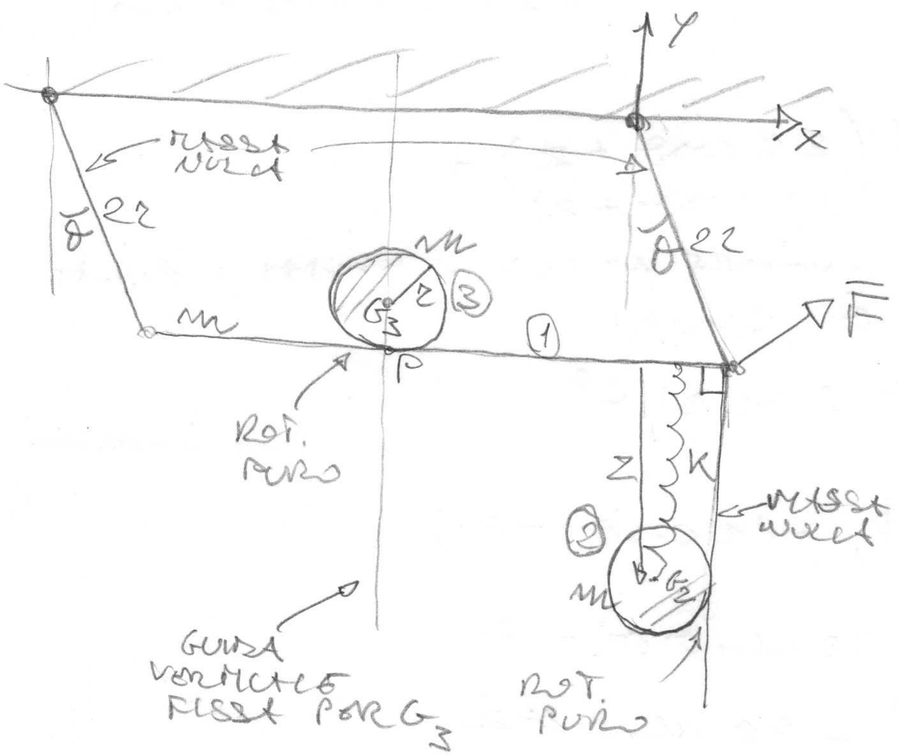
(θ, z) COORD. GENERALIZZATE



$mg = kr$

$$\bar{F} = -2kx\left(1+\frac{y}{r}\right)\hat{i} - \left(\pm 1 + \frac{x^2}{r^2}\right) kr\,\hat{j}$$

a) CALCOLARE V, PUNTI STAZIONARI, STABILITÀ

b) CALCOLARE T

c) PICCOLE OSCILLAZ.

NOTA: LA LUNGHEZZA DELL'ASTA NON SERVE

$$\bar{\nabla}\times\bar{F} = \left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right)\hat{k} = -2\frac{k}{r}x + 2kx\frac{1}{r} = 0$$

$\bar{F}$ È DEFINITA E C^1 OVUNQUE QUINDI IL POTENZIALE ESISTE.

$$U(x,y) = \int_\gamma \bar{F}\cdot d\bar{l} = \int_\sigma \bar{F}\cdot d\bar{l} + \int_\eta \bar{F}\cdot d\bar{l}$$

$$= \int_0^x F_x(x',0)\,dx' + \int_0^y F_y(x,y')\,dy' =$$

$$= \int_0^x (-2kx')\,dx' + \int_0^y -\left(\pm 1 + \frac{x^2}{r^2}\right) kr\,dy' =$$

$$= -kx^2 - \left(\pm 1 + \frac{x^2}{r^2}\right) kry = -kx^2\left(1+\frac{y}{r}\right) \mp kry$$

$$V_F = -U = kx^2\left(1+\frac{y}{r}\right) \pm kry$$

MA $x = 2r\sin\theta$

$y = -2r\cos\theta$

$$V_F(\theta) = 4kr^2\sin^2\theta\,(1-2\cos\theta) \mp k2r^2\cos\theta$$

$$V_1 = -mg\,2r\cos\theta = -2kr^2\cos\theta$$

$$V_1 = -mg\left(2r\cos\theta - r\right) = -2kr^2\cos\theta + \text{cost}$$

$$V_2 = \frac{1}{2}kz^2 - mg\left(2r\cos\theta + z\right) =$$
$$= \frac{1}{2}kz^2 - kr\left(2r\cos\theta + z\right)$$

L'ASTA HA VELOCITÀ ANGOLARE NULLA E TUTTI I PUNTI HANNO VELOCITÀ $2r\dot\theta$ QUINDI

$$T_1 = \frac{1}{2}m\left(2r\dot\theta\right)^2 = 2mr^2\dot\theta^2$$

$$G_2 = \left(2r\sin\theta + \text{cost},\ -2r\cos\theta - z\right)$$

$$\dot G_2 = \left(2r\cos\theta\,\dot\theta,\ 2r\sin\theta\,\dot\theta - \dot z\right)$$

$$\bar v_2^{\,2} = 4r^2\dot\theta^2 + \dot z^2 - 4r\sin\theta\,\dot\theta\,\dot z$$

$$T_2 = \frac{1}{2}m\left(4r^2\dot\theta^2 + \dot z^2 - 4r\sin\theta\,\dot\theta\,\dot z\right) + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot z}{r}\right)^2$$

$$= \frac{3}{4}m\dot z^2 + \frac{4}{2}m\left(r^2\dot\theta^2 - 2r\sin\theta\,\dot\theta\,\dot z\right)$$

$$G_3 = \left(\text{cost},\ -2r\cos\theta + r\right)$$

$$\dot G_3 = \left(0,\ 2r\sin\theta\,\dot\theta\right)$$

$$\bar v_3^{\,2} = 4r^2\sin^2\theta\,\dot\theta^2$$

$$\bar v(P) - \bar v(G_3) = \bar\omega_3 \times \left(P - G_3\right)$$

PRENDIAMO LA COMPONENTE x DI QUESTA EQ.

$$v_x(P) = \omega_3 r$$

MA $v_x(P) = 2r\dot\theta\cos\theta \Rightarrow \omega_3 = 2\dot\theta\cos\theta$

$$T_3 = \frac{1}{2}m\left(4r^2\sin^2\theta\,\dot\theta^2\right) + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(2\dot\theta\cos\theta\right)^2$$

$$= \frac{1}{2}mr^2\dot\theta^2\left(4\sin^2\theta + 2\cos^2\theta\right) =$$

$$= \frac{1}{2}mr^2\dot\theta^2\,2\left(1 + \sin^2\theta\right)$$

$$V = 4kr^2 \operatorname{sen}^2\theta\,(1-2\cos\theta) \mp k2r^2\cos\theta - 6kr^2\cos\theta + \frac{1}{2}kz^2 - krz$$

$$= 4kr^2 \operatorname{sen}^2\theta\,(1-2\cos\theta) - kr^2\cos\theta\,(6\pm 2) + \frac{1}{2}kz^2 - krz$$

$$\frac{\partial V}{\partial \theta} = 4kr^2(2\operatorname{sen}\theta\cos\theta)(1-2\cos\theta) + 8kr^2\operatorname{sen}^3\theta + kr^2\operatorname{sen}\theta\,(6\pm 2)$$

$$= \operatorname{sen}\theta\; kr^2\left(8\cos\theta(1-2\cos\theta) + 8\operatorname{sen}^2\theta + 6\pm 2\right)$$

$$\frac{\partial V}{\partial z} = k(z-r)$$

PUNTO STAZIONARIO $(\theta, z) = (0,0)$

$$\left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\substack{\text{punto}\\\text{staz}}} = \cos\theta\, kr^2\left(8\cos\theta(1-2\cos\theta) + 8\operatorname{sen}^2\theta + 6\pm 2\right)\Big|_{\substack{\text{punto}\\\text{staz}}} = kr^2 \begin{cases} 0 \\ -4 \end{cases}$$

$$\left.\frac{\partial^2 V}{\partial \theta\,\partial z}\right|_{P.S.} = 0 \qquad \frac{\partial^2 V}{\partial z^2} = k$$

QUINDI IL PRIMO CASO, ALMENO AL SECONDO ORDINE, E' INDIFFERENTE MENTRE IL SECONDO CASO E' INSTABILE.

$$8\cos\theta(1-2\cos\theta) + 8\operatorname{sen}^2\theta + 6\pm 2 =$$

$$= 8\cos\theta - 16\cos^2\theta + 8\operatorname{sen}^2\theta + 6 \pm 2$$

$$= 8\cos\theta - 24\cos^2\theta + 14 \pm 2$$

$$12x^2 - 4x - 7 \mp 1 = 0$$

$$\cos\theta_{1,2} = x_{1,2} = \frac{2 \pm \sqrt{4 + (7\pm 1)12}}{24}$$

FORNISCONO ALTRI PUNTI STAZIONARI