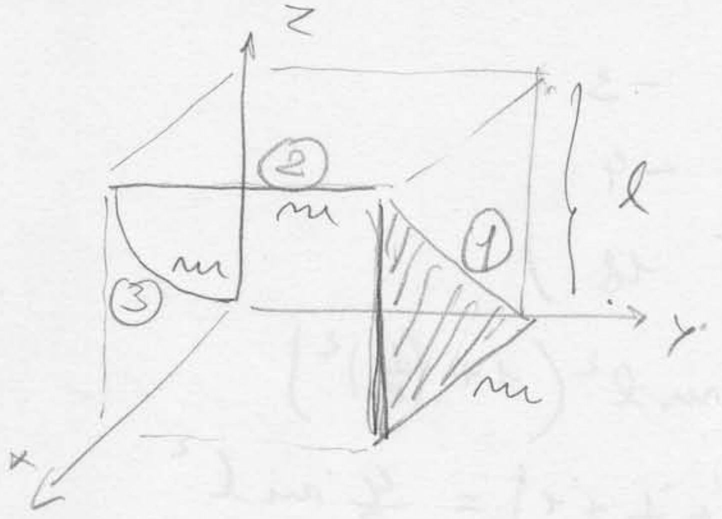


$$(1)\quad I_z = \int (x^2+y^2)\,dm = \int (x^2+l^2)\,dm$$

$$= l^2 m + \int x^2\,dm = l^2 m + \rho \int_0^l dx \int_0^x dz\; x^2$$

$$= l^2 m + \frac{m}{l^2/2}\int_0^l dx\; x^3 = l^2 m + \frac{m l^2}{2} = \frac{3}{2} m l^2$$

$$I_x = \int (z^2+y^2)\,dm = \int (z^2+l^2)\,dm$$

$$= l^2 m + \int z^2\,dm = l^2 m + \rho \int_0^l dx \int_0^x dz\; z^2$$

$$= l^2 m + \frac{m}{l^2/2}\int_0^l dx\; \frac{x^3}{3} = l^2 m + \frac{m l^2}{6} = \frac{7}{6} m l^2$$

$$I_y = \int (x^2+z^2)\,dm = \frac{m l^2}{2} + \frac{m l^2}{6} = \frac{2}{3} m l^2$$

$$I_{xy} = -\int xy\,dm = -l\int x\,dm = -l\,m\,x_G = -\frac{2}{3} m l^2$$

$$I_{zy} = -\int zy\,dm = -l\int z\,dm = -l\,m\,z_G = -\frac{1}{3} m l^2$$

$$I_{zx} = -\int zx\,dm = -\rho\int_0^l x\,dx\int_0^x z\,dz = -\frac{m}{l^2/2}\int_0^l x\,dx\;\frac{x^2}{2}$$

$$= -\frac{2m}{l^2}\,\frac{l^4}{4} = -\frac{m l^2}{4}$$

$$I^{(1)} = \frac{ml^2}{12}\begin{pmatrix} 14 & -8 & -3 \\ -8 & 8 & -4 \\ -3 & -4 & 18 \end{pmatrix}$$

(2)

$$I_x = I_z = \frac{1}{12} m l^2 + m l^2 \left(1 + \left(\frac{1}{2}\right)^2\right)$$
$$= m l^2 \left(\frac{1}{12} + \frac{1}{4} + 1\right) = \frac{4}{3} m l^2$$

$$I_y = 0 + m l^2 (1+1) = 2 m l^2$$

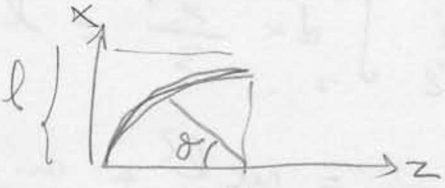
$$I_{xz} = - m l^2$$

$$I_{xy} = - m x_G y_G = - \frac{m l^2}{2}$$

$$I_{yz} = - m l^2/_2$$

$$I^{(2)} = \frac{ml^2}{6}\begin{pmatrix} 8 & -3 & -6 \\ -3 & 12 & -3 \\ -6 & -3 & 8 \end{pmatrix}$$

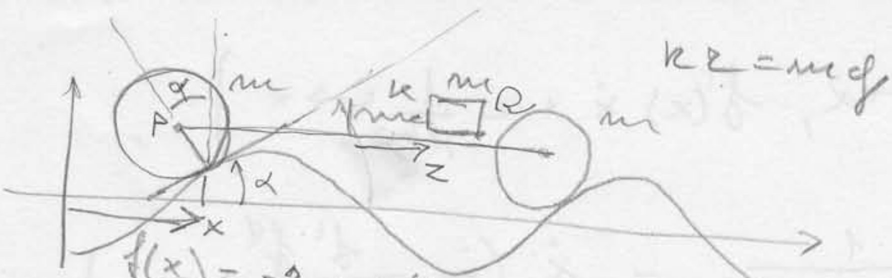
(3) $I_{yz} = I_{yx} = 0$ $\qquad$ $I_y = I_x + I_z$



$$I_z = \int x^2 \, dm = \frac{m}{\pi/2} \int_0^{\pi/2} x^2 \, d\theta = \frac{m}{\pi/2} \int_0^{\pi/2} l^2 \sin^2\theta \, d\theta$$
$$= \frac{2m}{\pi} l^2 \frac{\pi}{4} = \frac{m}{2} l^2$$

$$I_x = \int z^2 \, dm = \frac{m}{\pi/2} \int_0^{\pi/2} z^2 \, d\theta = \frac{m}{\pi/2} \int_0^{\pi/2} l^2 (1-\cos\theta)^2 \, d\theta$$
$$= \frac{2m}{\pi} l^2 \int_0^{\pi/2} (1 - 2\cos\theta + \cos^2\theta) \, d\theta$$
$$= \frac{2m}{\pi} l^2 \left\{ \frac{\pi}{2} - 2 + \frac{\pi}{4} \right\} = \frac{2m l^2}{\pi} \left\{ \frac{3}{4}\pi - 2 \right\}$$

$$I_{xz} = -\frac{2m}{\pi} l^2 \int_0^{\pi/2} \sin\theta (1-\cos\theta) \, d\theta = -\frac{2ml^2}{\pi}\left(1 - \frac{1}{2}\right) = -\frac{ml^2}{\pi}$$



$kr = mg$

$f(x) = -r\cos(x/2r)$.

$$P = \left(x - r\sin\alpha,\ f(x) + r\cos\alpha\right)$$

$$\tan\alpha = f'(x) = \frac{1}{2}\sin\left(\frac{x}{2r}\right)$$

$$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \tan^2\alpha\cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow \cos^2\alpha = \frac{1}{1+\tan^2\alpha} \Rightarrow \cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}} = \frac{1}{\sqrt{1+(f')^2}}$$

$$\Rightarrow \sin\alpha = \frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}} = \frac{f'}{\sqrt{1+(f')^2}}$$

$$V(x,z) = 3\left\{f(x) + r\cos\alpha\right\}mg + \frac{1}{2}kz^2$$

$$= 3r\left\{\cos\alpha - \cos\frac{x}{2r}\right\}mg + \frac{1}{2}kz^2$$

$$= 3r\left\{\frac{1}{\sqrt{1+\frac{1}{4}\sin^2\frac{x}{2r}}} - \cos\frac{x}{2r}\right\}kr + \frac{1}{2}kz^2$$

$$\frac{\partial V}{\partial x} = 3kr^2\left\{-\frac{\frac{1}{8r}\sin\left(\frac{x}{2r}\right)\cos\left(\frac{x}{2r}\right)}{\left(1+\frac{1}{4}\sin^2\frac{x}{2r}\right)^{3/2}} + \frac{1}{2r}\sin\left(\frac{x}{2r}\right)\right\}$$

$$= \frac{3}{2}kr\sin\left(\frac{x}{2r}\right)\left\{-\frac{1}{4}\frac{\cos\left(\frac{x}{2r}\right)}{\left(1+\frac{1}{4}\sin^2\left(\frac{x}{2r}\right)\right)^{3/2}} + 1\right\}$$

$$\frac{\partial V}{\partial z} = kz$$

LA PARENTESI GRAFFA È SEMPRE POSITIVA QUINDI PUNTI STAZIONARI

$$\begin{cases} x = n\pi & n \in \mathbb{Z} \\ z = 0 \end{cases}$$

$$\dot{P} = \left(\dot{x} - r \frac{d}{dt} \sin\alpha ,\ f'(x)\dot{x} + r \frac{d}{dt}\cos\alpha \right)$$

$$\frac{d}{dt}\cos\alpha = \dot{x} \frac{d}{dx} \frac{1}{\sqrt{1+(f')^2}} = \dot{x}\left(- \frac{f' f''}{\left(1+(f')^2\right)^{3/2}} \right)$$

$$= -\dot{x} \frac{\left(\frac{1}{2}\sin\frac{x}{2r}\right)\left(\frac{1}{4r}\cos\frac{x}{2r}\right)}{\left\{1+\left(\frac{1}{2}\sin\left(\frac{x}{2r}\right)\right)^2\right\}^{3/2}}$$

$$\frac{d}{dt}\sin\alpha = \frac{d}{dt}\left(\tan\alpha\cos\alpha\right) = \dot{x}\frac{d}{dx}\left(f' \frac{1}{\sqrt{1+(f')^2}}\right)$$

$$= -\dot{x}\frac{\left(\frac{1}{2}\sin\frac{x}{2r}\right)^2\left(\frac{1}{4r}\cos\frac{x}{2r}\right)}{\left\{1+\left(\frac{1}{2}\sin\left(\frac{x}{2r}\right)\right)^2\right\}^{3/2}}$$

$$+ \dot{x}\frac{\frac{1}{4r}\cos\frac{x}{2r}}{\left\{1+\left(\frac{1}{2}\sin\frac{x}{2r}\right)^2\right\}^{1/2}}$$

$$= \dot{x}\frac{\frac{1}{4r}\cos\frac{x}{2r}}{\left\{1+\left(\frac{1}{2}\sin\frac{x}{2r}\right)^2\right\}^{3/2}}$$

$$\dot{P}^2 = \dot{x}^2 + r^2\left\{\left(\frac{d}{dt}\sin\alpha\right)^2 + \left(\frac{d}{dt}\cos\alpha\right)^2\right\} + (f')^2\dot{x}^2$$
$$- 2r\dot{x}\frac{d}{dt}\sin\alpha + 2f'\dot{x}r\frac{d}{dt}\cos\alpha$$

$$T_{DISCO} = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\frac{\dot{P}}{r}\right)^2$$

$$Q = \left(x - r\sin\alpha + z + \text{Cost},\ f(x) + r\cos\alpha\right)$$
$$\dot{Q} = \left(\dot{x} - r\frac{d}{dt}\sin\alpha + \dot{z},\ f'\dot{x} + r\frac{d}{dt}\cos\alpha\right]$$
$$\dot{Q}^2 = \dot{P}^2 + \dot{z}^2 + 2\dot{x}\dot{z} - 2r\dot{z}\frac{d}{dt}\sin\alpha$$

$$T_{BLOCCO} = \frac{1}{2}m\dot{Q}^2$$