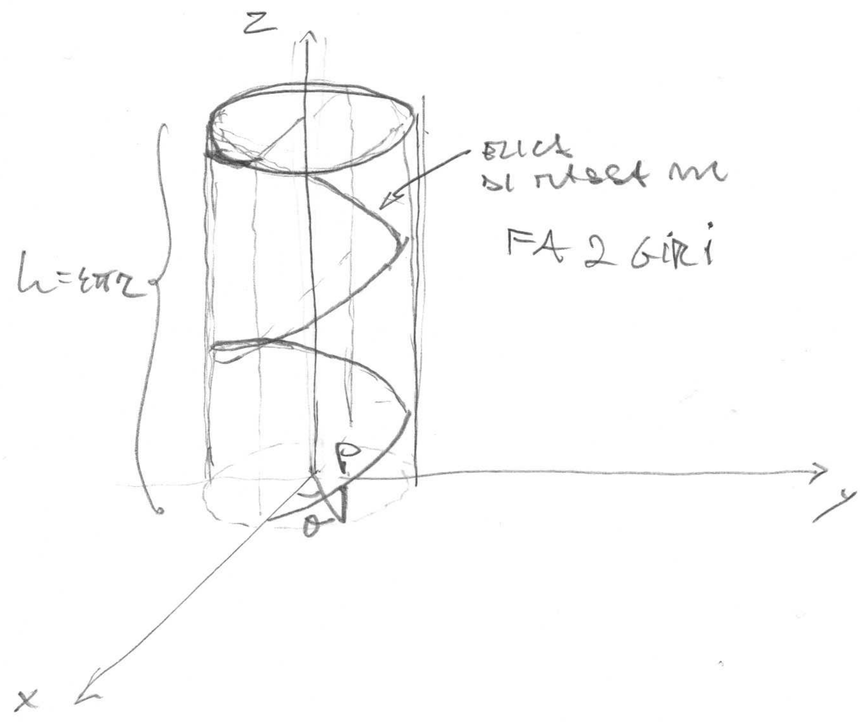


CALCOLARE

I_z, I_x, I_{xy}, I_{yz}

IL PUNTO P DELL'ELICA SI PUÒ PARAMETRIZZARE CON L'ANGOLO θ INDIVIDUATO DALLA SUA PROIEZIONE, CON $\theta \in [0, 4\pi]$ IN QUANTO L'ELICA FA 2 GIRI.

PER LA PROPORZIONE

$$\frac{z(P)}{\theta} = \frac{4\pi r}{4\pi} \Rightarrow z = r\theta$$

QUINDI

$$P(\theta) \begin{cases} x(\theta) = r\cos\theta \\ y(\theta) = r\sin\theta \\ z(\theta) = r\theta \end{cases}$$

L'ELEMENTO DI MASSA dm SI TROVA DALLA PROPORZIONE

$$\frac{dm}{d\theta} = \frac{m}{4\pi} \Rightarrow dm = \frac{m}{4\pi}\, d\theta$$

I_z SI TROVA FACILMENTE SCHIACCIANDO LUNGO Z. VIENE UN ANELLO DI MASSA m, QUINDI

$$I_z = m r^2$$

PER LA STESSA RAGIONE $I_{xy} = 0$ IN QUANTO L'ANELLO HA IL PIANO DI SIMMETRIA XZ.

$$I_x = \int (y^2+z^2)\, dm = \int_0^{4\pi} r^2 \left(\operatorname{sen}^2\theta + \theta^2\right) \frac{m}{4\pi}\, d\theta$$

$$= \frac{r^2 m}{4\pi} \left(\frac{4\pi}{2} + \int_0^{4\pi} \theta^2\, d\theta \right) = \frac{m r^2}{4\pi} \left(2\pi + \frac{(4\pi)^3}{3} \right)$$

$$= m r^2 \left(\frac{1}{2} + \frac{16}{3}\pi^2 \right)$$

$$I_{yz} = -\int yz\, dm = -\int_0^{4\pi} r^2\, \theta \operatorname{sen}\theta\, \frac{m}{4\pi}\, d\theta$$

$$= -\frac{m r^2}{4\pi} \int_0^{4\pi} \theta \operatorname{sen}\theta\, d\theta$$

$$= -\frac{m r^2}{4\pi} \left\{ -\theta \cos\theta \Big|_0^{4\pi} + \int_0^{4\pi} \cos\theta\, d\theta \right\}$$

$$= -\frac{m r^2}{4\pi} \left\{ -4\pi + 0 \right\} = + m r^2$$

COORDINATE: θ, x

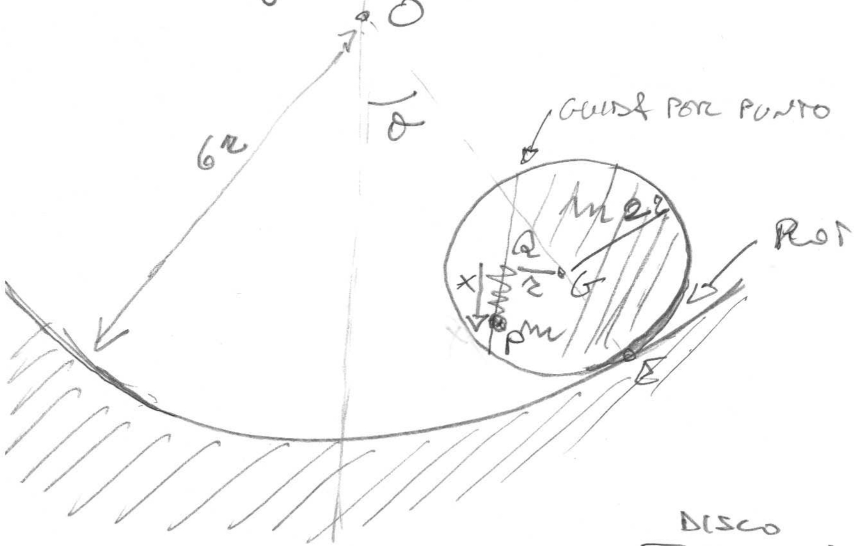
FORZE $mg = kr$

ELIMINARE g IN FAVORE DI K

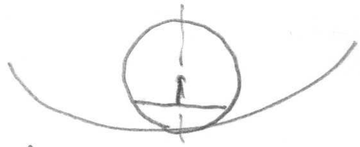
a) ENERGIA POTENZIALE
PUNTI STAZIONARI, STABILITÀ

b) ENERGIA CINETICA

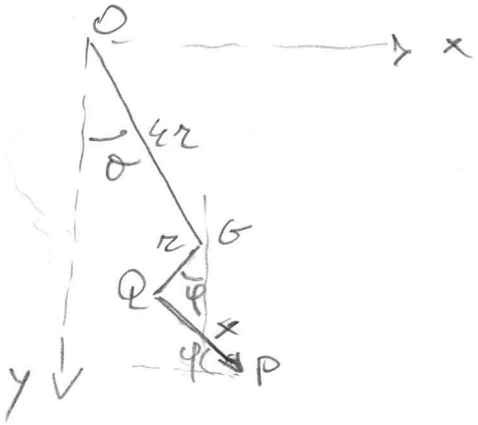
c) PICCOLE OSCILLAZIONI



STATO INIZIALE:



$$I_E^{DISCO} = \frac{1}{2} m (2r)^2 + m(2r)^2 = 6 m r^2$$



$$v_G = \dot\theta\, 4r$$

$$\omega = \frac{v_G}{\overline{GE}} = \frac{4r\dot\theta}{2r} = 2\dot\theta$$

$$\varphi = \int_0^t \omega\, dt = \int_0^t 2\dot\theta\, dt = 2\theta$$

$$G = \left(4r \sin\theta,\ 4r\cos\theta\right) = 4r\left(\sin\theta, \cos\theta\right)$$

$$Q = \left(4r\sin\theta - r\sin\varphi,\ 4r\cos\theta + r\cos\varphi\right)$$

$$P = \left(4r\sin\theta - r\sin\varphi + x\cos\varphi,\ 4r\cos\theta + r\cos\varphi + x\sin\varphi\right)$$

$$= \left(4r\sin\theta - r\sin(2\theta) + x\cos(2\theta),\ 4r\cos\theta + r\cos(2\theta) + x\sin(2\theta)\right)$$

$$\dot P = \left(4r\cos\theta\,\dot\theta - 2r\cos(2\theta)\,\dot\theta + \dot x\cos(2\theta) - 2x\sin(2\theta)\,\dot\theta,\ -4r\sin\theta\,\dot\theta - 2r\sin(2\theta)\,\dot\theta + \dot x\sin(2\theta) + 2x\cos(2\theta)\,\dot\theta\right)$$

$$V = +\frac{1}{2} k x^2 - mg\, 4r \cos\theta - mg\left(4r\cos\theta + r\cos(2\theta) + x \,\mathrm{sen}\, 2\theta\right)$$

$$= -kr\left\{ 8r\cos\theta + r\cos(2\theta) + x\,\mathrm{sen}(2\theta) \right\} + \frac{1}{2}kx^2$$

$$\frac{\partial V}{\partial \theta} = -kr\left\{ -8r\,\mathrm{sen}\theta - 2r\,\mathrm{sen}(2\theta) + 2x\cos(2\theta) \right\}$$

$$\frac{\partial V}{\partial x} = -kr\left\{ \mathrm{sen}(2\theta) \right\} + kx$$

QUINDI UN PUNTO STAZIONARIO È $\theta = 0$, $x = 0$

$$\frac{\partial^2 V}{\partial \theta^2} = -kr\left\{ -8r\cos\theta - 4r\cos(2\theta) - 4x\,\mathrm{sen}(2\theta) \right\}$$

$$\frac{\partial V}{\partial x \partial \theta} = -kr\, 2\cos 2\theta$$

$$\frac{\partial^2 V}{\partial x^2} = k$$

QUINDI $$B = k\begin{pmatrix} 12r^2 & -2r \\ -2r & 1 \end{pmatrix}$$ $\det B > 0$, $B_{11} > 0$ EQUILIBRIO STABILE

$$T = \frac{1}{2} I_G^{DISCO}\omega^2 + \frac{1}{2} m \dot{P}^2$$

$$\dot{P}^2 = 16r^2\dot\theta^2 + 4r^2\dot\theta^2 + \dot{x}^2 + 4x^2\dot\theta^2 + \text{TERMINI MISTI}$$

PER PICCOLE OSCILLAZIONI SOPRAVVIVONO SOLO I TERMINI MISTI

$$-16\, r^2 \cos\theta \cos(2\theta)\dot\theta^2 + 8r\cos\theta\,\dot\theta\dot{x}\cos(2\theta) - 4r\,\dot{x}\dot\theta\cos(2\theta)\cos(\theta)$$

$$\to -16r^2\dot\theta^2 + 8r\dot\theta\dot{x} - 4r\dot{x}\dot\theta$$

ETC. ...