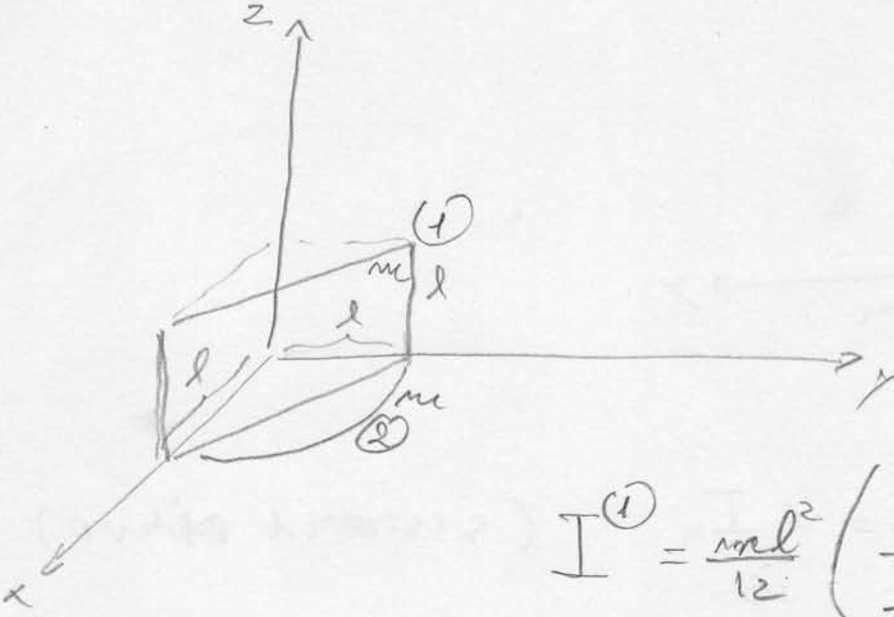
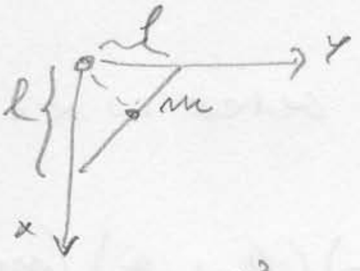


$$I^{(1)} = \frac{ml^2}{12}\begin{pmatrix} 8 & -2 & -3 \\ -2 & 8 & -3 \\ -3 & -3 & 8 \end{pmatrix}$$

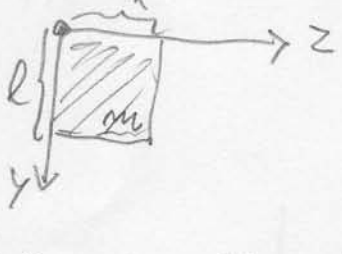
(1) I_z LO SI PUÒ CALCOLARE PER CONTRAZIONE COME



*

$$I_z = \frac{1}{12} m (\sqrt{2}\, l)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{1}{6} m l^2 + \frac{m l^2}{2} = \frac{2}{3} m l^2$$

PER I_x SEMPRE PER CONTRAZIONE LO SI PUÒ CALCOLARE COME



$$I_x = \frac{2}{3} m l^2$$

PER I_{YZ} IL RIFERIMENTO POGGIA SULLA STESSA FIGURA QUINDI USANDO HUYGENS-STEINER PER IL MOMENTO CENTRIFUGO $I_{yz} = -m\left(\frac{l}{2}\right)^2 = -\frac{m}{4} l^2$

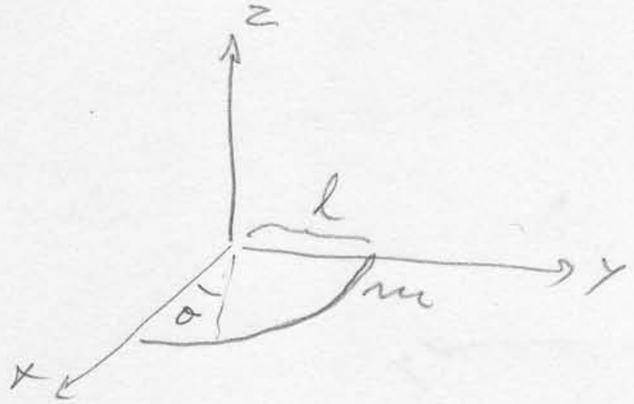
PER I_{xy} RICONSIDERIAMO *

$$I_{xy} = -\int xy\, dm = -\int x(l-x)\, dm$$

USIAMO $dm = \frac{m}{l} dx$

$$I_{xy} = -\frac{m}{l}\int_0^l x(l-x)\, dx = -\frac{m}{l}\left(\frac{l^3}{2} - \frac{l^3}{3}\right) = -\frac{m}{6} l^2$$

DALLE EVIDENTI SIMMETRIE SEGUE LA MATRICE.



$$I_z = I_x + I_y = 2 I_x \quad \text{(SISTEMA PIANO)}$$

$$I_z = m l^2 \quad \text{(TUTTI GLI ELEMENTI DI MASSA } dm \text{ DISTANO } l \text{ DALL'ASSE)}$$

$$I_x = I_y = \frac{1}{2} m l^2$$

$$I_{xz} = I_{xy} = 0 \quad \text{(IL SISTEMA GIACE IN } x = 0\text{)}$$

$$I_{xy} = -\int xy\, dm = -\int_0^{\pi/2} (l\cos\theta)(l \sin\theta) \left(\frac{m}{\pi/2} d\theta\right)$$

$$= -\frac{2}{\pi} m l^2 \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = -\frac{2}{\pi} m l^2 \int_0^1 \sin\theta \, d\sin\theta$$

$$= -\frac{1}{\pi} m l^2$$

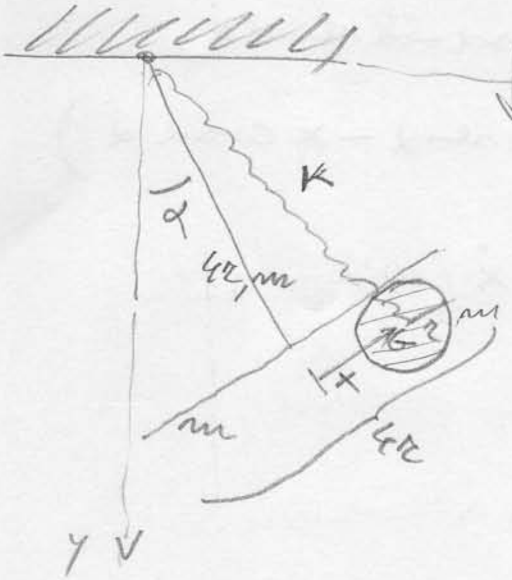
$$I^{(2)} = m l^2 \begin{pmatrix} \frac{1}{2} & -\frac{1}{\pi} & 0 \\ -\frac{1}{\pi} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

IL PIANO DI SIMMETRIA $x = y$ IMPLICA CHE IL VETTORE ORTOGONALE $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ È UN AUTOVETTORE

IL CORRISPONDENTE MOMENTO PRINCIPALE È

$$m l^2 \left(\frac{4}{3} + \frac{1}{\pi} \right)$$

COORDINATE LAGRANGIANE (α, x)



$$\frac{k}{m} = \frac{4g}{r}$$

a) CALCOLARE V, PUNTI STAZIONARI, STABILITÀ

b) CALCOLARE T

c) PICCOLE OSCILLAZIONI

$$(z_G, y_G) = \left(5r \sin\alpha + x\cos\alpha,\ 5r\cos\alpha - x\sin\alpha\right)$$

$$V = \frac{k}{2}\left(x^2 + (5r)^2\right) - mg\left(2r\cos\alpha + 4r\cos\alpha\right)$$

$$- mg\left(5r\cos\alpha - x\sin\alpha\right)$$

$$= \frac{2mg}{r}\left(x^2 + 25r^2\right) - mg\left(11r\cos\alpha - x\sin\alpha\right)$$

$$\frac{\partial V}{\partial x} = \frac{4mg}{r}x + mg\sin\alpha$$

$$\frac{\partial V}{\partial \alpha} = -mg\left(-11r\sin\alpha - x\cos\alpha\right)$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial \alpha} = 0 \Rightarrow \begin{cases} 4x = -r\sin\alpha \\ -11r\sin\alpha = x\cos\alpha \end{cases} \Rightarrow \begin{matrix} x = 0 \\ \alpha = 0 \end{matrix}$$

SIA $\beta = r\alpha$

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{\text{STAZION.}} = \frac{4mg}{r} \qquad \left.\frac{\partial^2 V}{\partial\beta\,\partial x}\right|_{\text{STAZ.}} = \frac{mg}{r} \qquad \left.\frac{\partial^2 V}{\partial\beta^2}\right|_{\text{STAZ.}} = 11\frac{mg}{r}$$

$$B = \frac{mg}{r}\begin{pmatrix} 4 & 1 \\ 1 & 11 \end{pmatrix} \qquad \begin{matrix} B_{xx} > 0 \\ \det B > 0 \end{matrix} \Rightarrow \text{STABILE}$$

$$\dot{G} = (\dot{x}_G, \dot{y}_G) = (5r\cos\alpha\,\dot{\alpha} + \dot{x}\cos\alpha - x\,\mathrm{sen}\,\alpha\,\dot{\alpha},$$
$$-5r\,\mathrm{sen}\,\alpha\,\dot{\alpha} - \dot{x}\,\mathrm{sen}\,\alpha - x\cos\alpha\,\dot{\alpha})$$

$$\dot{G}^2 = 25r^2\dot{\alpha}^2 + \dot{x}^2 + x^2\dot{\alpha}^2 + 10r\dot{\alpha}\dot{x}$$

$$T = \frac{1}{2}\dot{\alpha}^2\left(\frac{1}{3}m(4r)^2 + \frac{1}{12}m(4r)^2 + m(4r)^2\right)$$
$$+\frac{1}{2}m\left(25r^2\dot{\alpha}^2 + \dot{x}^2 + x^2\dot{\alpha}^2 + 10r\dot{\alpha}\dot{x}\right)$$
$$+\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{x}}{2} + \dot{\alpha}\right)^2$$
$$= \frac{289}{12}r^2\dot{\alpha}^2 m + \frac{3}{4}m\dot{x}^2 + \frac{1}{2}mx^2\dot{\alpha}^2$$
$$+\frac{11}{2}mr\dot{\alpha}\dot{x}\cos 2\alpha +$$

$$A = m\begin{pmatrix} 3/2 & 11/2 \\ 11/2 & \frac{289}{6} \end{pmatrix}$$