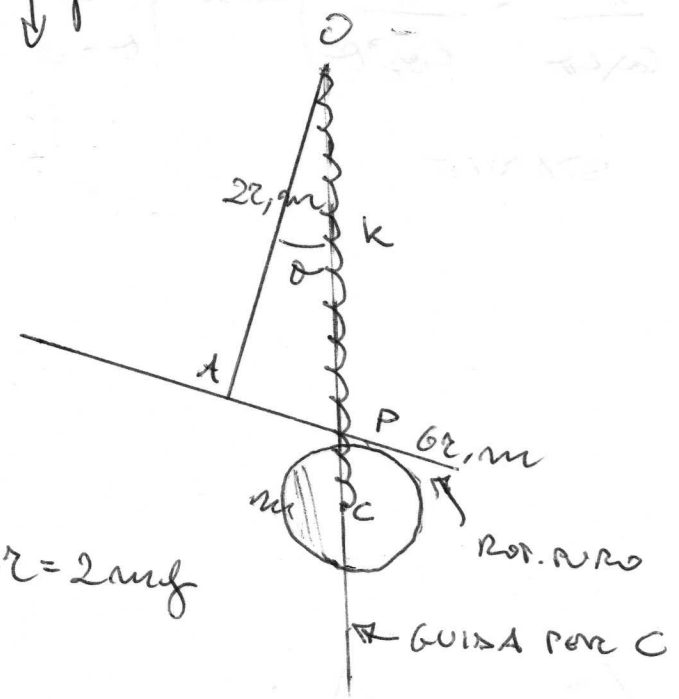
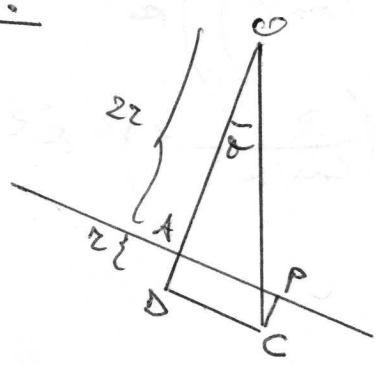
M-O



1) ENERGIA POTENZIALE PUNTI STAZIONARI STABILITÀ
2) SCRIVERE T
3) PICCOLE OSCILLAZIONI

SOL.



$\overline{AP} = 3r \operatorname{tg}\theta$

$\overline{OC} = \dfrac{3r}{\cos\theta}$

NOTA CHE PER I VINCOLI $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ QUINDI $\cos\theta > 0$.

$$V = -mgr\cos\theta - mg(2r)\cos\theta - mg\frac{3r}{\cos\theta} + \frac{1}{2}k\left(\frac{3r}{\cos\theta}\right)^2$$

$$= -\frac{kr}{2}3r\cos\theta - \frac{kr}{2}\frac{3r}{\cos\theta} + \frac{1}{2}k\frac{9r^2}{\cos^2\theta}$$

$$= \frac{3}{2}kr^2\left(-\cos\theta - \frac{1}{\cos\theta} + \frac{3}{\cos^2\theta}\right)$$

$$0 = \frac{\partial V}{\partial\theta} = \frac{3}{2}kr^2\left(-1 + \frac{1}{\cos^2\theta} - \frac{6}{\cos^3\theta}\right)(-\sin\theta)$$

$$= \frac{3}{2}kr^2\frac{1}{\cos^3\theta}\sin\theta\left(\cos^3\theta - \cos\theta + 6\right)$$

UNICA POSSIBILITÀ $\sin\theta = 0 \Rightarrow \theta = 0$

$$B = \left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=0} = \frac{3}{2}kr^2\left(1 - \frac{1}{\cos^2\theta} + \frac{6}{\cos^3\theta}\right)\cos\theta\Big|_{\theta=0}$$

$$= 9kr^2 \geq 0 \quad \text{STABILE}$$

$$v_C = \frac{d}{dt}\overline{OC} = \frac{3r}{\cos^2\theta}\sin\theta\,\dot{\theta}$$

$$\vec{\omega}_{02} = \vec{\omega}_{01} + \vec{\omega}_{12}$$

con 0 = RIF. INERZIALE
1 = RIF. PENDOLO
2 = RIF. DISCO

$$\left.\begin{aligned} \omega_{12} &= \frac{3}{\cos^2\theta}\dot{\theta} \\ \omega_{01} &= -\dot{\theta} \end{aligned}\right\} \Rightarrow \omega_{02} = \left(\frac{3}{\cos^2\theta} - 1\right)\dot{\theta}$$

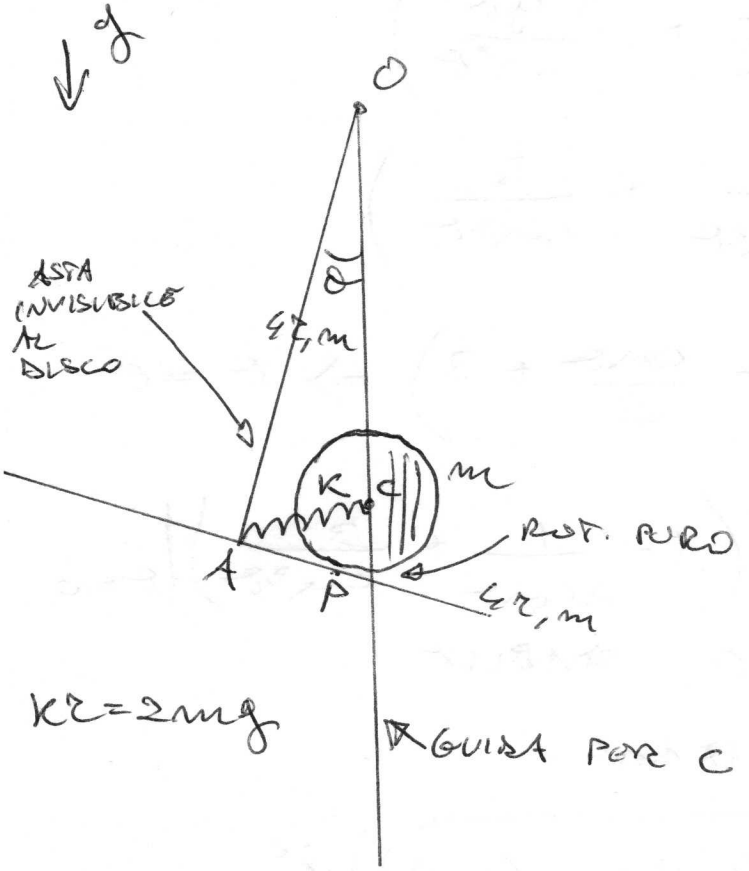
$$T = \frac{1}{2}\left(\overbrace{\frac{1}{3}m(2r)^2 + \left(\frac{1}{12}m(6r)^2 + m(2r)^2\right)}^{\frac{25}{3}mr^2}\right)\dot{\theta}^2$$

$$+ \frac{1}{2}m9r^2\frac{\sin^2\theta}{\cos^4\theta}\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{3}{\cos^2\theta} - 1\right)^2\dot{\theta}^2$$

$$A = \frac{31}{3}mr^2$$

$$\omega = \sqrt{\frac{B}{A}} = \sqrt{\frac{27}{31}\frac{k}{m}}$$

P-Z



1) ENERGIA POTENZIALE
~~2)~~ PUNTI STAZIONARI STABILITÀ
2) SCRIVERE T
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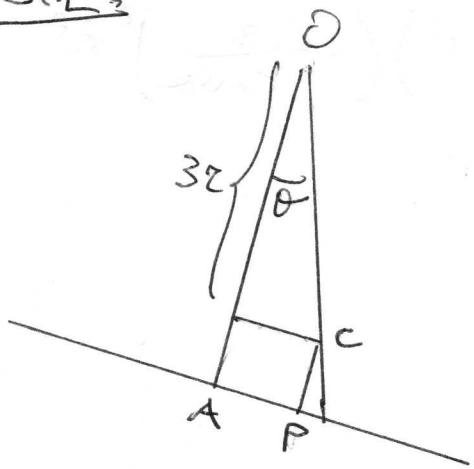
SOL:



$$\overline{OC} = \frac{3r}{\cos\theta}, \quad \overline{AP} = 3r\,\mathrm{tg}\,\theta$$

$$\overline{AC}^2 = \overline{AP}^2 + r^2$$

$$V = -mg(2r)\cos\theta - mg(4r)\cos\theta - mg\frac{3r}{\cos\theta} + \frac{1}{2}kr^2\left(9\,\mathrm{tg}^2\theta + 1\right)$$

$$= -3kr^2\cos\theta - \frac{3}{2}kr^2\frac{1}{\cos\theta} + \frac{1}{2}kr^2\left(9\,\mathrm{tg}^2\theta + 1\right)$$

$$= kr^2\left(-3\cos\theta - \frac{3}{2\cos\theta} + \frac{1}{2}\,\mathrm{tg}^2\theta\right) + \mathrm{cost.}$$

$$= \frac{\partial V}{\partial \theta} = kr^2\left(3\sin^2\theta - \frac{3}{2}\frac{\sin\theta}{\cos^2\theta} + 2\frac{\sin\theta}{\cos^2\theta}\right)$$

$$= 3kr^2\sin\theta\left(1 - \frac{1}{2\cos^2\theta} + \frac{3}{\cos^3\theta}\right)$$

$$= 3kr^2\frac{\sin\theta}{\cos^3\theta}\left(\cos^3\theta - \frac{\cos\theta}{2} + 3\right) \Rightarrow \theta = 0$$

$$B = \left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=0} = 3kr^2\cos\theta\left(1 - \frac{1}{2\cos\theta} + \frac{3}{\cos^3\theta}\right)\Big|_{\theta=0}$$

$$= \frac{21}{2}kr^2 > 0 \quad \text{STABILE}$$

$$T = \frac{1}{2}\left(\overbrace{\frac{1}{3}m(4r)^2 + \left(\frac{1}{12}m(4r)^2 + m(4r)^2\right)}^{\frac{4}{3}\cdot 17\,mr^2}\right)\dot\theta^2$$

$$+ \frac{1}{2}m\frac{2r^2}{\cos^4\theta}\sin^2\theta\,\dot\theta^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(1 + \frac{3}{\cos^2\theta}\right)^2\dot\theta^2$$

$$A = mr^2\left(\frac{4}{3}\cdot 17 + 8\right) = \frac{4}{3}\cdot 23\,mr^2$$

$$\omega = \sqrt{\frac{B}{A}} = \sqrt{\frac{63}{8\cdot 23}\frac{k}{m}}$$