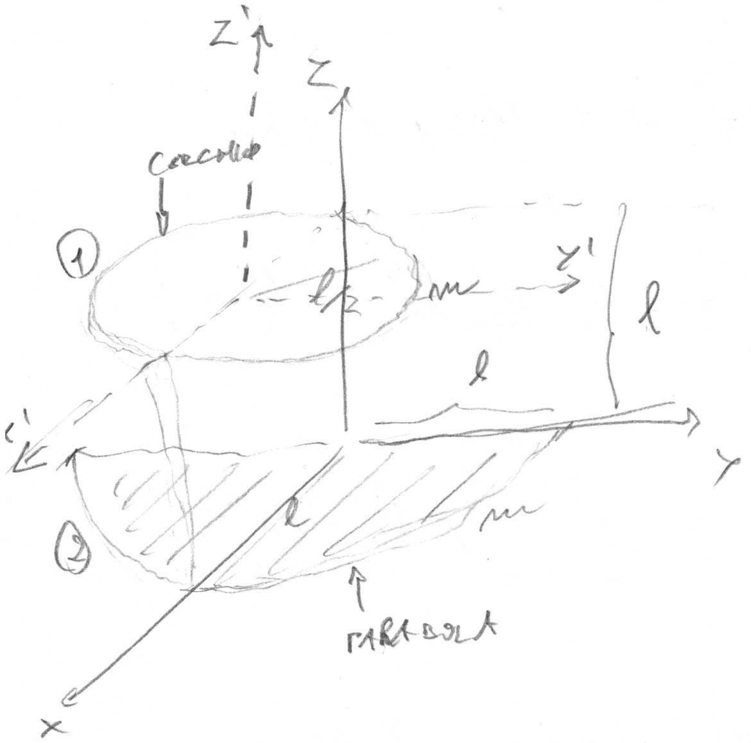

1) MATRICE D'INERZIA (1), (2) E TOTALE

2) SIMMETRIE, MOMENTI E ASSI PRINCIPALI

3) SE $\omega = (0,0,1)\ s^{-1}$ $m = 10\ g$, $l = 10$ cm TROVARE $\bar{L}, T, \bar{M}$

CORPO 1

USANDO HUYGENS-STEINER

$$I^{(1)} = \frac{ml^2}{8}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + ml^2 \begin{pmatrix} 1 & 0 & -1/2 \\ 0 & 5/4 & 0 \\ -1/2 & 0 & 1/4 \end{pmatrix} =$$

$$= \frac{ml^2}{8}\begin{pmatrix} 9 & 0 & -4 \\ 0 & 11 & 0 \\ -4 & 0 & 4 \end{pmatrix}$$

CORPO 2

$$m = \rho A \qquad A = \int dx\,dy = \int_{-l}^{l} dy \int_0^{l - y^2/l} dx$$

$$= \int_{-l}^{l} dy \left(l - y^2/l\right) = 2\int_0^l dy \left(l - y^2/l\right)$$

$$= 2\left\{ l^2 - \frac{y^3}{3l}\Big|_0^l \right\} = \frac{4}{3} l^2$$

$$\Rightarrow \rho = \frac{3}{4}\frac{m}{l^2}$$

$$I_x^{②} = 2\rho \int_0^{\ell} y^2 dy \int_0^{\ell - y^2/\ell} dx = 2\rho \int_0^{\ell} y^2 (\ell - y^2/\ell)\, dy$$

$$= 2\rho \left\{ \frac{\ell y^3}{3} - \frac{y^5}{5\ell} \right\}\Big|_0^{\ell} = 2\rho \frac{2}{15} \ell^4 = \frac{1}{5} m \ell^2$$

$$I_y^{②} = 2\rho \int_0^{\ell} dy \int_0^{\ell - y^2/\ell} x^2 dx = \frac{2\rho}{3} \int_0^{\ell} dy \left(\ell - \frac{y^2}{\ell}\right)^3$$

$$= \frac{1}{2} \frac{m}{\ell^2} \int_0^{\ell} dy \left\{ \ell^3 - 3\ell y^2 + 3\frac{y^4}{\ell} - \frac{y^6}{\ell^3} \right\}$$

$$= \frac{1}{2} \frac{m}{\ell^2} \left\{ \ell^4 - \frac{3\ell^4}{3} + \frac{3\ell^4}{5} - \frac{\ell^4}{7} \right\} = \frac{8}{35} m \ell^2$$

$$I_z^{②} = I_x^{②} + I_y^{②} = \frac{15}{35} m \ell^2 = \frac{3}{7} m \ell^2$$

PER IL CORPO 2 I PIANI XY E ZX SONO DI SIMMETRIA QUINDI I PERPENDICOLARI, CIOÈ L'ASSE Z E L'ASSE Y SONO PRINCIPALI E COSÌ LO È IL TERZO PERPENDICOLARE (O L'INTERSEZIONE DEI PIANI) CIOÈ L'ASSE X. QUINDI TUTTI I MOMENTI CENTRIFUGHI FANNO ZERO

$$I^{②} = \frac{m\ell^2}{35} \begin{pmatrix} 7 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 15 \end{pmatrix}$$

$$I = I^{①} + I^{②} = \frac{m\ell^2}{280} \begin{pmatrix} 371 & 0 & -140 \\ 0 & 449 & 0 \\ -140 & 0 & 260 \end{pmatrix}$$

PER IL CORPO TOTALE IL PIANO XZ È DI SIMMETRIA QUINDI IL VETTORE $\bar{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ È AUTOVETTORE

$$I \bar{v}_1 = \frac{m\ell^2}{280} 449 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{QUINDI} \quad \lambda_1 = 449, \quad I_1 = \frac{449}{280} m \ell^2$$

$$p(\lambda)=\det\begin{pmatrix} a-\lambda & & d \\ & b-\lambda & \\ d & & c-\lambda \end{pmatrix} = (b-\lambda)\left\{(a-\lambda)(c-\lambda)-d^2\right\}$$

$$\lambda^2-(a+c)\lambda+ac-d^2$$

$$\lambda_{1,2}=\frac{a+c\pm\sqrt{(a+c)^2-4(ac-d^2)}}{2}$$

$$=\frac{a+c\pm\sqrt{(a-c)^2+4d^2}}{2}=\frac{631\pm\sqrt{10721}}{2}$$

3) $\omega=\begin{pmatrix}0\\0\\1\end{pmatrix}s^{-1}$, $m=10^{-2}\,kg$, $l=0{,}1\,m$

$$\bar{L}=I(\bar{\omega})=\frac{ml^2}{280}s^{-1}\left(-140\,\hat{\imath}+260\,\hat{k}\right)$$

$$=10^{-4}\left(kg\,m^2s^{-1}\right)\left(-\frac{1}{2}\hat{\imath}+\frac{13}{14}\hat{k}\right)$$

$$T=\frac{1}{2}\bar{L}\cdot\bar{\omega}=\overbrace{\left(kg\left(\frac{m}{s}\right)^2\right)}^{J}10^{-4}\,\frac{13}{28}$$

$$\bar{M}=\bar{\omega}\times\bar{L}=\hat{k}\times\left(-\frac{1}{2}\hat{\imath}+\frac{13}{14}\hat{k}\right)10^{-4}\,Nm$$

$$=-\frac{1}{2}10^{-4}\,\hat{\jmath}\,Nm$$