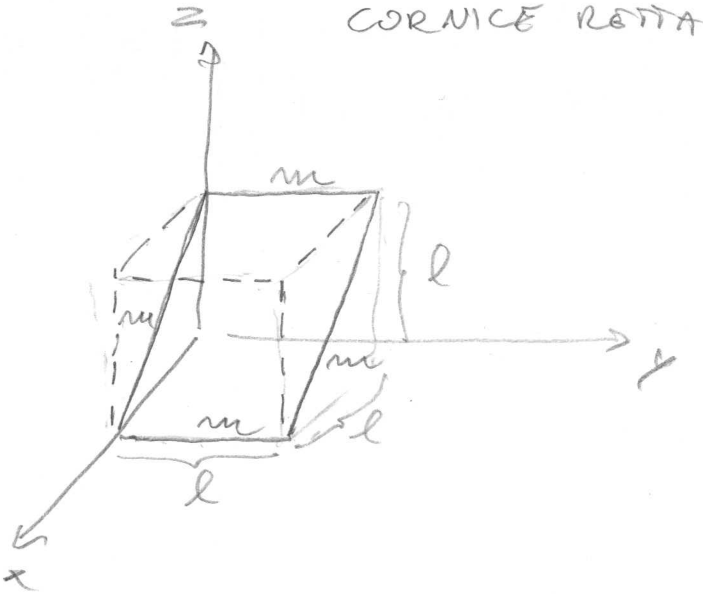
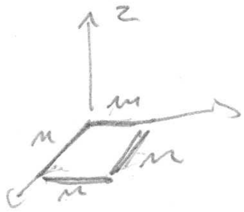
CORNICE RETTANGOLARE

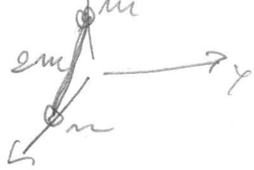
a) MATRICE DI INERZIA

b) DISCUTERE SIMMETRIE ASSI E MOMENTI PRINCIPALI

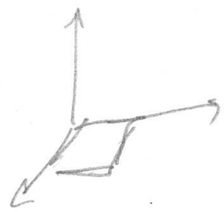
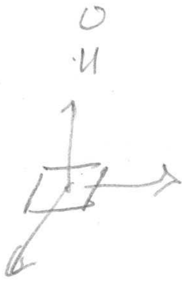


$I_z =$  $= 2\left(\frac{1}{3} m l^2\right) + 2\left\{\frac{1}{12} m l^2 + m l^2\left(1+\left(\frac{1}{2}\right)^2\right)\right.$

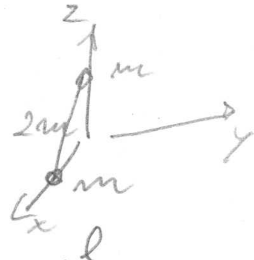
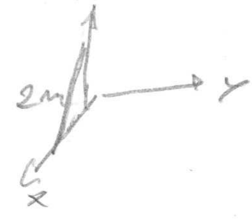
$= \frac{10}{3} m l^2 = I_x$

$I_y =$  $= 2(m l^2) + \frac{1}{12}(2m)(l\sqrt{2})^2 + 2m\left(\frac{l}{\sqrt{2}}\right)^2$

$= \frac{10}{3} m l^2$

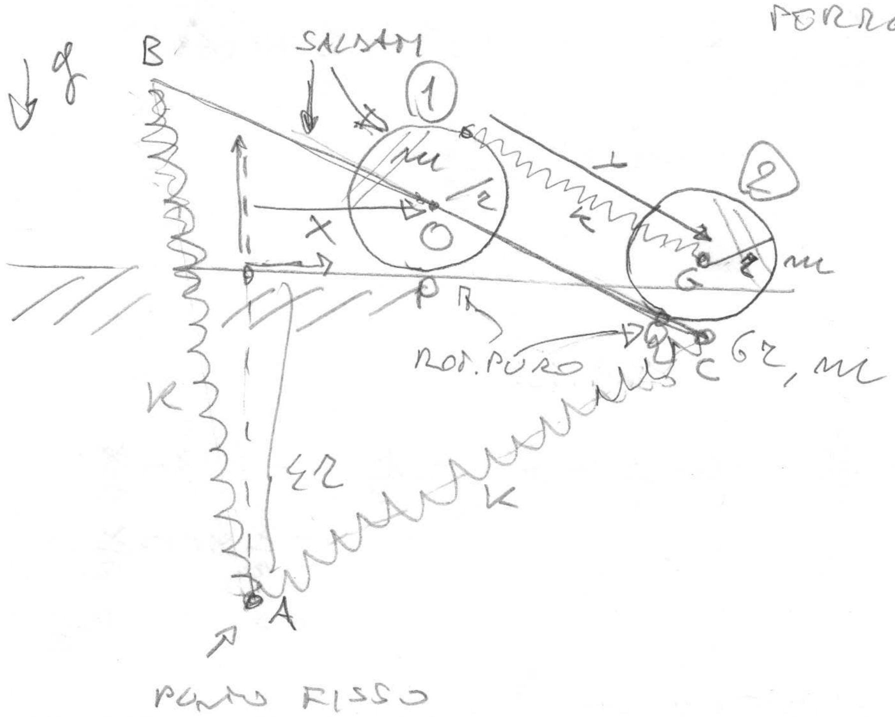
$I_{xy} =$  $=$  $-4m\left(\frac{l}{2}\right)^2 = -m l^2$

$= I_{zy}$

$I_{xz} =$  $=$  $= -\int xy \, dm$

$= -\int_0^l x(l-x)\frac{2m}{l} dx = -\frac{2m}{l}\left(\frac{l^3}{2} - \frac{l^3}{3}\right) = -\frac{1}{3} m l$

$$I = \frac{1}{3} m l^2 \begin{pmatrix} 10 & -3 & -1 \\ -3 & 10 & -3 \\ -1 & -3 & 10 \end{pmatrix}$$



PORRE $kr = mg$

SOSTITUIRE IN FAVORE DI r

TROVARE

a) $\begin{cases} V^1 \\ V^2 \end{cases}$

b) $\begin{cases} T^1 \\ T^2 \end{cases}$

c) PICCOLE OSCILLAZ.

$$O = (x, r)$$

$$Q = \left(x + y\cos x/r,\ r - y\sin x/r\right)$$

$$G = \left(x + y\cos x/r + r\sin\frac{x}{r},\ r - y\sin\frac{x}{r} + r\cos x/r\right.$$

$$C = \left(x + 3r\cos\frac{x}{r},\ r - 3r\sin x/r\right)$$

$$B = \left(x - 3r\cos\frac{x}{r},\ r + 3r\sin x/r\right)$$

$$A = (0\ -4r)$$

$$B - A = \left(x - 3r\cos\frac{x}{r},\ 5r + 3r\sin x/r\right)$$

$$C - A = \left(x + 3r\cos\frac{x}{r},\ 5r - 3r\sin x/r\right)$$

$$(B-A)^2 + (C-A)^2 = 2x^2 + 18r^2 + 25r^2$$

$$V^1 = kx^2 + \text{cost}$$

$$V^2 = kr\left(r - y\sin\frac{x}{r} + r\cos\frac{x}{r}\right) + \frac{1}{2}ky^2$$

$$T^1 = \frac{1}{2} I_P \left(\frac{\dot x}{r}\right)^2 = \frac{1}{2}\left(\frac{1}{2} m r^2 + m r^2 + \frac{1}{12} m (6r)^2 + m r^2\right)\left(\frac{\dot x}{r}\right)^2$$

$$= \frac{11}{4} m \dot x^2$$

$$\dot G = \left(\dot x + \dot y \cos\frac{x}{r} - \frac{\dot x}{r} y \sin\frac{x}{r} + \dot x \cos\frac{x}{r},\ -\dot y \sin\frac{x}{r} - \frac{\dot x}{r} y \cos\frac{x}{r} - \dot x \sin x/r\right)$$

posto $\theta = x/r$

$$\dot G^2 = \left(\dot x + \dot y \cos\theta - \frac{\dot x}{r} y \sin\theta + \dot x \cos\theta\right)^2 + \left(\dot y \sin\theta + \frac{\dot x}{r} y \cos\theta + \dot x \sin\theta\right)^2$$

$$= 2\dot x^2 + \dot y^2 + \left(\frac{\dot x y}{r}\right)^2 + 2\dot x\left(\dot y \cos\theta - \frac{\dot x}{r} y \sin\theta + \dot x \cos\theta\right) + 2\dot x \dot y$$

$$T^2 = \frac{1}{2} m \dot G^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\left(\frac{\dot y}{r} + \frac{\dot x}{r}\right)^2$$

$$V = k x^2 + \frac{1}{2} k y^2 + k r\left(-y \sin\frac{x}{r} + r \cos x/r\right) + \text{cost.}$$

$$\frac{\partial V}{\partial x} = k\left(2x - y \cos\frac{x}{r} - r \sin\frac{x}{r}\right)$$

$$\frac{\partial V}{\partial y} = k\left(y - r \sin\frac{x}{r}\right)$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0 \Rightarrow 2x - \underbrace{r \sin\frac{x}{r}}_{\leq x}\underbrace{\left(1 + \cos\frac{x}{r}\right)}_{\leq 2} = 0$$

unica sol $x = 0,\ y = 0$

$$\frac{\partial^2 V}{\partial x^2} = k(2-1) = k$$

$$\frac{\partial^2 V}{\partial x \partial y} = -k \qquad \Rightarrow B = k\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\frac{\partial^2 V}{\partial y^2} = k$$

$\det B = 0$, EQ - INSIFFRONCENTO