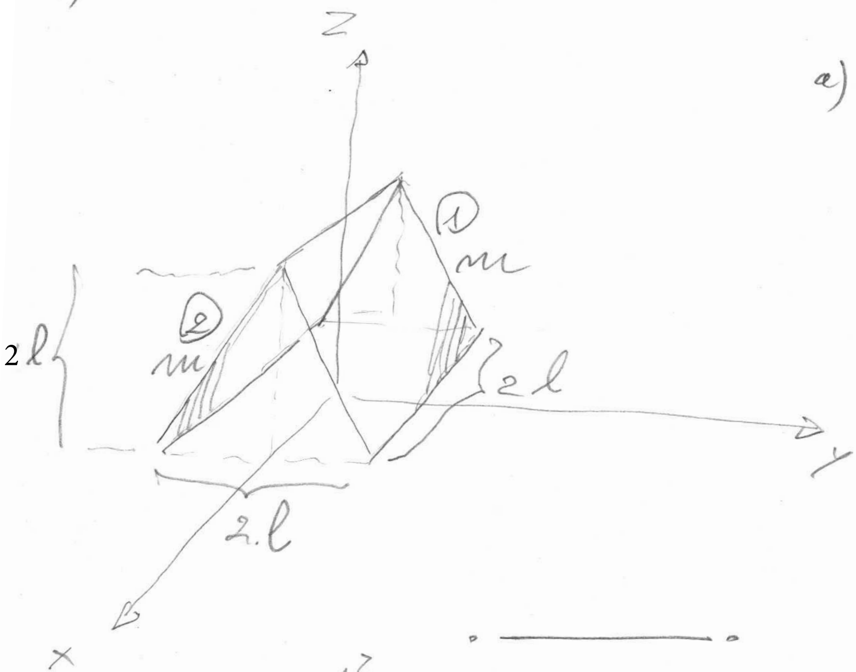
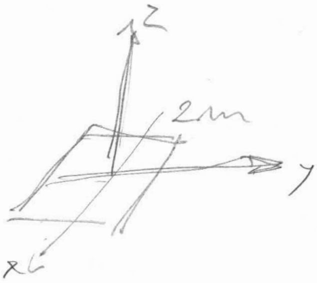
1)

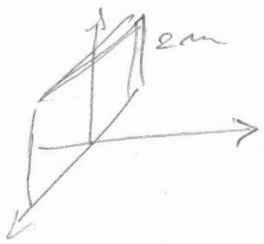
a) DETERMINARE I



$I_z =$ 

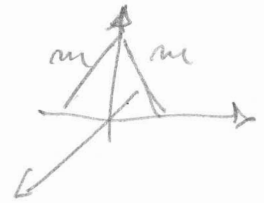
$= \frac{1}{12} 2m\left((2l)^2 + (2l)^2\right) = \frac{4}{3} m l^2$

$I_{xy} = 0$

$I_y =$ 

$= \frac{1}{12} 2m(2l)^2 + \frac{1}{3} 2m(2l)^2 = \frac{10}{3} m l^2$

$I_{xz} = 0$

$I_x =$ 

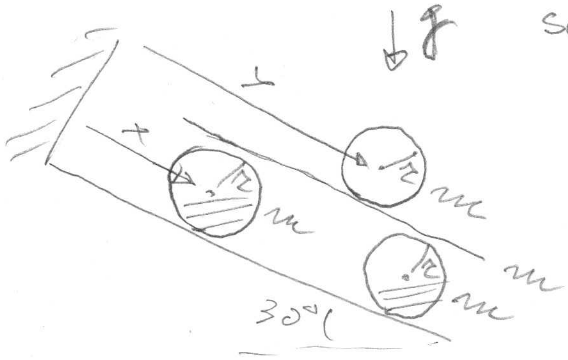
$= \frac{1}{3} 2m(2l)^2 + \frac{1}{12} 2m(2l)^2 = \frac{10}{3} m l^2$

$I_{zy} = 0$

$$I = \frac{2}{3} m l^2 \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

2)

ROT. PURO SU TUTTE LE SUPERFICI



a) DETERMINARE $\ddot{y}$

$$T = \underbrace{\frac{1}{2}(2m)\,\dot{x}^2 + \frac{1}{2}\left(\frac{1}{2}(2m)r^2\right)\left(\frac{\dot{x}}{r}\right)^2}_{\text{CILINDRI PIENI}}$$

$$+ \underbrace{\frac{1}{2}m(2\dot{x})^2}_{\text{ASTA}}$$

$$+ \underbrace{\frac{1}{2}m\dot{y}^2 + \frac{1}{2}(mr^2)\left(\frac{\dot{y}-2\dot{x}}{r}\right)^2}_{\text{CILINDRO CAVO}}$$

$$= \frac{1}{2}m\left(7\dot{x}^2 + \dot{y}^2 + (\dot{y}-2\dot{x})^2\right)$$

$$= \frac{1}{2}m\left(11\dot{x}^2 + 2\dot{y}^2 - 4\dot{x}\dot{y}\right)$$

$$V = \underbrace{-mg\frac{y}{2}}_{\text{CILINDRO CAVO}} \underbrace{-2mg\frac{x}{2}}_{\text{CILINDRI PIENI}} \underbrace{-mg\frac{2x}{2}}_{\text{ASTA}}$$

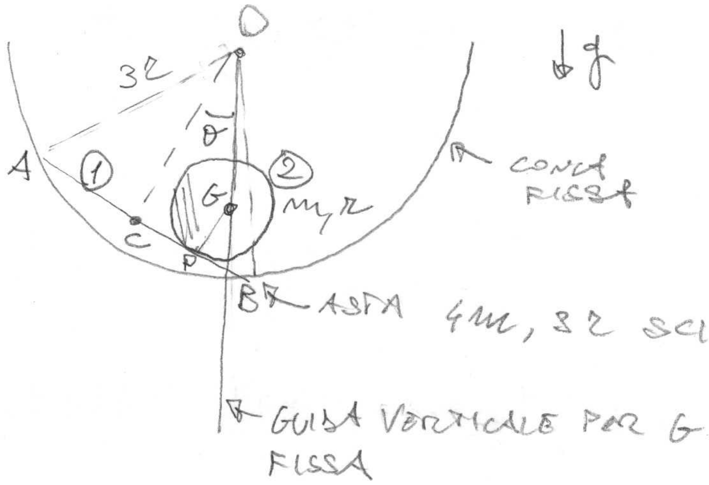
$$= -mg\left(2x + y/2\right)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \cancel{\frac{\partial T}{\partial q_i}} + \frac{\partial V}{\partial q_i} = 0$$

$$\begin{cases} \dfrac{d}{dt}\left(11m\dot{x} - 2m\dot{y}\right) - mg2 = 0 & \qquad 11\ddot{x} - 2\ddot{y} = 2g \\ \dfrac{d}{dt}\left(2m\dot{y} - 2m\dot{x}\right) - \dfrac{mg}{2} = 0 & \qquad 2\ddot{y} - 2\ddot{x} = g/2 \end{cases}$$

$$\Rightarrow 11\ddot{y} - \frac{11}{4}g - 2\ddot{y} = 2g \Rightarrow \ddot{y} = \frac{19}{36}g$$

3)



a) T

b) V

c) PICCOLE OSCILLAZIONI

θ = COORD. GENERALIZZATA

$$\overline{OC} = \frac{3r\sqrt{3}}{2}$$

$$V^{(1)} = -4mg\,\frac{3r}{2}\sqrt{3}\cos\theta$$

$$\frac{OC}{OB} = \frac{GP}{GB} \Rightarrow GB = \frac{OB}{OC}\,GP = \frac{r}{\cos\theta}$$

$$OG = OB - GB = \frac{OC}{\cos\theta} - \frac{r}{\cos\theta} = r\left(\frac{3}{2}\sqrt{3}-1\right)\frac{1}{\cos\theta}$$

$$V^{(2)} = -mgr\left(\frac{3}{2}\sqrt{3}-1\right)\frac{1}{\cos\theta}$$

$\frac{\partial V}{\partial\theta} \propto \frac{d\cos\theta}{d\theta} = \sin\theta$ PUNTO STAZIONARIO PER $\theta=0$

PER TROVARE b IN $V(\theta) = V(0) + \frac{1}{2}b\theta^2$

BASTA USARE $\cos\theta = 1-\frac{\theta^2}{2}+\dots$ E $\frac{1}{1+x} = 1-x+\dots$

QUINDI

$$V = \text{cost} + \frac{\theta^2}{2}\left\{4mg\,\frac{3r}{2}\sqrt{3} - mgr\left(\frac{3}{2}\sqrt{3}-1\right)\right\}$$

QUINDI $b = mgr\left(\frac{9}{2}\sqrt{3}+1\right) > 0$ EQ. STABILE

$$T^{(1)} = \frac{1}{2} I_O \dot{\theta}^2 = \frac{1}{2}\left(\frac{1}{12} 4m (3r)^2 + 4m\left(\frac{3r}{2}\sqrt{3}\right)^2\right)\dot{\theta}^2$$

$$= \frac{1}{2} 30 m r^2 \dot{\theta}^2$$

$$\omega_{disco} = \dot{\theta} + \dot{CP}/r$$

$$CP = CB - r \operatorname{tg}\theta = (OC - r)\operatorname{tg}\theta = \left(\frac{3\sqrt{3}}{2} - 1\right) r \operatorname{tg}\theta$$

$$\dot{CP} = \left(\frac{3\sqrt{3}}{2} - 1\right) r \frac{1}{\cos^2\theta}\dot{\theta}$$

$$\omega_{disco} = \dot{\theta}\left\{1 + \frac{1}{\cos^2\theta}\left(\frac{3\sqrt{3}}{2} - 1\right)\right\}$$

$$v_G = \dot{OG} = r\left(\frac{3\sqrt{3}}{2} - 1\right)\left(-\frac{1}{\cos^2\theta}\right)(-\operatorname{sen}\theta)\dot{\theta}$$

$$T^{(2)} = \frac{1}{2} m r^2 \left(\frac{3\sqrt{3}}{2} - 1\right)^2 \left(\frac{\operatorname{sen}\theta}{\cos^2\theta}\dot{\theta}\right)^2$$

$$+ \frac{1}{2}\left(\frac{1}{2} m r^2\right)\dot{\theta}^2 \left\{1 + \frac{1}{\cos^2\theta}\left(\frac{3\sqrt{3}}{2} - 1\right)\right\}^2$$

NEL PUNTO STAZIONARIO $T = \frac{1}{2} a \dot{\theta}^2$

$$a = m r^2\left\{30 + \frac{1}{2}\left(\frac{3\sqrt{3}}{2}\right)^2\right\} = m r^2\left\{30 + \frac{27}{8}\right\}$$

QUINDI LA PULSAZIONE È

$$\omega = \sqrt{\frac{b}{a}} = \sqrt{\frac{g}{r}}\sqrt{\frac{\frac{1}{2}\sqrt{3} + 1}{30 + 27/8}}$$