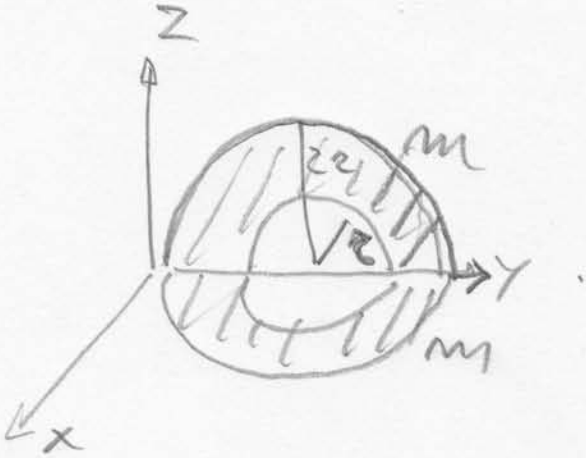


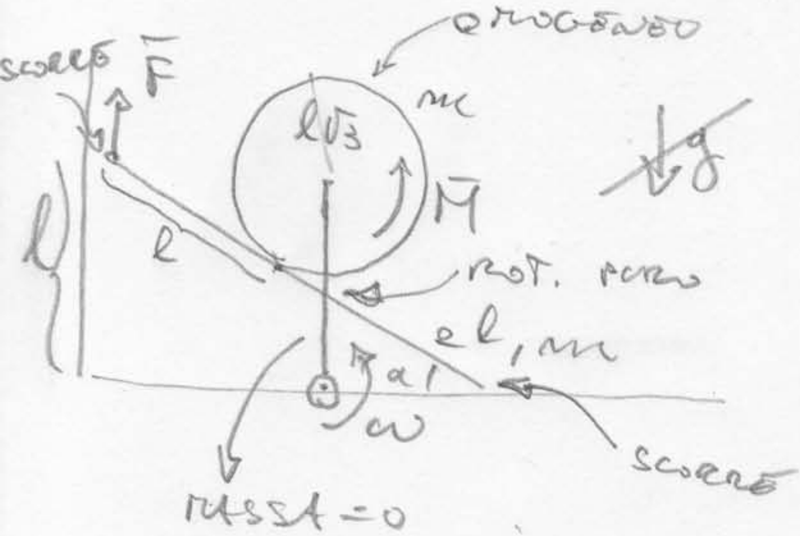
(a) MATRICE D'INERZIA

(b) SIMMETRIE, MOMENTI E ASSI PRINCIPALI

(c) SE $\bar{\omega} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} s^{-1}$

$m = 10\,g$, $r = 10\,cm$

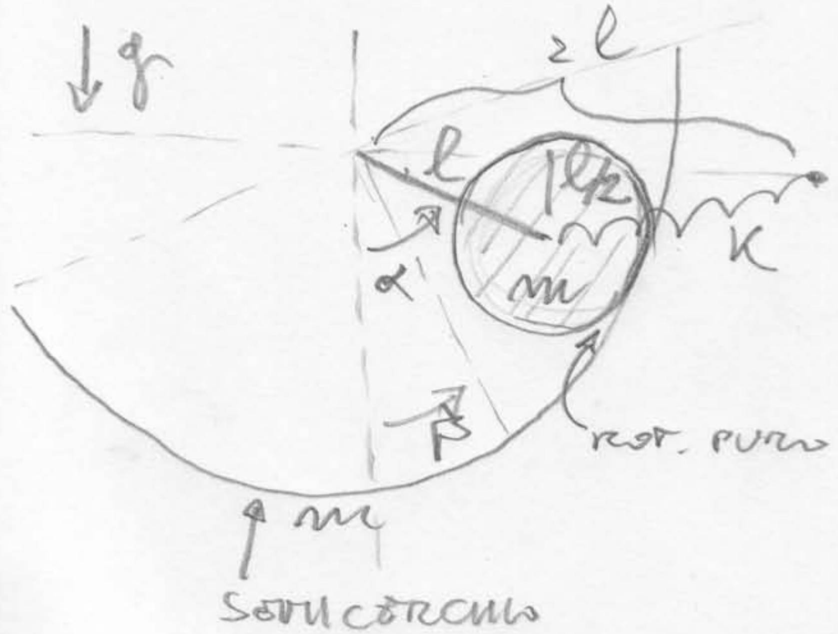
TROVARE $\bar{L}$ E T



(a) CENTRI ISTANTANEI DI ROTAZIONE

(b) DATO ω TROVARE T

(c) RELAZIONE TRA $\bar{F}$ E $\bar{M}$ PER EQUILIBRIO

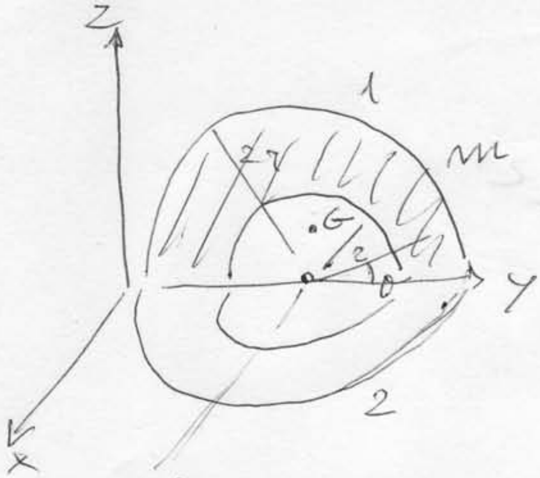


$k = \dfrac{mg}{2\sqrt{3}\,l}$

(a) TROVARE T E V

(b) PUNTI STAZIONARI E STABILITÀ

(c) MODI E PULSAZIONI PICCOLE OSCILLAZIONI



$$A_G = \frac{\pi 4 r^2}{2} = 2\pi r^2$$

$$A_P = \frac{\pi r^2}{2}$$

$$A = \frac{3}{2}\pi r^2 \qquad \rho = \frac{2m}{3\pi r^2}$$

$$M_G = \frac{4}{3} m$$

$$M_P = \frac{1}{3} m$$

$$\hat{I} = \frac{1}{2} M_G (2r)^2 - \frac{1}{2} M_P (r)^2$$

$$= \frac{1}{2}\frac{4}{3} m 4r^2 - \frac{1}{6} m r^2 = \left(\frac{8}{3} - \frac{1}{6}\right) m r^2$$

$$= \frac{15}{6} m r^2 = \frac{5}{2} m r^2$$

$$I_{yy} = \frac{5}{4} m r^2$$

$$I_{zz} = \frac{5}{4} m r^2 + m(2r)^2 = \frac{21}{4} m r^2$$

$$I_{xx} = \frac{2\hat{I} + (2m)(2r)^2}{2} = \hat{I} + 4mr^2$$

$$= \frac{13}{2} m r^2 = I_{yy} + I_{zz}$$

$$z_G = \frac{1}{m}\rho \int_r^{2r} dr' \int_0^{\pi} d\theta \; z\, r' = \frac{2}{3\pi r^2} \int_r^{2r} dr' \int_0^{\pi} d\theta \; r'^2 \operatorname{sen}\theta$$

$$= \frac{4}{3\pi r^2} \int_r^{2r} (r')^2 dr' = \frac{4}{3\pi r^2}\frac{1}{3}\left(8r^3 - r^3\right)$$

$$= \frac{28}{9\pi} r$$

$$I_{zy} = -m z_G y_G = -\frac{56}{9\pi} m r^2$$

$$I_1 = \begin{pmatrix} \frac{13}{2} & 0 & 0 \\ 0 & 5/4 & -\frac{56}{9\pi} \\ 0 & -\frac{56}{9\pi} & \frac{21}{4} \end{pmatrix} m r^2$$

OTTENGO I_2 CON $x \longleftrightarrow z$

$$I_2 = \begin{pmatrix} \frac{21}{4} & -\frac{56}{9\pi} & 0 \\ -\frac{56}{9\pi} & 5/4 & 0 \\ 0 & 0 & \frac{13}{2} \end{pmatrix} m r^2$$

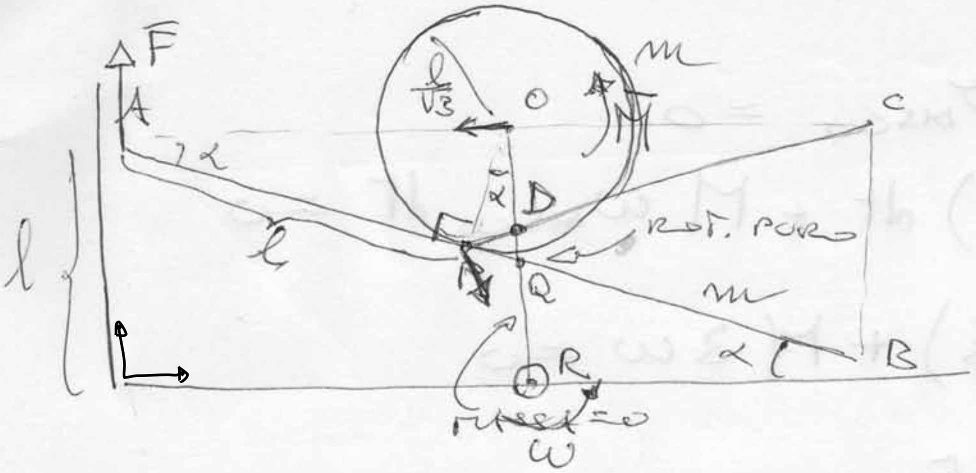
$$I = \begin{pmatrix} \frac{47}{4} & -\frac{56}{9\pi} & 0 \\ -\frac{56}{9\pi} & 5/2 & -56/9\pi \\ 0 & -\frac{56}{9\pi} & \frac{47}{4} \end{pmatrix} m r^2$$

IL VETTORE $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ È ORTOGONALE AL PIANO DI SIMMETRIA $x = z$ E QUINDI È UN AUTOVETTORE ALL'AUTOVALORE $47/4\, m r^2$

$$\det(I - \lambda \mathbb{1}) = \left(\frac{47}{4} - \lambda\right) \left\{ \left(\frac{5}{2} - \lambda\right)\left(\frac{47}{4} - \lambda\right) - 2\left(\frac{56}{9\pi}\right)^2 \right\}$$

$$\lambda^2 - \frac{57}{4}\lambda + \left\{ \frac{235}{4} - 2\left(\frac{56}{9\pi}\right)^2 \right\} = 0$$

$$\lambda_{1,2} = 57/4 \pm \sqrt{\left(\frac{57}{4}\right)^2 - 4\left\{235 - 8\left(\frac{56}{9\pi}\right)^2\right\}}$$



POICHÉ IL RAGGIO È $l/\sqrt{3}$ IL CENTRO O DEL DISCO GIACE SULLA RETTA CA DOVE C È IL CENTRO ISTANTANEO DI ROTAZIONE DELL'ASTA MASSIVA

nota che OP=OQ/cos 30=L/2 quindi RQ=L/2

CONSIDERANDO IL MOTO DEI PUNTI P E O SEGUE CHE IL CENTRO ISTANTANEO DI ROTAZIONE DEL DISCO È IN D

NOTA CHE $R = \left(l/\cos\alpha,\ 0\right)$

$$D = \left(l/\cos\alpha,\ \frac{l}{2}+\frac{l}{6}\right) = \left(\frac{2\,l\,\cancel{\sqrt3}}{\sqrt3\,\cancel{2}},\ \frac{2}{3}l\right)$$

NOTA CHE IL TRIANGOLO PDQ È EQUILATERO, IN QUANTO TUTTI GLI ANGOLI SONO 60°, E IL LATO È $\left(l/\sqrt3\right)/\sqrt3 = l/3$

$\omega_{ASTA}\, PC = \omega_{DISCO}\, PD$ $\qquad PC = RO = l$

$\qquad PD = l/3$

$\omega\, RO = \omega_{DISCO}\, DO$ $\qquad DO = l - \frac{2}{3}l = \frac{l}{3}$

QUINDI $\omega_{ASTA} = \omega_{DISCO}/3$, $\omega = \omega_{DISCO}/3$ $\Rightarrow$ $\omega_{DISCO} = 3\omega$, $\omega_{ASTA} = \omega$

$$T = \frac{1}{2}\left\{\frac{1}{12}(m)(2l)^2 + m l^2\right\}\omega^2 + \frac{1}{2}\left\{\frac{1}{2}m\left(\frac{l}{\sqrt3}\right)^2 + m\left(\frac{l}{3}\right)^2\right\}(3\omega)^2$$

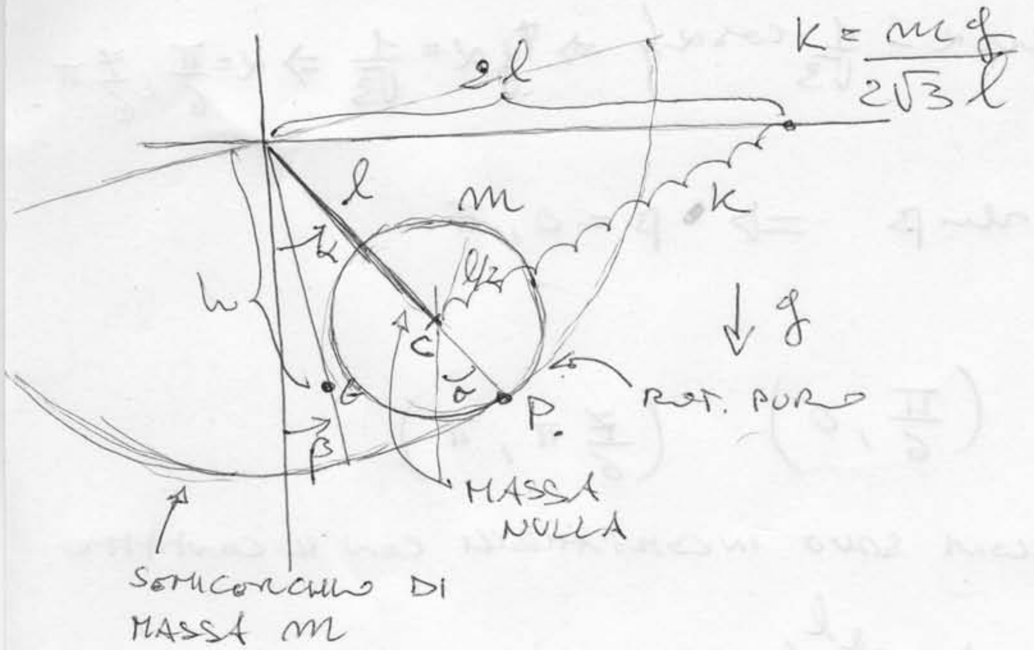
$$= \frac{2}{3} m l^2 \omega^2 + \frac{\cancel{3}}{2}\,\frac{5}{6} m l^2 \omega^2 = \frac{23}{3} m l^2 \omega^2$$

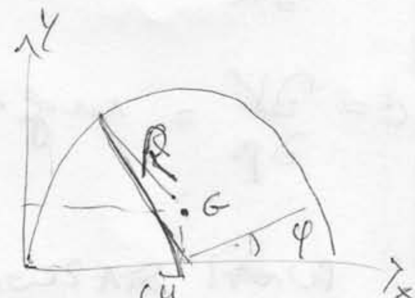
$$-F\, dY_A + M\, d\theta_{DISCO} = 0$$

$$-F\, \omega_{ASTA}(l\sqrt{3})\, dt + M\, \omega_{DISCO}\, dt = 0$$

$$-F\, \omega\,(l\sqrt{3}) + M\, 3\omega = 0$$

$$M = \frac{lF}{\sqrt{3}}$$





$$y_G = \frac{1}{\pi R}\int_0^{\pi} y R\, d\varphi = \frac{1}{\pi R}\int_0^{\pi} R^2 \sin\varphi\, d\varphi = \frac{2R}{\pi} = \frac{2}{\pi}\frac{3}{2} l$$

$$\left.\begin{aligned} v_P &= v_C + \dot\theta \frac{l}{2} = \dot\alpha\, l + \dot\theta \frac{l}{2} \\ v_P &= \dot\beta\, \frac{3}{2} l \end{aligned}\right\} \Rightarrow \dot\theta = 3\dot\beta - 2\dot\alpha$$

$$T = \frac{1}{2}\left(m\left(\frac{3}{2}l\right)^2\right)\dot\beta^2 + \frac{1}{2}m\left(l\dot\alpha\right)^2 + \frac{1}{2}\left(\frac{1}{2}m\left(\frac{l}{2}\right)^2\right)\dot\theta^2$$

$$= \frac{ml^2}{2}\left\{\frac{9}{4}\dot\beta^2 + \dot\alpha^2 + \frac{1}{8}\left(3\dot\beta - 2\dot\alpha\right)^2\right\}$$

$$= \frac{ml^2}{2}\left\{\frac{9}{4}\dot\beta^2 + \dot\alpha^2 + \frac{9}{8}\dot\beta^2 - \frac{3}{2}\dot\alpha\dot\beta + \frac{1}{2}\dot\alpha^2\right\}$$

$$= \frac{ml^2}{2}\left\{\frac{27}{4}\dot\beta^2 + \frac{3}{2}\dot\alpha^2 - \frac{3}{2}\dot\alpha\dot\beta\right\}$$

$$V = -mgl\cos\alpha - mgh\cos\beta + \frac{1}{2}K\left\{(2l)^2 + l^2 - 4l^2\sin\alpha\right\}$$

$$= -mgl\cos\alpha - mg\frac{3}{\pi}l\cos\beta - \frac{mg}{\sqrt{3}}l\sin\alpha + \text{cost.}$$

$$= mgl\left\{-\cos\alpha - \frac{3}{\pi}\cos\beta - \frac{1}{\sqrt{3}}\sin\alpha\right\} + \text{cost.}$$

$$\mathcal{L} = T - V$$

$$0=\frac{\partial V}{\partial \alpha}=mgl\left\{\operatorname{sen}\alpha-\frac{1}{\sqrt{3}}\cos\alpha\right\} \Rightarrow \operatorname{tg}\alpha=\frac{1}{\sqrt{3}} \Rightarrow \alpha=\frac{\pi}{6},\frac{7}{6}\pi$$

$$0=\frac{\partial V}{\partial \beta}=mgl\,\frac{3}{\pi}\operatorname{sen}\beta \Rightarrow \beta=0,\pi$$

PUNTI STAZIONARI $\left(\frac{\pi}{6},0\right)$ $\left(\frac{7}{6}\pi,\pi\right)$

LE ALTRE DUE OPZIONI SONO INCOMPATIBILI CON IL CONTATTO

$$\begin{pmatrix}\frac{\partial^2 V}{\partial\alpha^2} & \frac{\partial V}{\partial\beta\partial\alpha}\\ \frac{\partial^2 V}{\partial\alpha\partial\beta} & \frac{\partial^2 V}{\partial\beta^2}\end{pmatrix}=mgl\begin{pmatrix}\cos\alpha+\frac{1}{\sqrt{3}}\operatorname{sen}\alpha & 0\\ 0 & \frac{3}{\pi}\cos\beta\end{pmatrix}$$

SOLO NEL PRIMO PUNTO STAZIONARIO E' DEFINITO POSITIVO E VALE

$$B=mgl\begin{pmatrix}\frac{2}{\sqrt{3}} & 0\\ 0 & \frac{3}{\pi}\end{pmatrix}$$

$$\boxed{\omega_{1,2}=\sqrt{4\frac{g}{l}\eta_{1,2}} \qquad \bar{v}_{1,2}=\begin{pmatrix}1\\ 2-\frac{2}{3\sqrt{3}\eta_{1,2}}\end{pmatrix}}$$

INOLTRE $A=\frac{ml^2}{4}\begin{pmatrix}6 & -3\\ -3 & 27\end{pmatrix}$

$p(\lambda)=\det(B-\lambda A)$

SIA $\lambda=\frac{g}{l}4\eta$ $\det\begin{pmatrix}\frac{2}{\sqrt{3}}-6\eta & +3\eta\\ 3\eta & \frac{3}{\pi}-27\eta\end{pmatrix}$

$$0=\left(\frac{2}{\sqrt{3}}-6\eta\right)\left(\frac{3}{\pi}-27\eta\right)-9\eta^2$$

$$\frac{6}{\sqrt{3}\pi}-\left(\frac{18}{\pi}+\frac{54}{\sqrt{3}}\right)\eta+153\eta^2=0$$

$$\eta_{1,2}=\frac{\left(\frac{18}{\pi}+\frac{54}{\sqrt{3}}\right)\pm\sqrt{\left(\frac{18}{\pi}+\frac{54}{\sqrt{3}}\right)^2-\frac{24}{\sqrt{3}\pi}153}}{306}$$