{1}{2}kx^2$$

$$C = \left(-\frac{\sqrt{3}}{2}x, -\frac{x}{2} - y\right) \Rightarrow \dot{C}^2 = \frac{3}{4}\dot{x}^2 + \left(\frac{\dot{x}}{2} + \dot{y}\right)^2$$

$$V^{(3)} = -mg\left(\frac{x}{2} + y\right) + \frac{1}{2}ky^2 = -kr\left(\frac{x}{2} + y\right) + \frac{1}{2}ky^2$$

$$V^{(2)} = -mg\frac{3}{2}x + \text{cost} = -kr\frac{3}{2}x + \text{cost}$$

$$T^{(1)} = \frac{1}{2}\left(\frac{1}{2}mr^2 + mr^2\right)\left(\frac{\dot{x}}{r}\right)^2 = \frac{3}{4}m\dot{x}^2$$

$$G = \left(-\frac{\sqrt{3}}{2}x + \text{cost}, -\frac{3}{2}x + \text{cost}\right)$$

$$\dot{G} = \left(-\frac{\sqrt{3}}{2}\dot{x}, -\frac{3}{2}\dot{x}\right) \Rightarrow \dot{G}^2 = 3\dot{x}^2$$

$$T^{(2)} = \frac{1}{2}m3\dot{x}^2$$

$$T^{(3)} = \frac{1}{2}m\left\{\frac{3}{4}\dot{x}^2 + \left(\frac{\dot{x}}{2} + \dot{y}\right)^2\right\} + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{y} - \dot{x}}{r}\right)^2$$

$$V = \frac{1}{2}k(x^2+y^2) - kr\frac{5}{2}x - kry$$

$$\left.\begin{aligned} \frac{1}{k}\frac{\partial V}{\partial x} &= x - r\frac{5}{2} \\ \frac{1}{k}\frac{\partial V}{\partial y} &= y - r \end{aligned}\right\} \Rightarrow \text{PUNTO STAZ} \quad \begin{cases} x = \frac{5}{2}r \\ y = r \end{cases}$$

$$\frac{\partial^2 V}{\partial x^2} = k, \quad \frac{\partial^2 V}{\partial y^2} = k, \quad \frac{\partial^2 V}{\partial x \partial y} = 0 \Rightarrow B = k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T = \frac{9}{4}m\dot{x}^2 + \frac{1}{2}m\left\{\frac{3}{4}\dot{x}^2 + \left(\frac{\dot{x}}{2}+\dot{y}\right)^2\right\} + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{y}-\dot{x}}{2\ldots}\right.$$

$$= 3m\dot{x}^2 + \frac{3}{4}m\dot{y}^2 \Rightarrow A = m\begin{pmatrix} 6 & 0 \\ 0 & 3/2 \end{pmatrix}$$

$$\omega_1 = \sqrt{\frac{k}{6m}} \qquad \omega_2 = \sqrt{\frac{2k}{3m}}$$

GLI AUTOVETTORI SONO $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ E $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.