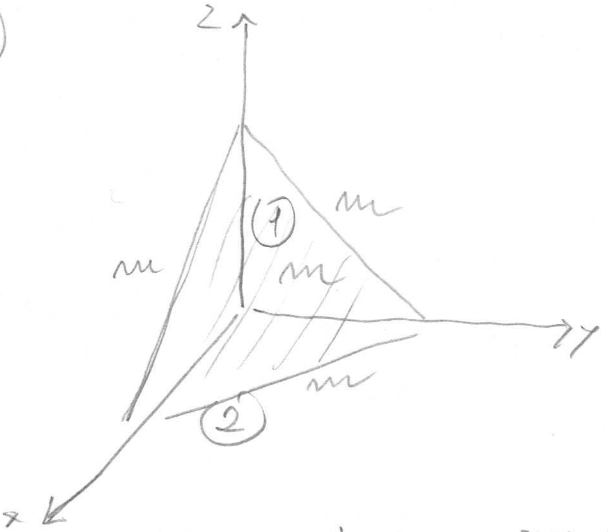
1)



a) Matrice di inerzia

b) Simmetrie, momenti e assi principali di inerzia

c)

SISTEMI PIANI — SOLIDO CONTIN.

$I_{zz}^{(1)} =$  $= 2$  $\ldots = 2\,\frac{1}{6} m l^2 = \frac{1}{3} m l^2$

$I_{xx}^{(1)} = I_{yy}^{(1)}$

$I_{xy}^{(1)} =$  $= -\frac{m}{l^2/2} \int_0^l x\,dx \int_0^{l-x} y\,dy =$

$$= -\frac{2m}{l^2} \int_0^l x\,dx\ \frac{(l-x)^2}{2} =$$

$$= -\frac{m}{l^2} \int_0^l dx \left(l^2 x - 2x^2 l + x^3 \right) = -\frac{m}{l^2} \left(\frac{l^4}{2} - \frac{2}{3} l^4 + \frac{l^4}{4} \right)$$

$$= -\frac{1}{12} m l^2 = I_{xz}^{(1)} = I_{zy}^{(1)}$$

$$I^{(1)} = \frac{1}{12} m l^2 \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$$

$$I_{yy}^{(2)} = \frac{1}{3} m l^2 = I_{xx}^{(2)} \qquad I_{zz}^{(2)} = I_{xx}^{(2)} + I_{yy}^{(2)} = \frac{2}{3} m l^2$$

$$I_{xz}^{(2)} = I_{yz}^{(2)} = 0$$

$$I_{xy}^{(2)} = -\frac{m}{l} \int_0^l x(l-x)\,dx = -\frac{m}{l} \left(\frac{l x^2}{2} - \frac{x^3}{3} \right) \Big|_0^l$$

$$= -\frac{m}{l} \left(\frac{l^3}{2} - \frac{l^3}{3} \right) = -\frac{1}{6} m l^2$$

$$I^{(2)} = \frac{1}{12} m l^2 \begin{pmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$I^{ASTE} = \frac{1}{12} m l^2 \begin{pmatrix} 16 & -2 & -2 \\ -2 & 16 & -2 \\ -2 & -2 & 16 \end{pmatrix}$$

$$I = \frac{1}{12} m l^2 \begin{pmatrix} 20 & -3 & -3 \\ -3 & 20 & -3 \\ -3 & -3 & 20 \end{pmatrix}$$

CI SONO TRE PIANI DI SIMMETRIA A 120° L'UNO DALL'ALTRO CHE SI INTERSECANO NELL'ASSE INDIVIDUATO DA

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

CHE QUINDI È PRINCIPALE. L'ELLISSOIDE È ROTONDO.

$$I \vec{v}_1 = I_1 \vec{v}_1 \quad \text{con} \quad I_1 = \frac{7}{6} m l^2 \quad (\lambda_1 = 14)$$

MA

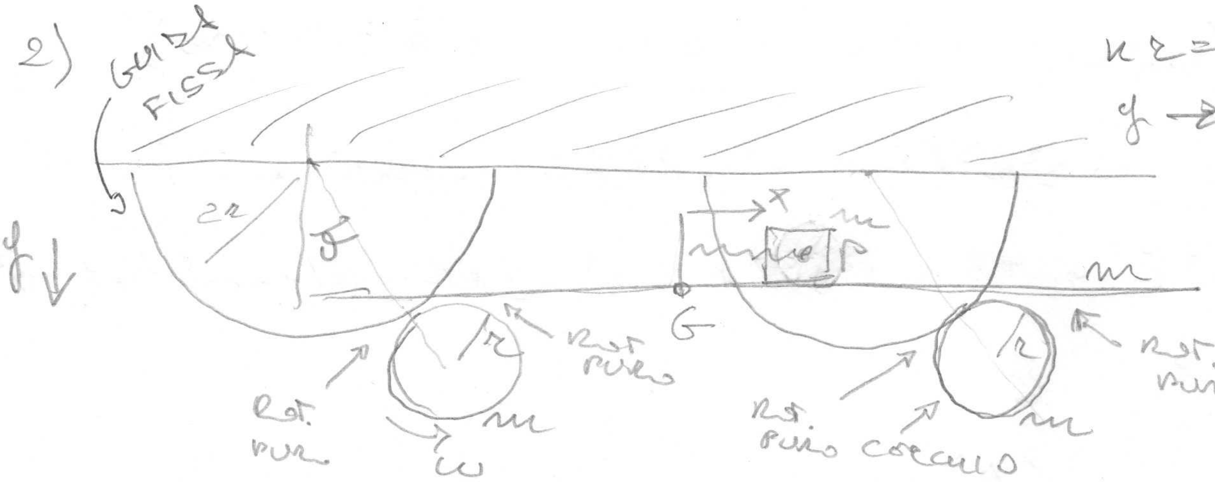
$$\lambda_1 + \lambda_2 + \lambda_3 = \mathrm{Tr} \begin{pmatrix} 20 & -3 & -3 \\ -3 & 20 & -3 \\ -3 & -3 & 20 \end{pmatrix} = 60$$

E $\lambda_2 = \lambda_3 \Rightarrow \quad \lambda_2 = \lambda_3 = 23$

$$\Rightarrow \quad I_2 = I_3 = \frac{23}{12} m l^2$$

$\vec{v}_2$ E $\vec{v}_3$ SONO UNA QUALUNQUE COPPIA DI VETTORI ORTOGONALI TRA LORO E A $\vec{v}_1$.

2)



$kr = mg$

$g \to k$

a) V, punti staz. stabilità

b) T

c) piccole oscillaz.

$$V = \frac{1}{2}kx^2 + 4mg(-3r\cos\theta)$$

$$\frac{\partial V}{\partial x} = kx$$

$$\frac{\partial V}{\partial \theta} = 12mrg\,\text{sen}\,\theta$$

$\to$ punti staz. $\begin{cases} x = 0 \\ \theta = 0 \end{cases}$

$$\frac{\partial^2 V}{\partial x^2} = k, \quad \frac{\partial^2 V}{\partial x \partial \theta} = 0, \quad \frac{\partial^2 V}{\partial \theta^2} = 12mgr\cos\theta$$

$$B = \begin{pmatrix} k & 0 \\ 0 & 12kr^2 \end{pmatrix}$$

$\omega = 3\dot\theta$

$G = (3r\,\text{sen}\,\theta - 3\theta r + \text{cost.},\ -3r\cos\theta + \text{cost})$

$P = G + (x, 0)$

$\dot G = (3r\cos\theta\,\dot\theta - 3r\dot\theta,\ 3r\,\text{sen}\,\theta\,\dot\theta)$

$\dot P = (3r\cos\theta\,\dot\theta - 3r\dot\theta + \dot x,\ 3r\,\text{sen}\,\theta\,\dot\theta)$

$\dot G^2 = 18r^2\dot\theta^2 - 18r^2\dot\theta^2\cos\theta = 18r^2\dot\theta^2(1-\cos\theta)$

$\dot P^2 = 18r^2\dot\theta^2(1-\cos\theta) + \dot x^2 + 6r\dot\theta\dot x(\cos\theta - 1)$

$$T = 2\,\frac{1}{2}\left(mr^2+mr^2\right)\left(3\dot{\theta}\right)^2 + \frac{1}{2}m\;18r^2\dot{\theta}^2(1-\cos\theta)$$

$$+\frac{1}{2}m\left\{18r^2\dot{\theta}^2(1-\cos\theta) - 6r\dot{\theta}\dot{x}(1-\cos\theta) + \dot{x}^2\right\}$$

$$= 18mr^2\dot{\theta}^2 + 18mr^2\dot{\theta}^2(1-\cos\theta) - 3r\dot{\theta}\dot{x}m(1-\cos\theta) + \frac{1}{2}m\dot{x}^2$$

$$A = m\begin{pmatrix} 1 & 0 \\ 0 & 36r^2 \end{pmatrix}$$

$$\omega_1 = \sqrt{\frac{k}{m}}\;,\quad \omega_2 = \sqrt{\frac{k}{3m}}$$

I MODI DI OSCILLAZIONE DI x E θ SONO GIÀ DISACCOPPIATI.