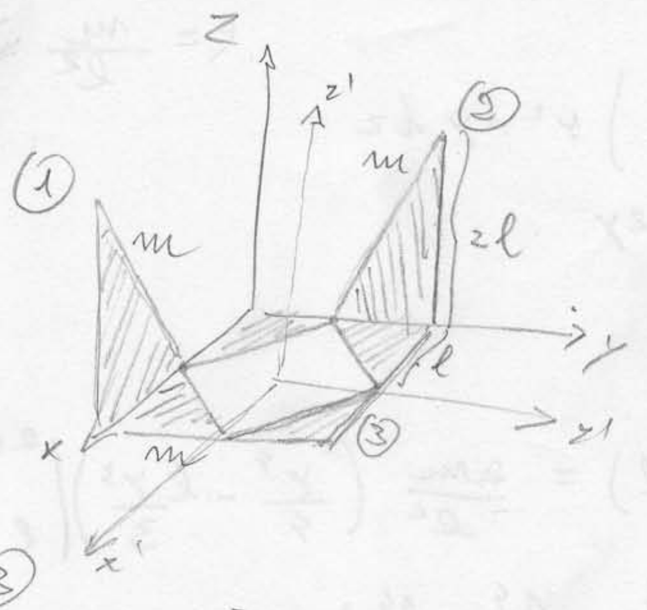
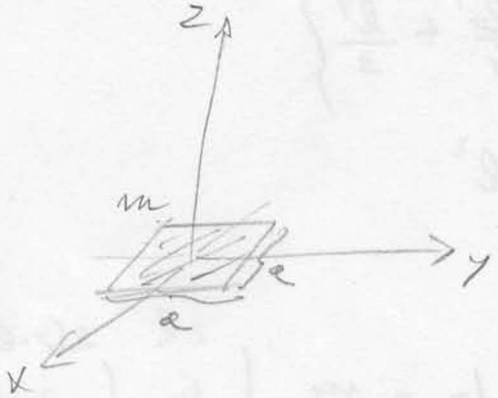
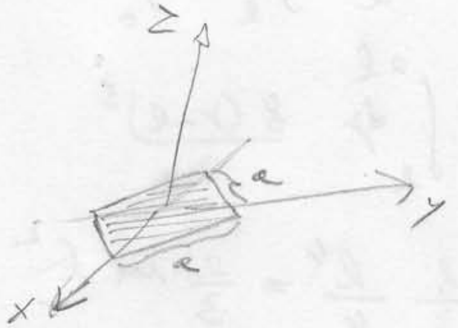




$$I_x = \frac{1}{12} m a^2 = I_y$$

$$I_z = \frac{1}{6} m a^2$$

$$I_{xy} = I_{yz} = I_{zx} = 0$$



STESSI DI PRIMA PERCHÉ PER SIMMETRIA L'ELLISSOIDE È TONDO

$$I^{(3)}_{x'} = \frac{2m}{12}(2l)^2 - \frac{m}{12}(\sqrt{2}\, l)^2 = \frac{1}{2} m l^2 = I^{(3)}_{y'}$$

$$I^{(3)}_{x} = I^{(3)}_{x'} + m l^2 = \frac{3}{2} m l^2 = I^{(3)}_{y} \quad , \quad I^{(3)}_{z} = 3 m l^2$$

$$I^{(3)}_{xy} = I^{(3)}_{x'y'} - m l^2 = - m l^2$$

$$I^{(3)} = \frac{ml^2}{2}\begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$\rho = \frac{m}{\ell^2}$$

$$I_{zz}^{(2)} = \int y^2\, dm = \rho \int y^2\, dy\, dz$$

$$= \frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, y^2 \int_0^{-2\ell+2y} dz$$

$$= \frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, y^2\, 2(y-\ell) = \frac{2m}{\ell^2} \left(\frac{y^4}{4} - \ell \frac{y^3}{3} \right)\Bigg|_{\ell}^{2\ell}$$

$$= \frac{2m}{\ell^2} \left\{ 4\ell^4 - \frac{8}{3}\ell^4 - \frac{\ell^4}{4} + \frac{\ell^4}{3} \right\}$$

$$= 2m\ell^2 \left\{ \frac{17}{12} \right\} = \frac{17}{6} m \ell^2$$

$$I_{yy}^{(2)} = \int z^2\, dm = \frac{m}{\ell^2} \int z^2\, dy\, dz = \frac{m}{\ell^2} \int_{\ell}^{2\ell} dy \int_0^{2(y-\ell)} z^2\, dz$$

$$= \frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, \frac{z^3}{3}\Bigg|_0^{2(y-\ell)} = \frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, \frac{8(y-\ell)^3}{3}$$

$$= \frac{m}{\ell^2} \frac{8}{3} \frac{(y-\ell)^4}{4}\Bigg|_{\ell}^{2\ell} = \frac{m}{\ell^2} \frac{8}{3} \frac{\ell^4}{4} = \frac{2}{3} m \ell^2$$

$$I_{yz}^{(2)} = -\int yz\, dm = -\frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, y \int_0^{2(y-\ell)} z\, dz$$

$$= -\frac{m}{\ell^2} \int_{\ell}^{2\ell} dy\, y\, 2(y-\ell)^2 = -\frac{2m}{\ell^2} \int_{\ell}^{2\ell} dy\, y(y-\ell)^2$$

$$= -\frac{2m}{\ell^2} \int_{\ell}^{2\ell} dy \left[(y-\ell)^3 + \ell (y-\ell)^2 \right]$$

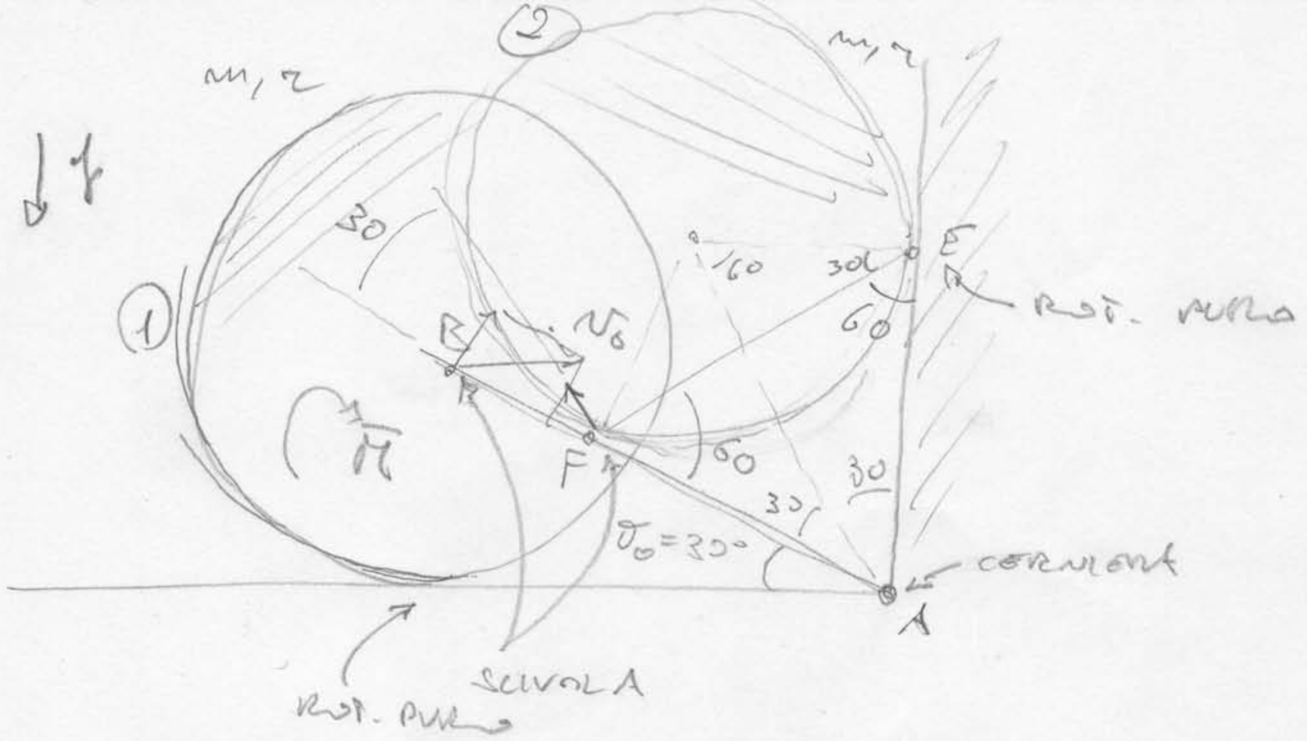
$$= -\frac{2m}{\ell^2} \left[\frac{(y-\ell)^4}{4} + \ell \frac{(y-\ell)^3}{3} \right]\Bigg|_{\ell}^{2\ell}$$

$$= -\frac{2m}{\ell^2} \left[\frac{\ell^4}{4} + \frac{\ell^4}{3} \right] = -\frac{7}{6} m \ell^2$$

$$\underline{I}^{(2)} = \frac{1}{6} m l^2 \begin{pmatrix} 21 & 0 & 0 \\ 0 & 4 & -7 \\ 0 & -7 & 17 \end{pmatrix}$$

$$\underline{I}^{(1)} = \frac{1}{6} m l^2 \begin{pmatrix} 4 & 0 & -7 \\ 0 & 21 & 0 \\ -7 & 0 & 17 \end{pmatrix}$$

$$\underline{I} = \frac{1}{6} m l^2 \begin{pmatrix} 34 & -6 & -7 \\ -6 & 34 & -7 \\ -7 & -7 & 52 \end{pmatrix}$$



a) DATI v_0 E ϑ_0 AL TEMPO $t=0$ TROVARE T

b) TROVARE $\bar{M}$ PER EQUILIBRIO.

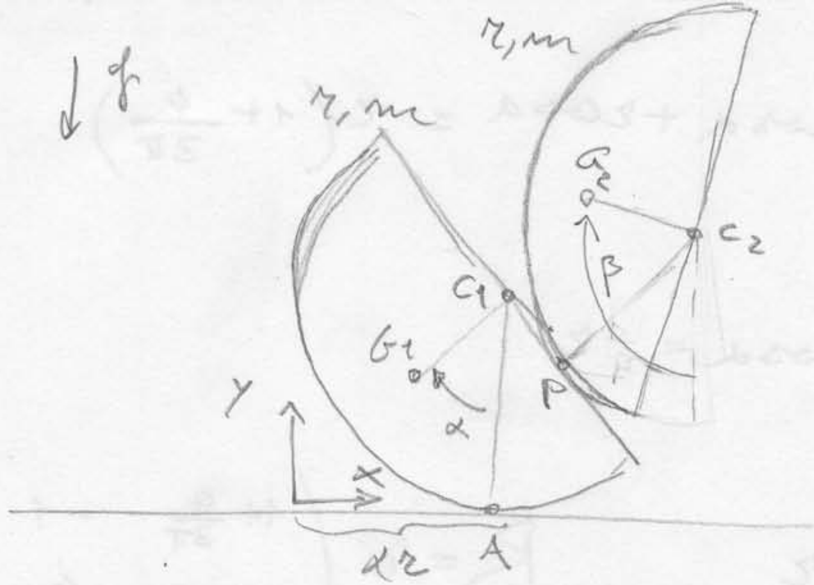
CHIARAMENTE $\omega_1 = v_0/r$.
IL PUNTO MATERIALE IN B APPARTENENTE ALL'ASTA HA VELOCITA' $v_0 \sin\vartheta_0 = v_0/2$ QUINDI POICHE' $\overline{BA} = 2r$ SI HA $\omega_{ASTA} = \frac{1}{4}\frac{v_0}{r}$

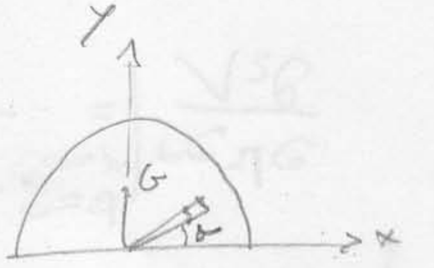
LA CONDIZIONE DI SCIVOLAMENTO SU F CI DICE CHE LA VELOCITA' DI F COME PUNTO MATERIALE DELL'ASTA O DEL CILINDRO DEVE AVERE LA STESSA COMPONENTE NORMALE (NOTA FEA E' EQUILATERO)

$$\omega_2 \cancel{\overline{FE}} \sin 30° = \omega_{ASTA} \cancel{\overline{FA}} \Rightarrow \omega_2 = \frac{1}{2}\frac{v_0}{r}$$

$$T = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\frac{v_0}{r}\right)^2 + \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\frac{1}{2}\frac{v_0}{r}\right)^2 = \frac{15}{16}mv_0^2$$

$$M\omega_1 dt = mg\,\omega_2 r\,dt \Rightarrow M = mgr/2$$





$$Y_G = \frac{2}{\pi r^2}\int y\, r\, dr\, d\theta = \frac{2}{\pi r^2}\int_0^r r^2\, dr \int_0^\pi \operatorname{sen}\theta\, d\theta = \frac{2}{\pi r^2}\frac{r^3}{3}\,2 = \frac{4}{3\pi} r$$

$$C_1 = (\alpha r, r)$$

$$G_1 = \left(\alpha r - \frac{4}{3\pi} r \operatorname{sen}\alpha,\ r - \frac{4r}{3\pi}\cos\alpha\right)$$

$$\overline{C_1P} = (\beta-\alpha) r$$

$$P = \left(\alpha r + (\beta-\alpha) r\cos\alpha,\ r - (\beta-\alpha) r \operatorname{sen}\alpha\right)$$

$$C_2 = \left(\alpha r + (\beta-\alpha) r\cos\alpha + r\operatorname{sen}\alpha,\ r - (\beta-\alpha) r\operatorname{sen}\alpha + r\cos\alpha\right)$$

$$G_2 = \left(\alpha r + (\beta-\alpha) r\cos\alpha + r\operatorname{sen}\alpha - \frac{4r}{3\pi}\operatorname{sen}\beta,\ r - (\beta-\alpha) r\operatorname{sen}\alpha + r\cos\alpha - \frac{4r}{3\pi}\cos\beta\right)$$

$$V = mg\left(Y_{G_1} + Y_{G_2}\right) = mg\left(2r + r\left(1 - \frac{4}{3\pi}\right)\cos\alpha - (\beta-\alpha) r\operatorname{sen}\alpha - \frac{4r}{3\pi}\cos\beta\right)$$

$$\frac{\partial V}{\partial \alpha} = -r\left(\cancel{1} - \frac{4}{3\pi}\right)\operatorname{sen}\alpha - (\beta-\alpha) r\cos\alpha + \cancel{r\operatorname{sen}\alpha} = 0$$

$$\frac{\partial V}{\partial \beta} = \frac{4r}{3\pi}\operatorname{sen}\beta - r\operatorname{sen}\alpha = 0$$

$\alpha = \beta = 0$ è sol., DIFFICILE FAR VEDERE UNICITÀ!

$$\left.\frac{\partial^2 V}{\partial \alpha^2}\right|_{\substack{\alpha=0\\ \beta=0}} = \frac{4r}{3\pi}\cos\alpha + r\cos\alpha = r\left(1+\frac{4}{3\pi}\right)$$

$$\left.\frac{\partial^2 V}{\partial \beta\,\partial \alpha}\right|_{\substack{\alpha=0\\ \beta=0}} = -r\cos\alpha = -r$$

$$\left.\frac{\partial^2 V}{\partial \beta^2}\right|_{\substack{\alpha=0\\ \beta=0}} = \frac{4r}{3\pi} \qquad B = r\begin{pmatrix} 1+\frac{4}{3\pi} & -1 \\ -1 & \frac{4}{3\pi} \end{pmatrix}$$

$\det B < 0$ EQUILIBRIO INSTABILE

$$T = \frac{1}{2} m \dot{G}_1^2 + \frac{1}{2} m \dot{G}_2^2 + \frac{1}{2}\left(\frac{1}{2} m r^2 - m\left(\frac{4r}{3\pi}\right)^2\right)\left(\dot{\beta}^2 + \dot{\alpha}^2\right)$$

IL PROBLEMA DELLE PICCOLE OSCILLAZIONI NON HA SENSO QUINDI È INUTILE CALCOLARSI A.