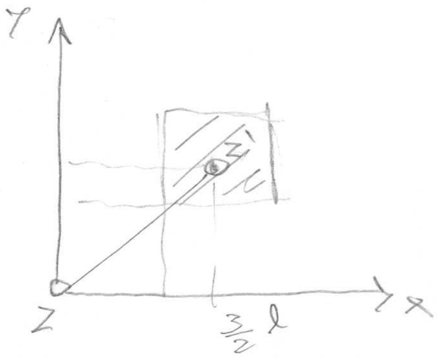
②



$$I_z = I_{z'} + m\left(\sqrt{2}\,\frac{3}{2}l\right)^2$$
$$= \frac{1}{12} m l^2 \cdot 2 + \frac{9}{2} m l^2$$
$$= \frac{14}{3} m l^2$$

$$I_x = I_y = \frac{1}{12} m l^2 + m\left(\left(\frac{3}{2}l\right)^2 + l^2\right) = \frac{10}{3} m l^2$$

$$I_{xy} = -m\left(\frac{3}{2}l\right)^2 = -\frac{9}{4} m l^2$$

$$I_{xz} = -\sum_i m_i x_i z_i = -l \sum_i m_i x_i =$$
$$= -l m x_G = -\frac{3}{2} m l^2 = I_{yz}$$

$$I^{(2)} = \frac{ml^2}{12}\begin{pmatrix}40 & -27 & -18\\ -27 & 40 & -18\\ -18 & -18 & 56\end{pmatrix}$$

①

$$I_x = \frac{1}{3} m l^2$$

$$I_z = \frac{1}{12} m l^2 + m\left(\frac{3}{2}l\right)^2 = \frac{7}{3} m l^2$$

$$I_y = I_x + I_z = \frac{8}{3} m l^2$$

$$I_{xy} = I_{zy} = 0$$

$$I_{xz} = -\int xz\,dm = -\int_l^{2l} x(2l-x)\,\frac{m}{l}\,dx$$
$$= -\frac{m}{l}\int_l^{2l}(2lx - x^2)\,dx = -\frac{m}{l}\left(lx^2 - \frac{x^3}{3}\right)\Big|_l^{2l}$$
$$= -\frac{m}{l}\left\{l^3\left(4-\frac{8}{3}\right) - l^3\left(1-\frac{1}{3}\right)\right\} =$$
$$= -ml^2\left\{\frac{4}{3} - \frac{2}{3}\right\} = -\frac{2}{3} m l^2$$

$$I^{(1)} = \frac{ml^2}{3}\begin{pmatrix}1 & 0 & -2\\ 0 & 8 & 0\\ -2 & 0 & 7\end{pmatrix} = \frac{ml^2}{12}\begin{pmatrix}4 & 0 & -8\\ 0 & 32 & 0\\ -8 & 0 & 28\end{pmatrix}$$

$$I^{(3)} = \frac{ml^2}{12}\begin{pmatrix} 32 & 0 & 0 \\ 0 & 4 & -8 \\ 0 & -8 & 28 \end{pmatrix}$$

$$I = \frac{1}{12} ml^2 \begin{pmatrix} 76 & -27 & -26 \\ -27 & 76 & -26 \\ -26 & -26 & 112 \end{pmatrix}$$

$x=y$ è di simmetria quindi $\vec{v} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ è un autovettore che individua la direzione dell'asse principale d'inerzia

$$I\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \frac{103}{12} ml^2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \frac{103}{12} ml^2$$ è un momento principale

se $\vec{\omega} = (1,2,0)\ s^{-1}$

$m = 10\,g = 10^{-2}\,kg$

$l = 20\,cm = 0{,}2\,m$

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \frac{1}{12} ml^2 \begin{pmatrix} \cdots \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} s^{-1} =$$

$$= \frac{ml^2}{12}\begin{pmatrix} 22 \\ 125 \\ -78 \end{pmatrix} s^{-1} = \frac{1}{3} 10^{-4} \begin{pmatrix} 22 \\ 125 \\ 78 \end{pmatrix} \frac{kg\,m^2}{s}$$

$$T = \frac{1}{2}(L_1\, L_2\, L_3)\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \frac{136}{3} 10^{-4}\, J$$