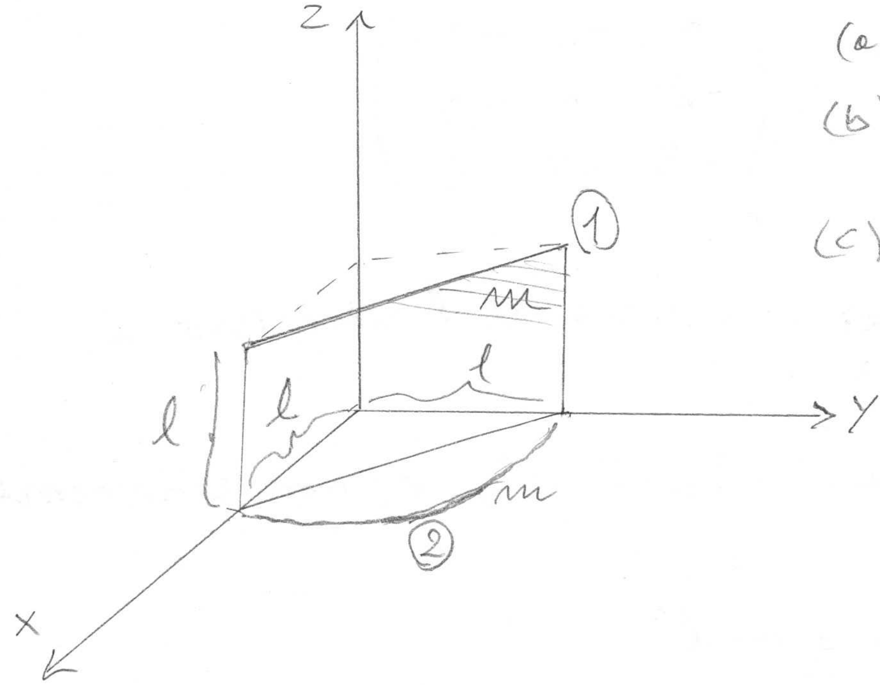
2)



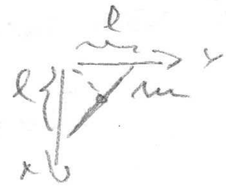
(a) MATRICE DI INERZIA

(b) SIMMETRIE, MOMENTI E ASSI PRINCIPALI

(c) SE $\vec{\omega} = (2,1,0)\ s^{-1}$ CALCOLARE $\vec{L}$ E T.

①

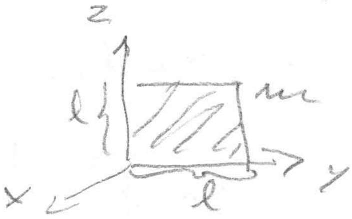
I_z PER COMPRESSIONE E' EQUIVALENTE A



QUINDI

$$I_z = \frac{1}{12} m \left(\sqrt{2}\, l\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{2}{3} m l^2$$

I_x PER COMPRESSIONE E' EQUIVALENTE A



*

CIO' E' UN SISTEMA PIANO IN CUI

$$I'_y = I'_z = \frac{1}{3} m l^2 \quad \text{(USA COMPRESSIONE)}$$

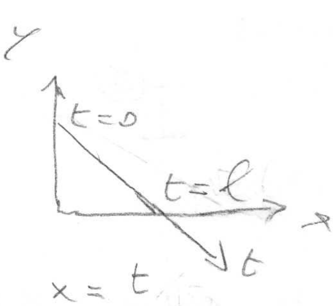
QUINDI $$I_x = \frac{2}{3} m l^2$$

PER SIMMETRIA $I_y = \frac{2}{3} m l^2$

$$I_{xy} = -\int xy\, dm = -\int_0^l t(l-t)\frac{m}{l}\, dt$$

$$= \frac{m}{l}\int_0^l (t^2 - lt)\, dt = \frac{m}{l}\left(\frac{t^3}{3} - l\frac{t^2}{2}\right)\Big|_0^l$$

$$= -\frac{1}{6} m l^2$$



$x = t$

$y = l - t$

$dm = \frac{m}{l} dt$

I_{yz} SI PUO' CALCOLARE COMPRIMENDO, PASSANDO A *

PER IL TEOREMA DI HUYGENS-STEINER E'

$$I_{yz} = -m\, l^2/4$$

ANALOGAMENTE $$I_{xz} = -m\, l^2/4$$

QUINDI

$$I^{(1)} = \frac{ml^2}{12}\begin{pmatrix} 8 & -2 & -3 \\ -2 & 8 & -3 \\ -3 & -3 & 8 \end{pmatrix}$$

(2)

SICCOME TUTTA LA MASSA È A DISTANZA l DALL'ASSE z

$$I_z = ml^2$$

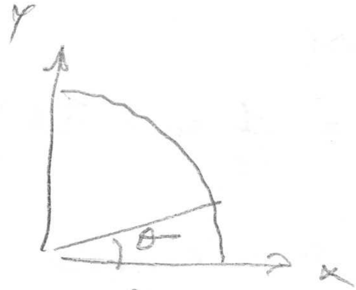
MA È UN SISTEMA PIANO, $I_z = I_x + I_y$, E PER SIMMETRIA $I_x = I_y$ QUINDI

$$I_x = I_y = \frac{1}{2} ml^2$$

POICHÉ STA SUL PIANO $z=0$

$$I_{xz} = I_{yz} = 0$$

CALCOLIAMO I_{xy}



$$I_{xy} = -\int xy\,dm = -\frac{2m}{\pi} l^2 \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta$$

$$= -\frac{2ml^2}{\pi}\int_0^1 \sin\theta\, d\sin\theta$$

$$= -\frac{ml^2}{\pi}$$

$x = l\cos\theta$

$y = l\sin\theta$

$dm = \frac{m}{\pi/2}\, d\theta$

QUINDI

$$I^{(2)} = \frac{ml^2}{2}\begin{pmatrix} 1 & -\frac{2}{\pi} & 0 \\ -\frac{2}{\pi} & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

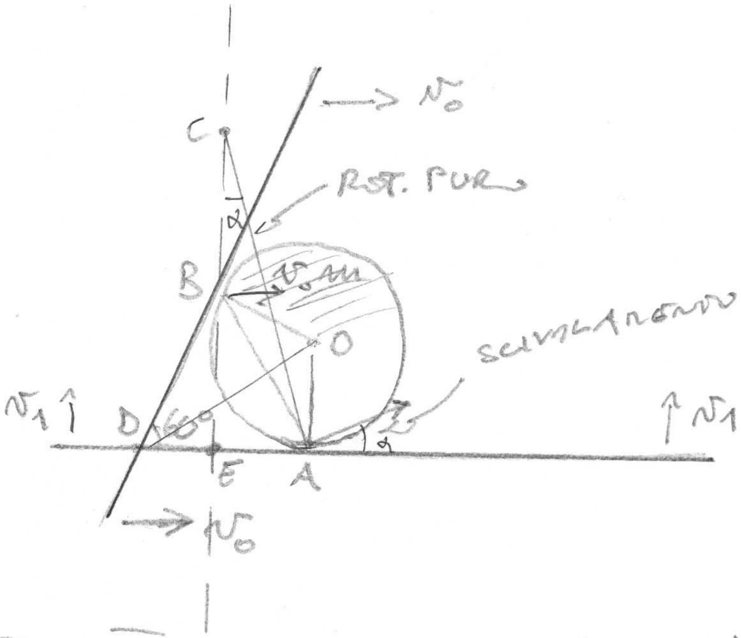
b) IL PIANO $x=y$ È DI SIMMETRIA QUINDI $v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

È AUTOVETTORE DI

$$I = I^{(1)} + I^{(2)} = \frac{ml^2}{12}\begin{pmatrix} 14 & -2-\frac{12}{\pi} & -3 \\ -2-\frac{12}{\pi} & 14 & -3 \\ -3 & -3 & 20 \end{pmatrix}$$

ALL'AUTOVALORE $I_1 = \left(\frac{4}{3} + \frac{1}{\pi}\right) ml^2$

2)



a) DETERMINARE CENTRO IST. ROT DISCO

b) DETERMINARE T

USARE $v_0 = \sqrt{3}\, v_1$

$\overline{DB} = \overline{DA}$ E SONO A 60° QUINDI DBA È TRIANGOLO EQUILATERO. PER ROT. PURO B, VISTO COME PUNTO MATERIALE DEL DISCO HA VELOCITÀ v_0, QUINDI PER CHASLES IL CENTRO DI IST. ROT. C STA SULLA RETTA VERTICALE PASSANTE PER B.

INDIPENDENTEMENTE DA DOVE SI TROVI C LA COMPONENTE VERTICALE DI v_A È

$$\omega\, \overline{CA} \sin\alpha = \omega\, \overline{EA}$$

← NON DIPENDE DA POSIZIONE DI C

LA CONDIZIONE DI SCIVOLAMENTO IN A RICHIEDE CHE QUESTA SIA UGUALE A v_1 QUINDI $\omega = v_1/\overline{EA} = v_1/(\sqrt{3}r/2)$

$= 2v_1/(\sqrt{3}r)$

LA DISTANZA $\overline{CB}$ CHE INDIVIDUA C È

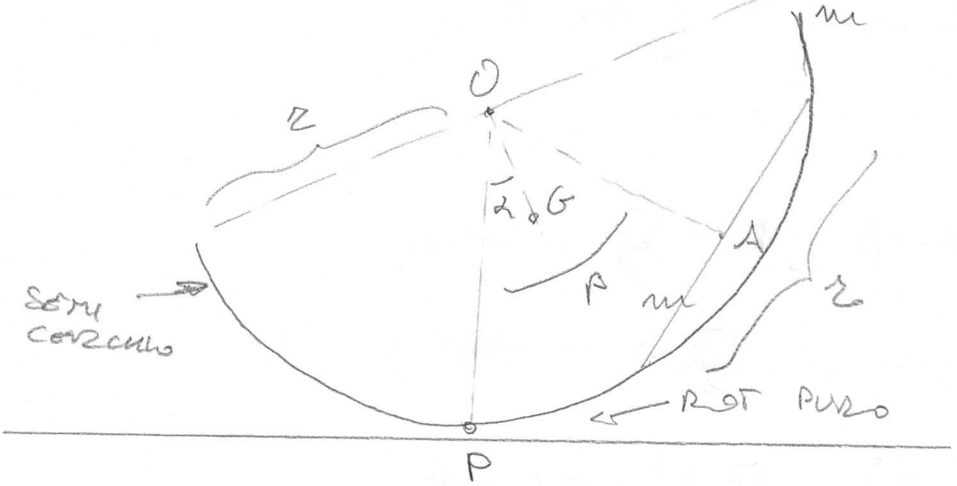
$$\overline{CB} = \frac{v_0}{\omega} = \frac{\sqrt{3}r}{2}\,\frac{v_0}{v_1} = \frac{3}{2}r$$

$$I_C = I_O + m\left\{\left(\frac{3}{2}r + \frac{r}{2}\right)^2 + \left(\frac{\sqrt{3}r}{2}\right)^2\right\} = \frac{1}{2}mr^2 + \left\{4r^2 + \frac{3}{4}r^2\right\}m$$

$$= \frac{21}{4}mr^2$$

$$T = \frac{1}{2}I_C\,\omega^2 = \frac{21}{8}mr^2\left(\frac{2v_1}{\sqrt{3}r}\right)^2 = \frac{7}{2}m v_1^2$$

3)



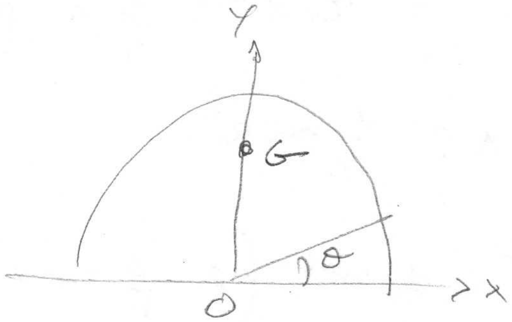
a) V, PUNTI STAZIONARI STABILITA'

b) CALCOLARE T

c) PICCOLE OSCILLAZ.

$$Y_G = \frac{1}{m}\int y\,dm = \frac{1}{m}\left(\frac{m}{\pi}\right)\int_0^\pi r\sin\theta\,d\theta$$

$$= \frac{r}{\pi}\left(-\cos\theta\right)\Big|_0^\pi = \frac{2r}{\pi}$$



$$I_O = I_G + mr^2\frac{4}{\pi^2} \Rightarrow I_G = mr^2\left\{1-\frac{4}{\pi^2}\right\}$$

$$I_P = I_G + m\left\{(\overline{OG}\sin\alpha)^2 + (r-\overline{OG}\cos\alpha)^2\right\} = mr^2\left\{1-\frac{4}{\pi^2}+\left(\frac{2}{\pi}\sin\alpha\right)^2 + \left(1-\frac{2}{\pi}\cos\alpha\right)^2\right\}$$

$$\overline{OA} = \frac{r}{2}\sqrt{3}$$

$$= mr^2\left\{2-\frac{4}{\pi}\cos\alpha\right\}$$

$$V = -mg\left(\overline{OG}\cos\alpha + \overline{OA}\cos\beta\right)$$

$$= -mg\left\{r\frac{2}{\pi}\cos\alpha + \frac{r}{2}\sqrt{3}\cos\beta\right\}$$

$$= -mgr\left\{\frac{2}{\pi}\cos\alpha + \frac{\sqrt{3}}{2}\cos\beta\right\}$$

$$\frac{\partial V}{\partial\alpha} = mgr\frac{2}{\pi}\sin\alpha \quad , \quad \frac{\partial^2 V}{\partial\alpha^2} = mgr\frac{2}{\pi}\cos\alpha$$

$$\frac{\partial V}{\partial\beta} = mgr\frac{\sqrt{3}}{2}\sin\beta \quad , \quad \frac{\partial^2 V}{\partial\beta^2} = mgr\frac{\sqrt{3}}{2}\cos\beta$$

$$\frac{\partial V}{\partial\alpha} = \frac{\partial V}{\partial\beta} = 0 \Rightarrow \begin{matrix}\alpha = 0, \cancel{\pi} \\ \beta = 0, \cancel{\pi}\end{matrix} \qquad \frac{\partial V}{\partial\alpha\,\partial\beta} = 0$$

$$B = \mathrm{Hess}\,V\Big|_{\alpha,\beta=0} = mgr\begin{pmatrix}\frac{2}{\pi} & 0 \\ 0 & \frac{\sqrt{3}}{2}\end{pmatrix} > 0$$

STABILE

$$T = \frac{1}{2} I_P \dot{\alpha}^2 + \frac{1}{2} m \dot{A}^2 + \frac{1}{2} I_A \dot{\beta}^2$$

$$A = \left(-r\alpha + \frac{\sqrt{3}}{2} r \,\mathrm{sen}\,\beta \,,\; r - \frac{\sqrt{3}}{2} r \cos\beta \right)$$

$$\dot{A} = \left(-r\dot{\alpha} + \frac{\sqrt{3}}{2} r \cos\beta\, \dot{\beta} \,,\; \frac{\sqrt{3}}{2} r \,\mathrm{sen}\,\beta\, \dot{\beta} \right)$$

$$\dot{A}^2 = r^2 \dot{\alpha}^2 + \frac{3}{4} r^2 \dot{\beta}^2 - \sqrt{3}\, r^2 \cos\beta\, \dot{\alpha}\dot{\beta}$$

$$I_A = \frac{1}{12} m r^2$$

$$T = \frac{1}{2} m r^2 \left(2 - \frac{4}{\pi}\cos\alpha \right) \dot{\alpha}^2 + \frac{1}{2} m r^2 \left(\dot{\alpha}^2 + \frac{3}{4}\dot{\beta}^2 - \sqrt{3}\cos\beta\, \dot{\alpha}\dot{\beta} \right)$$
$$+ \frac{1}{2}\left(\frac{1}{12} m r^2 \right) \dot{\beta}^2$$
$$= \frac{1}{2} m r^2 \left(\left(3 - \frac{4}{\pi}\cos\alpha \right) \dot{\alpha}^2 + \frac{5}{6}\dot{\beta}^2 - \sqrt{3}\cos\beta\, \dot{\alpha}\dot{\beta} \right)$$

$$T\Big|_{\alpha=\beta=0} = \frac{1}{2} m r^2 \left\{ \left(3 - \frac{4}{\pi} \right) \dot{\alpha}^2 + \frac{5}{6}\dot{\beta}^2 - \sqrt{3}\,\dot{\alpha}\dot{\beta} \right\}$$

$$\Rightarrow \quad A = m r^2 \begin{pmatrix} 3 - \frac{4}{\pi} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 5/6 \end{pmatrix}$$