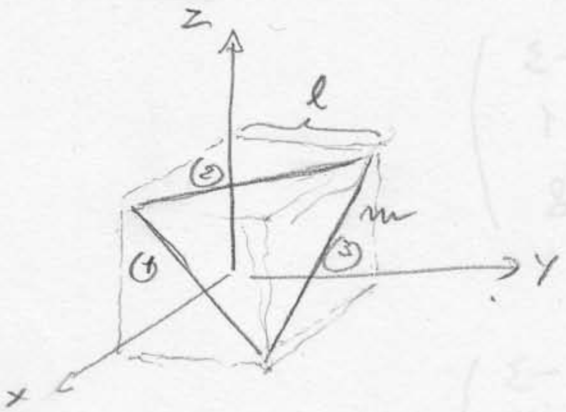
(1)

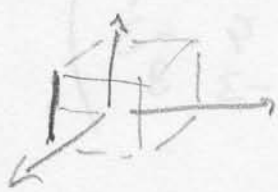


(a) MATRICE D'INERZIA

(b) SIMMETRIE, ASSI E MOMENTI PRINCIPALI

$$I_x^{(1)} = \frac{1}{12} m (\sqrt{2} l)^2 + m \left(\frac{l}{\sqrt{2}}\right)^2 = m l^2 \left(\frac{1}{6} + \frac{1}{2}\right) = \frac{2}{3} m l^2$$

PER $I_y^{(1)}$ PROIETTO



$$I_y^{(1)} = \frac{1}{12} m l^2 + m \left(\left(\frac{l}{2}\right)^2 + l^2\right) = m l^2 \left(\frac{1}{12} + \frac{5}{4}\right) = \frac{4}{3} m l^2$$

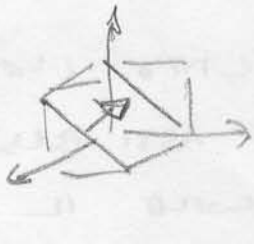
$$\parallel$$
$$I_z^{(1)}$$

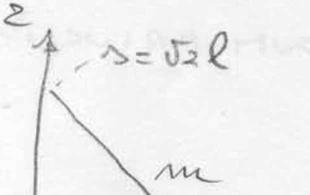
PER $I_{xz}^{(1)}$ USO STESSA PROIEZIONE E HUYGENS-STEINER

$$I_{xz}^{(1)} = - m \, l \frac{l}{2} = -\frac{1}{2} m l^2$$

$$\parallel$$
$$I_{xy}^{(1)}$$

PER $I_{yz}^{(1)}$ POSSO PROIETTARE LUNGO L'ASSE x





$$I_{yz}^{(1)} = - \int yz \, dm = -\rho \int yz \, ds$$

$$= -\frac{m}{\sqrt{2} l} \int_0^{\sqrt{2} l} \left(l - \frac{s}{\sqrt{2}}\right) \frac{s}{\sqrt{2}} \, ds$$

$$= -\frac{m}{l} \int_0^l (l - x) x \, dx = -\frac{m}{l} \left(\frac{l x^2}{2} - \frac{x^3}{3}\right)\Big|_0^l$$

$$= -\frac{m}{l} l^3 \left(\frac{1}{2} - \frac{1}{3}\right) = -\frac{1}{6} m l^2$$

QUINDI
$$I^{(1)} = \frac{ml^2}{6}\begin{pmatrix} 4 & -3 & -3 \\ -3 & 8 & -1 \\ -3 & -1 & 8 \end{pmatrix}$$

$$I^{(2)} = \frac{ml^2}{6}\begin{pmatrix} 8 & -1 & -3 \\ -1 & 8 & -3 \\ -3 & -3 & 4 \end{pmatrix}$$

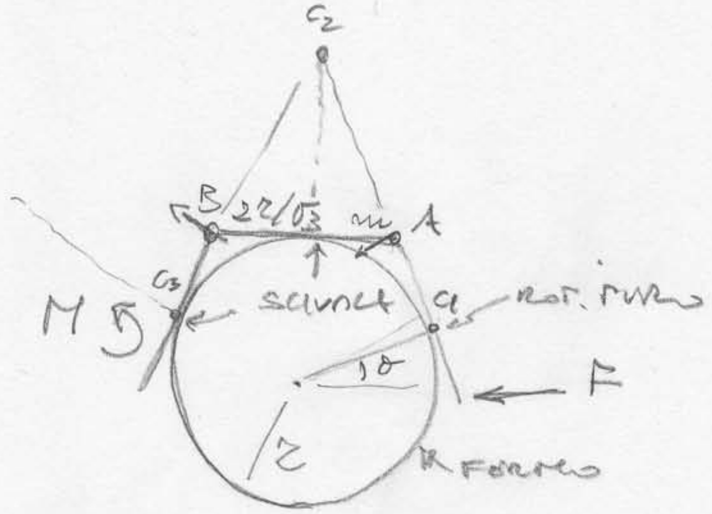
$$I^{(3)} = \frac{ml^2}{6}\begin{pmatrix} 8 & -3 & -1 \\ -3 & 4 & -3 \\ -1 & -3 & 8 \end{pmatrix}$$

$$I = \frac{ml^2}{6}\begin{pmatrix} 18 & -7 & -7 \\ -7 & 18 & -7 \\ -7 & -7 & 18 \end{pmatrix}$$

(b) ROTAZIONE DI 120° INTORNO A $(1,1,1)$ QUINDI QUESTO VETTORE È UN ASSE PRINCIPALE CON MOMENTO PRINCIPALE

$$\frac{ml^2}{6}(18-7-7) = \frac{2}{3}ml^2$$

INOLTRE L'ELLISSOIDE È TONDO QUINDI GLI ALTRI DUE ASSI BASTA SCEGLIERLI ORTOGONALI A QUESTO. SICCOME IL SISTEMA È PIANO I MOMENTI PRINCIPALI SONO $\frac{1}{3}ml^2$



(a) CENTRI ISTANT. ROT.

(b) SE A $t=0$, $\dot{\theta}=\omega$ TROVARE T

(c) RELAZIONE TRA F e M PER EQUILIBRIO

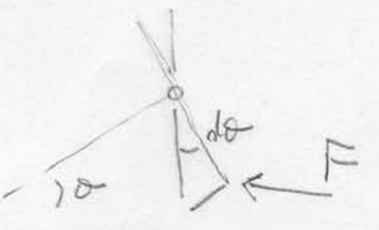
I CENTRI ISTANTANEI SONO C_1, C_2, C_3.

$$v_A = \omega\, r/\sqrt{3} = \omega_2\, 2r/\sqrt{3} \Rightarrow \omega_2 = \omega/2$$

$$v_B = \omega_2\, 2r/\sqrt{3} = \omega_3\, r/\sqrt{3} \Rightarrow \omega_3 = 2\omega_2 = \omega$$

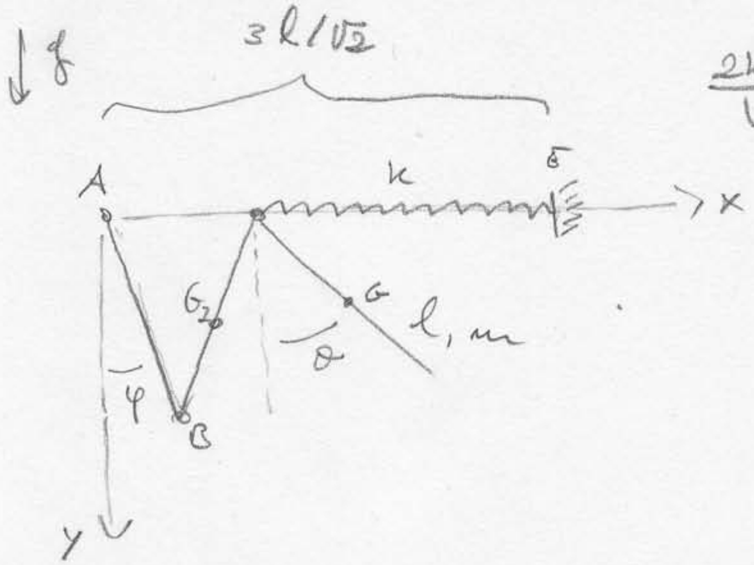
$$T = 2\cdot\frac{1}{2}\left(\frac{1}{12} m \left(\frac{2r}{\sqrt{3}}\right)^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{12} m \left(\frac{2r}{\sqrt{3}}\right)^2 + m r^2\right)\left(\frac{\omega}{2}\right)^2$$

$$= \frac{1}{9} m r^2 \omega^2 + \frac{5}{4}\frac{1}{9} m r^2\omega^2 = \frac{1}{4} m r^2 \omega^2$$



$$F \frac{r}{\sqrt{3}} |d\theta| \cos 30^\circ = M |d\theta|$$

$$\frac{F r}{2} = M$$



$$\frac{2kl}{\sqrt{2}} = mg$$

$$G = \left(2l \operatorname{sen}\varphi + \frac{l}{2} \operatorname{sen}\vartheta \, , \; \frac{l}{2} \cos\vartheta \right)$$

$$\dot{G} = l \left(2\cos\varphi \, \dot\varphi + \frac{1}{2} \cos\vartheta \, \dot\vartheta \, , \; -\frac{1}{2} \operatorname{sen}\vartheta \, \dot\vartheta \right)$$

$$\dot{G}^2 = l^2 \left\{ 4\cos^2\varphi \, \dot\varphi^2 + 2\cos\varphi \cos\vartheta \, \dot\varphi \dot\vartheta + \frac{1}{4} \dot\vartheta^2 \right\}$$

$$V = -mg \frac{l}{2} \cos\varphi - mg \frac{l}{2} \cos\varphi - mg \frac{l}{2} \cos\vartheta + \frac{1}{2} k d^2$$

$$= -mg \frac{l}{2} \left(2\cos\varphi + \cos\vartheta \right) + \frac{1}{2} k \left\{ \frac{3l}{\sqrt{2}} - 2l \operatorname{sen}\varphi \right\}^2$$

$$= -\frac{kl^2}{\sqrt{2}} \left(2\cos\varphi + \cos\vartheta \right) + \frac{1}{2} k \left\{ \frac{9l^2}{2} - \frac{12}{\sqrt{2}} l^2 \operatorname{sen}\varphi + 4l^2 \operatorname{sen}^2\varphi \right\}$$

$$= -\frac{kl^2}{\sqrt{2}} \left(2\cos\varphi + \cos\vartheta + 6 \operatorname{sen}\varphi - 2\sqrt{2} \operatorname{sen}^2\varphi \right) + \text{cost.}$$

$$\frac{\partial V}{\partial \vartheta} = + \frac{kl^2}{\sqrt{2}} \operatorname{sen}\vartheta$$

$$\frac{\partial V}{\partial \varphi} = -\frac{kl^2}{\sqrt{2}} \left(-2 \operatorname{sen}\varphi + 6\cos\varphi - 4\sqrt{2} \operatorname{sen}\varphi \cos\varphi \right)$$

$$\frac{\partial V}{\partial \vartheta} = 0 \Rightarrow \vartheta = 0, \cancel{\pi}$$

$$\frac{\partial V}{\partial \varphi} = 0 \Rightarrow \varphi = \frac{\pi}{4}$$

$$\frac{\partial^2 V}{\partial \vartheta^2} = \frac{k l^2}{\sqrt{2}} \cos\vartheta$$

$$\frac{\partial^2 V}{\partial \varphi^2} = -\frac{k l^2}{\sqrt{2}} \left(-2\cos\varphi - 6 \operatorname{sen}\varphi - 4\sqrt{2}\cos^2\varphi + 4\sqrt{2} \operatorname{sen}^2\varphi \right)$$

$$\frac{\partial^2 V}{\partial \vartheta \partial \varphi} = 0$$

$$B = \frac{k l^2}{2} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 8 \end{pmatrix}$$

$$G_2 = \left(\frac{3}{2} l \operatorname{sen}\varphi, \frac{l}{2} \cos\varphi \right)$$

$$\dot{G}_2 = \frac{l}{2} \left(3\cos\varphi\, \dot{\varphi}, -\operatorname{sen}\varphi\, \dot{\varphi} \right)$$

$$\dot{G}_2^2 = \frac{l^2}{4} \left(9\cos^2\varphi + \operatorname{sen}^2\varphi \right) \dot{\varphi}^2$$

$$T = \frac{1}{2}\left(\frac{1}{3} m l^2\right)\dot{\varphi}^2 + \frac{1}{2} m \dot{G}_2^2 + \frac{1}{2}\left(\frac{1}{12} m l^2\right)\dot{\varphi}^2$$

$$+ \frac{1}{2} m \dot{G}^2 + \frac{1}{2}\left(\frac{1}{12} m l^2\right)\dot{\vartheta}^2$$

$$= \frac{1}{6} m l^2 \dot{\varphi}^2 + \frac{1}{24} m l^2 \dot{\varphi}^2 + \frac{1}{24} m l^2 \dot{\vartheta}^2 + \frac{l^2}{8} m \overbrace{\left(9\cos^2\varphi + \operatorname{sen}^2\varphi\right)}^{8\cos^2\varphi + 1} \dot{\varphi}^2$$

$$+ \frac{m l^2}{2} \left\{ 4\cos^2\varphi\, \dot{\varphi}^2 + 2\cos\varphi\cos\vartheta\, \dot{\varphi}\dot{\vartheta} + \frac{1}{4}\dot{\vartheta}^2 \right\}$$

$$= \frac{1}{24} m l^2 \left\{ 8\,(8\cos^2\varphi + 1)\,\dot{\varphi}^2 + \dot{\vartheta}^2 + 48\cos^2\varphi\,\dot{\varphi}^2 + 24\cos\varphi\cos\vartheta\,\dot{\varphi}\dot{\vartheta} + 3\dot{\vartheta}^2 \right\}$$

$$= \frac{1}{24} m l^2 \left\{ \left(72\cos^2\varphi + 8\right)\dot{\varphi}^2 + 4\dot{\vartheta}^2 + 24\cos\varphi\cos\vartheta\,\dot{\varphi}\dot{\vartheta} \right\}$$

$$A = \frac{1}{12} m l^2 \begin{pmatrix} 4 & 12/\sqrt{2} \\ 12/\sqrt{2} & 44 \end{pmatrix} = \frac{1}{3} m l^2 \begin{pmatrix} 1 & 3/\sqrt{2} \\ 3/\sqrt{2} & 11 \end{pmatrix}$$